Exchange market pressure
in interest rate rules

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Abstract
Many central banks pursue some kind of an exchange rate objective. For a given objective we derive what variables the central bank should look at when setting the interest rate to implement that objective. Exchange market pressure (EMP), the tendency of the exchange rate to change, emerges as the key variable. The resulting interest rate rule can implement many regimes, from floating to intermediate to fixed rates, and does so exactly, even after a structural change. It can be applied to many models, and we illustrate it in a New Keynesian model for a small open economy.

Key words: DSGE, exchange market pressure, exchange rate regime, fixed exchange rate, monetary policy, open economy Taylor rule.

JEL classification: E43; E52; F31; F33.

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1 Introduction

Most central banks engage in some form of exchange rate management, particularly in small open and emerging economies; IMF (2017). What are the variables the central bank should look at when setting its policy instrument to achieve the exchange rate objective? We focus on this question, and to address it we derive an interest rate rule from the exchange rate objective. So we do not add preselected variables to an existing rule — as that cannot answer our research question — but we let the derivation reveal what variables to add, and how. We concentrate on the interest rate instrument, as that provides the simplest framework for introducing our idea, but the approach is also applicable to official forex intervention, with the same main insights.

We take the exchange rate objective as given. So we do not derive the degree of exchange rate management central banks should pursue. Instead, we start from what they actually pursue. In this sense the paper is not normative but positive.

This also allows us to keep the analysis general. In fact, we are able to derive our results with only few assumptions, so that the insights are not model specific but hold in many settings. The objective can be a fixed or some intermediate exchange rate regime (or the float), regarding the level or change of the exchange rate.

The derivation reveals that exchange market pressure (EMP) is a key variable for the central bank. EMP is the tendency of the exchange rate to change, where positive (negative) EMP means depreciation (appreciation) pressure.

The derivation also leads to a new interest rate rule that implements the exchange rate objective exactly, for many regimes and models. The rule has two main novelties. First, it extends a domestically-oriented rule, such as the Taylor rule, by adding EMP in deviation from the exchange rate change that is acceptable according to the objective. Excess pressure implies a high interest rate. This paper thus connects two strands of the literature, that on interest rate rules and EMP, and stresses that EMP, which is

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1Engel (2014) concludes that welfare-based optimal monetary policy analysis in open-economy models is still in the early stages, but that the analysis to date suggests a role for exchange rates in an optimal rule. Their importance depends on the economic structure. Schmitt-Grohé and Uribe (2016) show that in case of downward nominal wage rigidity optimal exchange rate policy calls for large devaluations during crises to ensure full employment. Davis et al. (2018) show that as central bank credibility falls and thereby the ability to commit to future policy, a highly open economy will quickly find it optimal to set the interest rate to peg the nominal exchange rate as the single mandate. Buffie et al. (2018) analyze less developed countries that pursue inflation targeting. In a float, currency substitution causes a high risk of indeterminacy (multiple equilibria), as well as escalation of inflation shocks. Both problems disappear by tight management of the nominal exchange rate.

2The formal definition, taken from the EMP literature, follows in Section 2.3.1. The EMP literature was initiated by Girton and Roper (1977) and has been further developed by Weymark (1995) and Klaassen and Jager (2011), among others. Interesting applications include the Financial Stress Index of IMF (2009), Frankel and Xie (2010), and Aizenman et al. (2017).
often used in applied work, also matters for theory and policy.

The second novelty of our rule is that the coefficient for EMP depends on the interest rate effectiveness to ward off depreciation, as determined by the model: for given EMP, the policy maker should mitigate the use of the interest rate if it is more effective. If not, the regime breaks down. This is a natural property but still a novel feature of our rule. It also helps us to reveal two structural parameters that underlie the coefficient for EMP, namely the effectiveness and the degree of exchange rate management.

What do central banks actually do? Some are explicit on this and consider what they call “pressure.” For example, Danmarks Nationalbank (2017) writes that “in situations with upward or downward pressure on the krone, Danmarks Nationalbank unilaterally changes its interest rates in order to stabilise the krone.” Likewise, the Hong Kong Monetary Authority (2009) describes its “automatic interest rate adjustment ... against downward pressure on the exchange rate.” The idea is that high selling pressure requires a high interest rate. Calvo and Reinhart (2002) provide further evidence.

Still, there is no formalization of this actual policy. We show that EMP is close to what central bankers mean by pressure. Hence we formalize their policy, and actual policy confirms the realism of our approach. In fact, we derive that it is natural to have EMP in a rule. In this sense, central banks with an exchange rate objective should consider EMP. So our derivation provides theoretical support to actual policy.

The traditional approach to model the interest rate for a central bank with an exchange rate objective is to add the exchange rate gap (actual minus target value) to a Taylor rule, as in Monacelli (2004), or to the foreign interest rate, as in Benigno et al. (2007). This has contributed to valuable insights, in other papers as well. However, we cannot use such an approach of adding a preselected variable to answer our research needs.

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3Hong Kong has a currency board system based on an automatic interest rate adjustment mechanism. In case of downward pressure, the central bank purchases Hong Kong dollars from banks so as to increase market interest rates and thereby capital inflows and achieve exchange rate stability. Because of its focus on the interest rate, we use Hong Kong as an example in this paper. An alternative would be to model policy as unsterilized intervention, but then the main idea relevant for us would be similar: the central bank responds to pressure and exploits the interest rate.

4Frankel (2014) suggests having EMP in a forex intervention rule but notes that this has not been formalized yet. Mohanty (2013) reports that in a BIS survey among central banks almost 80% said that curbing speculative pressures on the exchange rate was the most important priority. This emphasizes the usefulness of Frankel’s suggestion. Our work is such a formalization, albeit for the interest rate instead of the intervention instrument Frankel focuses on.

5An alternative to a rule is to simply pin down the exchange rate at the target level, as in De Paoli (2009). That can suffice for some analyses of the fixed rate. But the focus in our paper is on approaches that deliver a regime endogenously and that work for many more regimes, giving additional insights.

6The former rule has also been used by Engel and West (2005), Corsetti and Müller (2015), and Galí and Monacelli (2016), for example. The latter rule has been applied by Benigno (2004) and Born et al. (2013), among others. These (and our) papers focus on the nominal exchange rate. Some authors add the real rate to an interest rate rule, as in Clarida et al. (1998) and Ghosh et al. (2016).
question. After all, we want to know what variables to include, and how. That is why we derive our rule.

The derivation also makes our rule better founded. Still, one can compare the rules. There are two key differences, both favoring our rule. First, we have EMP instead of the actual exchange rate gap. For example, if the gap is zero, the existing rules imply that the central bank abstained from using the interest rate, but we allow for the possibility that the central bank sets a high rate to offset depreciation pressure so as to keep the exchange rate on target. As central bankers indicate that they look at pressure, using EMP increases realism. EMP also makes that our rule covers more exchange rate regimes, relies on weaker assumptions, and implements the objective exactly.

Second, our rule has separate parameters for the interest rate effectiveness and the exchange rate regime. These structural parameters are hidden in the traditional approach. As a consequence, from our rule it is always clear what the regime is, and in case of a structural change, our rule automatically accommodates so that the objective keeps on being implemented. This consistency is attractive for theoretical analysis.

The structure of the rest of the paper is as follows. In Section 2 we derive the interest rate rule and show the relevance of EMP. Section 3 discusses its characteristics. In Section 4 we set out the New Keynesian DSGE model for a small open economy to illustrate our method and derive the interest rate rule and EMP for that model. Section 5 illustrates their characteristics using a simulation study. Section 6 concludes.

2 Interest rate rule

We consider a two-country setting. The domestic monetary authorities, being the central bank throughout this paper, pursue some degree of exchange rate management as one of the policy goals (a perfectly free float is a valid special case). Foreign authorities do not try to control the exchange rate.

For a given exchange rate regime, the goal is to derive the variables the central bank should look at when setting the interest rate to implement that regime. To avoid that our answer is driven by specific modeling assumptions, we derive it in a general framework. Section 4 illustrates it for a New Keynesian small open-economy model.

The intuition of our derivation is as follows. The exchange rate is a function of the interest rate and other variables (the New Keynesian model yields a specific example). Solving for the interest rate and substituting the exchange rate by the objective (such as a target rate) gives the interest rate rule that implements the regime. The rest of the rule then reveals the variables that are relevant for the central bank. To properly extract the interest rate, we first have to appropriately specify the exchange rate function.
2.1 Exchange rate function

Let $s_t$ be the (logarithm of the nominal) exchange rate at time $t$, which is the domestic currency price of one unit of foreign currency. Many variables can matter for the determination of $s_t$, and this section groups them.

The interest rate is $i_t$ and can affect $s_t$ in three ways. First, there is a direct effect. For example, a high $i_t$ attracts capital and thus lowers $s_t$. Second, $i_t$ matters via expectations. A high $i_t$ could signal that many speculators attack the home currency in case of a fixed exchange rate, increasing the probability that the home currency will be devaluated, attracting other speculators, and that may cause an actual increase of $s_t$. The third channel is another indirect effect, but it goes through all endogenous variables that are not expectations. For example, a high $i_t$ weakens current consumption, reducing the current home price level, increasing foreign demand for home goods, and appreciating the home currency.

In the derivation of the rule below we need to be able to change $i_t$ and compute the full impact on $s_t$ under the condition that expectations remain constant. We thus take the first and third channels together, called the contemporaneous effect of $i_t$ on $s_t$. The impact of $i_t$ via expectations, the second channel, is split off, and the expectations together with all other exchange rate determinants form the vector $E_t$. This $(i_t, E_t)$-separation is not restrictive. It just splits the full impact of $i_t$ on $s_t$ into two groups.\(^7\)

The variables in $E_t$ depend on expectations, predetermined, and contemporaneous variables, but the separation implies that the latter no longer include the impact of $i_t$. So contemporaneous variables such as goods prices, interest rates concerning other maturities than the one underlying $i_t$, national income, and fiscal policy are first cleaned for $i_t$ by moving the $i_t$ dependencies to the $i_t$ argument, and then the remainder enters $E_t$. For example, consider good prices. The third channel mentioned above, that a high $i_t$ lowers goods prices and appreciates the currency, is captured by the $i_t$ argument. What remains in the $E_t$ argument is, for example, that lower expected future income weakens current consumption, causing lower prices and appreciation, and that exogenous technological progress via lower prices causes appreciation.

The particular model at hand determines what the contemporaneous channels are and what is in $E_t$. We assume that it is possible to solve $s_t$ from the model as a function $s$ of $i_t$ and $E_t$:

$$s_t = s(i_t, E_t), \quad (1)$$

\(^7\)The analogy in mathematics would be that the separation enables us to express the total derivative regarding $i_t$ as the sum of the partial derivative regarding $i_t$ that accounts for all contemporaneous channels, and the partial derivative regarding $E_t$ times the total derivative of $E_t$ regarding $i_t$. 

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where the \( i_t \) argument captures the interest rate impact via all contemporaneous channels, and the \( E_t \) argument picks up all other mechanisms. Section 4.2 derives the function for a specific model, in particular formula (26), where (28) specifies \( E_t \).

### 2.2 Derivation of the rule

We want to develop an interest rate rule that implements a given exchange rate objective. A natural idea to achieve this would be to take exchange rate function (1), solve it for the interest rate \( i_t \), and substitute the exchange rate \( s_t \) by the objective \( s_o \).

However, it is neither clear that solving for \( i_t \) is possible, nor what value to take for \( s_o \) in the float or another non-fixed exchange rate regime. Our derivation is related to the natural idea but offers a way around both problems.

In case of a floating exchange rate, the central bank sets the interest rate without considering the exchange rate. We denote the interest rate rule in this situation by \( i^d_t \), the domestically-oriented interest rate rule, such as the standard Taylor rule. The rule is complete and yields a direct link to existing rules not dealing with exchange rate policy, thus providing a natural starting point.

For any other regime, the rule for \( i_t \) differs from that for \( i^d_t \), and \( i_t - i^d_t \) has to capture the exchange rate objective. To quantify the exchange rate effect of \( i_t - i^d_t \), we take \( s(i_t, E_t) - s(i^d_t, E_t) \).

The latter term combines \( i^d_t \) with \( E_t \), where \( E_t \) is—as always in the paper—based on \( i_t \), so on the actual exchange rate regime. For an \( s \)-function that is linear in the second argument, taking \( E_t \) is without loss of generality, but in case of non-linearity one could have opted for another value instead of \( E_t \), such as the value under the float; but that would make the rule for \( i_t \) depend on variables of a non-existent regime and thus infeasible to use in practice.\(^8\)

Let

\[ s^d_t = s(i^d_t, E_t). \]

(2)

So \( s^d_t \) is not the counterfactual exchange rate that has interest rate \( i^d_t \) and the \( E_t \)-vector consistent with that rate, which would boil down to the exchange rate under a float. As such, \( s^d_t \) is not an equilibrium rate. We call it the intermedial exchange rate. The separation in Section 2.1 implies that the substitution of \( i_t \) by \( i^d_t \) in (2) takes out the

\(^8\)Taking \( s(i_t, E_t) - s(i^d_t, \tilde{E}_t) \), that is, combining \( i^d_t \) with some other value \( \tilde{E}_t \) instead of \( E_t \), would change (3) into \( s(i_t, E_t) - s(i^d_t, \tilde{E}_t) = w (i_t - \tilde{i}_t) + \partial s / \partial E (\tilde{k}_t)(E_t - \tilde{E}_t) \), where we use the mean value theorem and \( \tilde{k}_t \) is an intermediate vector on the line segment between \( (i_t, E_t) \) and \( (i^d_t, \tilde{E}_t) \). For an \( s \)-function that is linear in its second argument, \( s(i_t, E_t) - s(i^d_t, \tilde{E}_t) - \partial s / \partial E (\tilde{k}_t)(E_t - \tilde{E}_t) \) equals \( s(i_t, E_t) - s(i^d_t, E_t) \), so that we are back in (3), implying that taking \( \tilde{E}_t \) instead of \( E_t \) is equivalent. For a non-linearity, the resulting rule for \( i_t \) would depend on \( \tilde{E}_t \), so on values of a state of the economy that is not in place; that would make the rule hard to use in practice.
central bank defense of the currency via all contemporaneous channels. We thus view \( s^d_t \) as a summary of forex market conditions.

The difference \( s(i_t, E_t) - s(i^d_t, E_t) = s_t - s^d_t \) is due to the deviation of \( i_t \) from \( i^d_t \). So, using the simplifying assumption that the \( s \)-function is linear in its first argument,

\[
s_t = s^d_t - w \left( i_t - i^d_t \right), \tag{3}
\]

where

\[
w = -\frac{\partial s}{\partial i} (., E_t), \tag{4}
\]

where we leave out the time subscript for notational simplicity.

The scalar \( w \) is the effectiveness of the interest rate to counteract depreciation, so it transforms the interest rate deviation \( i_t - i^d_t \) into avoided depreciation units \( s^d_t - s_t \). We impose \( w \neq 0 \), as using the interest rate for exchange rate purposes would otherwise be useless from the outset.\(^9\)

The advantage of (3) is that we can easily solve for \( i_t \). We now propose the rule

\[
i_t = i^d_t + \frac{1}{w} \left( s^d_t - s^o_t \right). \tag{5}
\]

The intuition is that a high \( s^d_t \) reflects that investors intend to sell the currency, and to the extent that it exceeds the exchange rate objective \( s^o_t \), the central bank has to set a high interest rate. In case of the float, \( s^o_t \) is set at \( s^d_t \), so the rule gives \( i_t = i^d_t \), as should be. Other regimes are handled by other choices for \( s^o_t \), as described in Section 2.4. For example, objective (8) implies rule (9), which in our illustration becomes (32).

Of course, substituting this rule into (3) gives \( s_t = s^o_t \). So for a given exchange rate objective our rule implements that objective exactly, by construction.

### 2.3 EMP as the key determinant

Intuitively, exchange market pressure (EMP) represents the reluctance of investors to hold the domestic currency at the forex market. This reluctance tends to affect the exchange rate, and that may trigger the central bank to act. This resembles the idea of our interest rate rule. Indeed, we will show that the rule implies a prominent role for EMP in exchange rate policy.

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\(^9\)One usually considers \( w \) to be positive, that is, an interest rate increase appreciates the currency, particularly because counteracting impacts of \( i_t \) via the expectations in \( E_t \) only enter via the other argument of the \( s \)-function. Our model in Section 4 confirms this sign. In intuitive explanations below we will do as if \( w \) is positive, but we do not impose it in the derivation.
2.3.1 EMP definition

The idea of the EMP concept is to split the actual (relative) depreciation of the home currency, resulting from the interplay of investors and authorities, into a part reflecting the reluctance of investors to hold the currency, called EMP, and the policy-based part, which usually intends to counteract EMP. EMP applies to any exchange rate regime and can be positive as well as negative, where the latter means there is pressure on the currency to appreciate. One example is a fixed exchange rate that is under attack by speculators and where the attack is successfully offset by policy. Then EMP is positive, the policy-based counteracting depreciation is negative and offsets EMP exactly.

More formally, \( EMP_t \) is defined as the relative depreciation of the home currency at time \( t \) in the absence of exchange rate policy, while keeping expectations at the levels determined by actual policy. This is the standard definition in the EMP literature, due to Weymark (1995). It does not depend on a model, so \( EMP_t \) is uniquely defined.\(^{10}\)

We here apply it to the setting where the interest rate is the policy instrument.

One key element in the definition is the absence of exchange rate policy. The interest rate rule in this situation is \( i^d_t \), the domestically-oriented rule introduced in Section 2.2. Without the second key element in the EMP definition, the condition on expectations, the use of \( i^d_t \) would make EMP like the depreciation under a floating exchange rate regime. But that is not what EMP intends to capture; EMP is about the reluctance of investors to hold the currency in the actual regime. So EMP uses the same values for the expectations as \( E_t \) defined earlier. Exchange market pressure is thus defined as

\[
EMP_t = s^d_t - s^d_{t-1}.
\]

So \( EMP_t \) is a function of fundamentals excluding the actual interest rate \( i_t \).

2.3.2 EMP in the rule

Rewriting rule (5) using (6) shows that the rule depends on \( EMP_t - (s^d_t - s^d_{t-1}) \). Given the derivation of the rule, we conclude that EMP emerges in a natural way as the key

\(^{10}\)A model can be used to derive how \( EMP_t \) is a function of fundamentals, so the model provides the microfoundations of \( EMP_t \). This function varies across models, and Weymark (1995) and (6) with (31) in our paper provide examples.

In the EMP literature, the term “EMP” not only refers to EMP itself, but also to a measure of EMP. EMP measures vary across papers and include the current value of the policy instrument. In contrast, in our paper “EMP” always means EMP itself, not the measure, and we use EMP to explain the current value of the policy instrument. Note that EMP measures often include the change in foreign reserves divided by money supply. Klaassen and Jager (2011) explain why that is in line with the EMP definition (for a central bank using forex intervention as instrument) and how our EMP formula below (with the interest rate as instrument) is also consistent with the EMP definition.
determinant of the interest rate if a central bank wants to implement an exchange rate objective. In this sense, our derivation reveals the insight that central banks should look at EMP. So our rule guides policy.

The rule says that the central bank has to set $i_t > i^d_t$ to ward off $EMP_t$ insofar pressure exceeds the target depreciation $s_t^d - s_{t-1}$. The magnitude of $i_t - i^d_t$ is the amount of excess pressure converted into interest rate units by dividing by the effectiveness $w$ of the interest rate instrument.

It is contemporaneous pressure that matters, not expected future pressure. This marks a difference with the inclusion of, say, expected inflation in some Taylor rules. The latter are typically used to model central bank policy to control inflation between today and a year ahead, say. Such a focus on the future is not what matters in exchange rate management. The obvious example concerns the fixed rate: if today’s interest rate does not offset the pressure to move away from the target today, there will be an immediate breakdown of the peg, irrespective of expected future developments. Hence, today’s $EMP_t$ is what matters for $i_t$.

2.3.3 Justifying and being supported by actual policy

As argued in the introduction, central bankers consider what they call “pressure” on their currency at the forex market when implementing exchange rate management. A high $s^d_t$ reflects that investors intend to sell the currency. That mimics what central bankers mean by pressure. The EMP variable relates $s^d_t$ to the lagged rate $s_{t-1}$ and the EMP literature calls this pressure. That is in line with the phrase “downward pressure” used by Danmarks Nationalbank (2017) and Hong Kong Monetary Authority (2009), and with He et al. (2011) from the HKMA, who write that they monitor “foreign exchange market pressure.” This advocates a formalization of the word “pressure” by

$$\text{pressure} = EMP_t.$$  \hfill (7)  

Because pressure matters in actual policy, our rule not only guides policy, but it also justifies and is supported by actual policy.

2.4 The rule for specific exchange rate regimes

Rule (5) can be combined with many exchange rate objectives $s^d_t$. The current section applies it to six regimes. Five are inspired by practice, namely the float, fixed rate, crawling peg without band, peg with possibly time-varying band, and a policy that moderates the rate of change (called “leaning against the wind”). IMF (2017) shows
that these regimes cover the majority of the countries. Examples include the United States, Bulgaria, Nicaragua, China, and Brazil, respectively, albeit that we examine only one type of policy to implement the regime, that is, interest rate policy.

The other regime is a weighted combination of the fixed and floating exchange rate regimes, which we introduce because it will be convenient in theoretical analyses and we will focus on it in the rest of the paper to simplify the exposition.

In the float the central bank does not try to affect the exchange rate, so any tendency for the rate to move to a particular value is ignored by its interest rate policy. More formally, \( s^t = s^d_t \). Rule (5) then sets \( i_t = i^d_t \), as expected.

For the fixed exchange rate the question at hand is what \( i_t \) the central bank should choose to ensure the exchange rate equals the target \( s^t \). Substituting \( s^t = s^d_t \) into (5) gives the interest rate that hits this target.

The weighted fixed-floating exchange rate is a weighted average of the fixed and floating exchange rate regimes, where the weight \( \mu \in [0, 1] \) denotes the degree of exchange rate management. The regime and the interest rule implementing it are

\[
\text{Policy objective: } s^o_t = (1 - \mu) s^d_t + \mu s^t \\
\text{Interest rate rule: } i_t = i^d_t + \frac{1}{w} \mu \left( s^d_t - s^t \right).
\]

For \( \mu = 0 \) this confirms the rule for the float. The higher \( \mu \), the more \( i_t \) responds to a given \( s^d_t - s^t \), meaning tighter exchange rate management. For \( \mu = 1 \) the system represents the fixed rate. Note that \( \frac{1}{w} \mu \) discloses the two structural parts that underlie the coefficient for \( s^d_t \), namely the model-determined interest rate effectiveness \( w \) and the degree of exchange rate management \( \mu \) chosen by the policy maker. So (9) disentangles a Taylor-rule type of coefficient into two structural parameters.

The crawling peg generalizes the fixed rate by having a time-varying target: \( s^o_t = s^d_t \).

In the peg with band the exchange rate must lie in a band \([s_t, \bar{s}_t]\). One example, inspired by Krugman (1991), is where \( s^o_t = s^d_t \) if \( s^d_t \in [s_t, \bar{s}_t] \), but once the exchange rate tends to leave the band, the central bank uses the interest rate to make sure that the exchange rate settles at the nearest boundary, so \( s^o_t = s_t \) if \( s^d_t < s_t \), and \( s^o_t = \bar{s}_t \) if \( s^d_t > \bar{s}_t \). A special case is the one-sided band, as applied in Switzerland until 2015 and in the Czech Republic until 2017, where \( s_t \) restricts appreciation but \( \bar{s}_t \) is infinite.

In the “leaning against the wind” regime the central bank aims at mitigating the change in the exchange rate. So it counteracts the wind \( s^d_t - s_{t-1} \). This regime follows from the weighted fixed-floating regime by using \( s_{t-1} \) instead of \( s^t \). Despite the practical relevance of leaning against the wind, the rest of this paper focuses on the exchange rate level. Substituting \( s^t \) below by \( s_{t-1} \) gives the features of our rule for the change,
and it shares the same features.

3 Characteristics of the rule and relation to the literature

Because the interest rate rules in the existing literature cannot be used to address our research question, we have derived a new rule. Still, it is instructive to compare the outcome to the existing rules and see how EMP makes them different.

3.1 The rules to be compared

As explained in the introduction, the traditional approaches rely on the gap between the actual exchange rate and its target. The first approach, due to Monacelli (2004), adds this exchange rate gap to a standard Taylor rule, formalized by

\[ i_t = i^d_t + \varphi_s (s_t - s^d_t) , \]

where \( \varphi_s \geq 0 \).

Second, the rule by Benigno et al. (2007) adds the gap to the foreign interest rate,

\[ i_t = i^*_t + \varphi_s^{BBG} (s_t - s^d_t) , \]

where \( \varphi_s^{BBG} > 0 \), and adds two assumptions. That is, the rule assumes uncovered interest parity (UIP), and that the home central bank credibly forces investors to convert the foreign into the home currency if the latter depreciates beyond some value; a similar commitment is imposed on the foreign central bank if the home currency appreciates.

Our rule, in the special case of the weighted fixed-floating regime, is (9). So we have \( \frac{1}{w} \mu \), a new variable \( s^d_t \), and we impose neither UIP, nor the convertibility assumptions.

3.2 Implementing the exchange rate objective

The central bank wants to implement its objective, the weighted fixed-floating regime indicated by \( \mu \). We now examine whether the three rules fulfill this basic requirement.

Substituting the Monacelli (2004) rule (10) into (3) shows that the resulting \( s_t \) is a weighted average of \( s^d_t \) and \( s^d_t \), with weight \( \frac{w \varphi_s}{1 + w \varphi_s} \) on the latter. The objective \( s^o_t \), however, has weight \( \mu \). We see that the objective is missed (unless \( \varphi_s \) happens to be \( \frac{1}{w} \mu \frac{1}{1-\mu} \)), which is not surprising as Monacelli does not focus on implementing a specific regime. The fixed exchange rate is not covered but is the limiting case where \( \varphi_s \to \infty \).

Benigno et al. (2007) focus on the fixed rate, and they aim at implementing it without an infinite response parameter. Their rule (11) says that the central bank commits
to raising $i_t$ above $i^*_t$ if $s_t$ tends to exceed $s^t$. The authors show that without the convertibility assumptions the exchange rate would explode to plus or minus infinity with positive probability, and they prove that in equilibrium the threat of the convertibility restrictions implies $s_t = s^t$ and that the central bank will never actually need to raise $i_t$, that is, $i_t = i^*_t$. This holds for any $\varphi_{BBG} > 0$, reflecting the dominance of the convertibility assumptions.

Our rule (9) implements the regime exactly, for all $\mu$, as shown in Section 2.2. So it is the only rule that fulfills the basic requirement. This is achieved by offsetting exactly the right amount of pressure. Even if we just consider the fixed rate, our rule outperforms. After all, it can implement that regime with a finite parameter, improving on Monacelli (2004), and our rule needs neither the convertibility assumptions, nor UIP, and is thus more realistic than Benigno et al. (2007).

### 3.3 Pressure instead of the actual gap

Taylor rules typically include the gap the central bank wants to close. The traditional rules follow this method by having the actual gap $s_t - s^t$, but we do not. Our rule is the result of a general derivation from which $s^t$, and thus $EMP_t$, emerges as the key determinant. Moreover, the relevance of pressure is confirmed by actual central bank policy. Both support our rule.

The merits of using $s^t$ instead of $s_t$ can be illustrated by considering the fixed exchange rate regime. If investors’ supply weakens the currency in the sense that there is a tendency for the currency to depreciate to $s^t > s_t$, then our rule says that the central bank has to set $i_t > i^t$ to ward off the pressure. For a successful defense, the outcome is $s_t = s^t$. So our rule can explain $i_t > i^t$ if the actual gap $s_t - s^t = 0$.

In contrast, by relying on the actual gap, traditional approaches imply that there is no need for using the interest rate. This is another indication that it is better to use pressure than the actual gap, as in reality exchange rates close to target can come together with substantial use of the interest rate.\footnote{For example, consider the Annual Reports of the central banks of Denmark and Hong Kong. In February 1993 (ERM turbulence) Danmarks Nationalbank increased interest rates and succeeded in keeping the krone exchange rate stable. In 2000 (Danish referendum on euro participation) and 2008 (global financial crisis) it also defended the krone by interest rate hikes. In 2015 it set a negative interest rate to fend off appreciation pressure. Hong Kong experienced four speculative attacks in 1997-8 (East Asian financial turmoil). The central bank acted to increase market interest rates; see footnote 3 for the mechanism. It also increased the savings deposit rate. The exchange rate remained stable. In 2007 (US sub-prime mortgage problems), 2008 (Lehman collapse), and 2009 a lower interest rate counteracted appreciation pressures.}
3.4 Interest rate effectiveness and structure of the economy

The traditional rules use $\varphi_s$ or $\varphi_s^{BBG}$ to capture the impact of the gap $s_t - s^t$ on $i_t$. This is a fixed parameter, like a standard Taylor-rule parameter. It is set independently of the structure of the economy under consideration, including the exchange rate function.

To illustrate the consequences, consider a change in financial openness that makes the interest rate more effective for exchange rate purposes (higher $w$), so that one would expect a less aggressive interest rate response for a given $s_t - s^t$.

Traditional rules, however, imply the same $i_t$ as before the structural change, due to the unchanged $\varphi_s$ and $\varphi_s^{BBG}$. The consequence when using the Monacelli (2004) rule is that after the structural change the actual regime is one of tighter exchange rate management, so that the central bank misses its objective. For the Benigno et al. (2007) rule, the impact of keeping $\varphi_s^{BBG}$ constant is none, because the convertibility assumptions determine the outcome, irrespective of the value of the response parameter.

In our rule, the impact of a given pressure on $i_t$ depends on $w$, as the larger $w$ weakens the required interest rate reaction, as expected. Recall that $w$ is the effectiveness of the interest rate instrument in achieving the exchange rate objective. It seems straightforward that this should matter for a rule, but only our rule includes it. The effectiveness is determined by the structure of the economy, in particular the exchange rate function. By deriving our rule in close relation with that function, we automatically and correctly account for $w$ and changes thereof.

3.5 Observability

Having $s^d_t$ in our interest rate rule implies that the computation of $i_t$ requires knowledge of the functional form of the $s$-function in (1) and its determinants. In a theoretical model that is no problem. Indeed, in Section 4.4 we will calculate $s^d_t$ for a DSGE model.

In practice, central bankers use indicators to monitor pressure and thus $s^d_t$. For example, the Hong Kong Monetary Authority uses forward exchange rates, prices of currency options, balance of payment statistics on capital flows, and market surveys; see He et al. (2011). Fundamentals suggested by theoretical models, such as the one discussed below, can extend this list and thus help policy.

Still, traditional rules have an observable variable, $s_t$, and that may seem an advantage over our rule, which has $s^d_t$. However, this advantage is illusory. To show this, substitute rule (9) for $i_t - i^d_t$ in (3), and use the resulting $s^d_t$ in (9). That gives for $\mu \neq 1$

$$i_t = i^d_t + \frac{1}{w} \frac{\mu}{1 - \mu} (s_t - s^t).$$  (12)
So in the special case of the weighted fixed-floating regime, our approach can be reformulated such that $s_t$ appears instead of $s^d_t$, implying that the presence of the observable $s_t$ is no true advantage of the traditional rule. Even more so, having $s_t$ entails the costs that $i_t$ is no longer identified for $\mu = 1$ and that the rule misses the essence of policy, that the central bank should look at pressure. Both costs are avoided by using $s^d_t$.

3.6 Disentangling the reduced-form coefficient $\varphi_s$

Taylor-rule parameters such as $\varphi_s$ in the Monacelli (2004) rule (10) are typically viewed as policy-choice parameters. However, special case (12) shows that

$$\varphi_s = \frac{1}{w} \frac{\mu}{1 - \mu},$$

(13)

so that not only the policy choice $\mu$, but also the effectiveness $w$ matters (for $\mu > 0$).

So $\varphi_s$ is a reduced-form coefficient for which our approach gives the two underlying structural parameters. This explains why for the traditional rule the regime is not identified, and why a change in the economy that alters $w$ modifies the unknown regime in an unknown way.

Our rule resolves these identification issues. Interestingly, as $w$ is determined by existing parameters only, disentangling $\varphi_s$ does not increase the number of model parameters. We have simply used information in the exchange rate function, which is not exploited in the traditional rule.

3.7 Reality check: estimating the de facto regime

Because representation (12) implies

$$\frac{1}{w} \frac{\mu}{1 - \mu} = \frac{\text{stdev} \{i_t - i^d_t\}}{\text{stdev} \{s_t\}},$$

(14)

we can use data on $i_t - i^d_t$ and $s_t$ to estimate the left hand side by the ratio of sample standard deviations and then, for a given $w$, estimate the de facto degree of exchange rate management $\mu$. This offers a simple check of the realism of our approach.

To operationalize this, we assume that $i^d_t$ is a linear function of domestic producer price inflation $\pi_{H_t}$ with coefficient 1.5, following Monacelli (2004). One way to obtain a value of $w$ is by specifying a model and compute it from the model parameters. We will do that in the subsequent sections. For now, we set $w = 1.62$, the value of the baseline economy in the simulation section 5.

We examine five countries, namely Australia, Canada, New Zealand, Denmark, and
Hong Kong, the countries taken from the simulation section and the Introduction. The first three have an official inflation targeting policy, while the latter two pursue an exchange rate target. We use 15 years of quarterly data, from 2000 through 2014. This sample is for illustration only, and we leave a broad empirical study for future research.\footnote{The variables for quarter $t$ are measured as follows. For $i_t$, we take the three-month interbank interest rate, calculated as the period average of the daily rates in the quarter. Given period-average PPI values, we use year-on-year inflation for $\pi_{Ht}$ and thus $i_t^d$. Then we express $i_t$ and $i_t^d$ at a quarterly basis; all interest rates in the paper are at this basis, so not annualized. The rate $s_t$ is the log of the average daily price of one dollar (euro for Denmark). All data have been obtained from Datastream.}

![Diagram showing the degree of exchange rate management](image)

Figure 1: Estimating the degree of exchange rate management.

The estimates of $\text{stdev}\{i_t - i_t^d\}/\text{stdev}\{s_t\}$ are 0.02, 0.02, 0.02, 4.03, and 5.52 for the respective countries. Figure 1 illustrates the implied $\mu$. For Australia, Canada, New Zealand the estimated $\mu$ is 0.03. For Denmark we obtain 0.87, and for Hong Kong 0.90, meaning that their regimes can be characterized as an about 90% fixed and 10% floating exchange rate regime.\footnote{The value of $w$ that underlies the $\mu$ estimates is based on the core parameter values in Table 1. These are estimates. To quantify the reliability of $w$, we use the information on the posterior distributions of the core parameters, as reported by Justiniano and Preston (2010), to estimate the posterior distribution of $w$. The resulting 95% credible interval for $w$ is [1.39, 2.19]. The implied intervals for $\mu$ are [0.03, 0.04] for Australia, Canada, and New Zealand, [0.85, 0.90] for Denmark, and [0.88, 0.92] for Hong Kong. These are narrow, so that we simply focus on the point estimates.} All five are in line with the IMF (2017) de facto classification.
We conclude from this simple analysis that our pressure-based interest rate rule can deliver useful insights into structural parameters such as $\mu$, which are not identified when using traditional rules.

4 Illustration in a log-linearized DSGE model

In Section 2 we have derived the key role for EMP and this has resulted in a new interest rate rule. The approach is generally applicable and the resulting insights are valid in many models. The current section presents one specific model for illustration. Hence, we keep the model simple to focus on our contributions.\textsuperscript{14} Various extensions are possible but left for future study.

We take a two-country rational expectations New Keynesian model where the home country is a small open economy, in the spirit of De Paoli (2009). Many other elements and derivations will be standard, as described by Gali (2008), although our new explicit formula for the exchange rate and its derivation contribute to the literature.

4.1 The model: non-policy block

Web Appendix A, available from our homepages, specifies the non-policy part of the model and derives the zero-inflation and zero-depreciation, symmetric and efficient steady state. We log-linearize the equations around that steady state and use the log-linearized version from now on. The relevant equations are (15)-(25), derived in Web Appendix B. Lowercase Latin letters denote the logarithm of variables, except for the interest rate, and an asterisk refers to the foreign country or currency. Table 1 defines all parameters, gives their ranges, and shows the values used in the simulation section 5.

\begin{align*}
\text{Labor supply} & : \gamma \ell_t + \sigma c_t = w_t - p_t \quad (15) \\
\text{Consumption Euler} & : \sigma c_t = \sigma \mathbb{E}_t \{ c_{t+1} \} - (i_t - \mathbb{E}_t \{ \pi_{t+1} \} - \delta) \quad (16) \\
\text{Real marginal cost} & : mc_{Ht} = \log (1 - 1/\theta) + w_t - a_t - p_{Ht} \quad (17) \\
\text{Calvo-based pricing} & : \pi_{Ht} = \beta \mathbb{E}_t \{ \pi_{H,t+1} \} + \kappa mc_{Ht} \log (1 - 1/\theta) \quad (18) \\
\text{International risk sharing} & : \sigma (c_t - c^*_t) = s_t + p^*_t - p_t \quad (19) \\
\text{Labor market equilibrium} & : \ell_t = y_t - a_t \quad (20) \\
\text{Law of one price} & : p_{Ft} = p^*_{Ft} + s_t \quad (21)
\end{align*}

\textsuperscript{14}Several model assumptions we will make, such as producer currency pricing, are relevant for the optimality of the exchange rate regime. However, recall that we take the regime as given, so from this point of view those model assumptions are not restrictive. In fact, our rule has been derived in the general setup of Section 2, so we can safely impose the assumptions to simplify the exposition.
Goods market equilibrium: \[ y_t = \nu c_t + (1 - \nu) c^*_t + (1 - \nu^2) \eta (p_{Ft} - p_{Ht}) \] (22)
Goods market eq. abroad: \[ y^*_t = c^*_t \] (23)
CPI: \[ p_t = \nu p_{Ht} + (1 - \nu) p_{Ft} \] (24)
CPI abroad: \[ p^*_t = p^*_{Ft} \] (25)

The world is populated with a continuum of households, where the population in the home country \( H \) lies in the segment \([0, n]\), while that of the rest of the world \( F \) is in \([n, 1]\). Households live forever and have identical preferences, both within and across countries. They derive utility from the consumption of domestic and foreign goods, with home bias in preferences, and disutility from supplying labor to firms. They live in cashless economies. For simplicity, capital markets are complete, both domestically and internationally, with frictionless trade in assets.

Households maximize expected lifetime utility, where expectations \( E_t \) are conditional on the information available in period \( t \). Optimization yields labor supply equation (15) and consumption Euler equation (16), where \( \ell_t \) is labor supply in period \( t \), \( c_t \) is consumption, \( w_t \) is the wage rate, \( p_t \) is the consumer price index (CPI), \( i_t \) is the interest rate set by the central bank, and \( \pi_t = p_t - p_{t-1} \) is CPI inflation.

Firms specialize in the production of one firm-specific good. Domestic firms produce the varieties in \([0, n]\) and foreign firms those in \([n, 1]\). Each firm uses labor supplied by the households and a linear technology, where \( a_t \) is labor productivity, which is common across home firms and evolves exogenously according to some stationary stochastic process. The firm receives an employment subsidy that renders the steady state efficient. Real marginal cost \( mc_{Ht} \), expressed in terms of the producer price index (PPI) \( p_{Ht} \), thus becomes (17).

The firm sells its good under monopolistic competition. It sells at home and abroad without trade frictions. Prices are set in the producer’s currency, and they are sticky a la Calvo (1983). Hence, \( p_{Ht} \) depends on its lag and the price chosen by firms that are allowed to reset the price. Profit maximization then yields PPI inflation \( \pi_{Ht} = p_{Ht} - p_{H,t-1} \) based on (18), showing the importance of real marginal cost \( mc_{Ht} \), which enters the formula in deviation from its steady-state value.

Equilibrium concerns three markets. First, the asset market is in equilibrium if the perfect international risk sharing relation (19) holds, given symmetric initial conditions, where \( s_t + p^*_t - p_t \) is the real exchange rate. Second, labor market equilibrium is (20), where \( y_t \) is domestic output. Third, for the goods market, free international trade implies the law of one price, so that import price index \( p_{Ft} \) follows from (21), where \( p^*_{Ft} \) is the foreign PPI in foreign currency. The goods market also clears for all varieties.
Table 1: Model parameters

<table>
<thead>
<tr>
<th>Par.</th>
<th>Range</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Core parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>(0, 1)</td>
<td>0.99</td>
<td>subjective discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$&gt; 0$</td>
<td>1.17</td>
<td>inverse of Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$&gt; 0$</td>
<td>1.20</td>
<td>inverse of elasticity of intertemporal substitution for consumption</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$&gt; 0$</td>
<td>0.68</td>
<td>elasticity of subst. between home &amp; foreign goods</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$&gt; 1$</td>
<td>8.00</td>
<td>elasticity of subst. between varieties produced within a country</td>
</tr>
<tr>
<td>$\omega$</td>
<td>(0, 1)</td>
<td>0.72</td>
<td>Calvo fraction of firms not allowed to change prices (stickiness)</td>
</tr>
<tr>
<td>$n$</td>
<td>(0, 1)</td>
<td>$\rightarrow 0$</td>
<td>size of the home economy</td>
</tr>
<tr>
<td>$\nu$</td>
<td>(0, 1]</td>
<td>0.75</td>
<td>home bias in preferences</td>
</tr>
<tr>
<td>$\varphi_\pi \geq 0$</td>
<td></td>
<td>2.06</td>
<td>inflation impact on interest rate in Taylor rule</td>
</tr>
<tr>
<td>$\mu$</td>
<td>[0, 1]</td>
<td>—</td>
<td>degree of exchange rate management</td>
</tr>
<tr>
<td><strong>Additional parameters for simulation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>(-1, 1)</td>
<td>0.81</td>
<td>AR(1) coefficient in labor productivity process</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>$\geq 0$</td>
<td>0.52</td>
<td>standard deviation of labor productivity shock (in %)</td>
</tr>
<tr>
<td>$\sigma_a^* \geq 0$</td>
<td></td>
<td>0.12</td>
<td>standard deviation of foreign monetary policy shock (in %)</td>
</tr>
<tr>
<td><strong>Derived parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>(0, 1)</td>
<td>0.01</td>
<td>$= -\log (\beta)$: subjective discount rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>[0, 1]</td>
<td>$\rightarrow 0.75$</td>
<td>$= 1 - (1 - n)(1 - \nu)$: share of home goods in home consumption</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>[0, 1]</td>
<td>$\rightarrow 0$</td>
<td>$= n (1 - \nu)$: share of home goods in foreign consumption</td>
</tr>
<tr>
<td>$\tau$</td>
<td>[0, 1]</td>
<td>0.13</td>
<td>$= 1 - \frac{\theta - 1}{\nu}$: employment subsidy</td>
</tr>
<tr>
<td>$\kappa_{mc} &gt; 0$</td>
<td>0.11</td>
<td></td>
<td>marginal cost impact on PPI inflation in (18)</td>
</tr>
<tr>
<td>$\xi_c &gt; 0$</td>
<td>2.08</td>
<td>$= \sigma + \gamma \nu$</td>
<td>consumption effect on product wage</td>
</tr>
<tr>
<td>$\xi_{tot} \geq 0$</td>
<td>0.60</td>
<td>$= 1 - \nu + \gamma (1 - \nu^2) \eta$: terms-of-trade effect on product wage</td>
<td></td>
</tr>
<tr>
<td>$w &gt; 1$</td>
<td>1.62</td>
<td>$= \frac{\kappa_{mc} \xi_c}{\sigma} + \frac{1 + \kappa_{mc} \xi_{tot}}{\nu}$: effectiveness of $i_t$ to counteract deprec.</td>
<td></td>
</tr>
</tbody>
</table>

Foreign parameters $\beta^*, \gamma^*, \sigma^*, \theta^*, \omega^*, \varphi_\pi^*, \rho_a^*$, and $\sigma_a^*$ equal their home counterparts.

The values of the core and additional parameters for simulation have been taken from Justiniano and Preston (2010). The authors estimate a small open-economy model for three countries vis-à-vis the United States, namely for Australia, Canada, and New Zealand, using Bayesian techniques, though they calibrate the values for $\beta$, $\theta$, and $\nu$. We take the average of their three posterior medians.
To mimic that the domestic country is small, we now take the limit $n \to 0$. That gives goods market clearing at home (22) and abroad (23). The former captures that higher prices for imports relative to domestically produced goods (higher terms of trade $\text{tot}_t = p_{Ft} - p_{Ht}$) cause substitution towards domestic goods, stimulating domestic production. The limit also implies that home CPI in (24) follows from home PPI and the import price index, and that foreign CPI $p^*_t$ is simply foreign PPI, as (25) shows.

4.2 Exchange rate function

To close the model, we have to determine the interest rate rule that implements a given exchange rate objective. As shown in Section 2, the rule depends on exchange rate function (1). Its arguments are $i_t$ and the vector $E_t$. The $i_t$ argument captures the interest rate impact on the exchange rate via all contemporaneous channels, as defined in Section 2.1, while $E_t$ accounts for everything else. So we now derive the equilibrium exchange rate in the $(i_t, E_t)$-form as determined by the model just introduced.

The derivation starts from the fact that the exchange rate $s_t$ clears the asset market, so $s_t$ solves risk sharing (19). Web Appendix B presents a streamlined derivation of how the remaining equations of the model matter for the equilibrium $s_t$. This gives (26), albeit not yet in $(i_t, E_t)$-form.

To shape the result in $(i_t, E_t)$-form, realize that all predetermined, exogenous, and foreign variables are unaffected by $i_t$, so they have to be put in the $E_t$ vector. The recursive nature of (16) implies that $E_t \{c_{t+1}\}$ is determined by expectations of future variables, so there is no contemporaneous effect of $i_t$, making $E_t \{c_{t+1}\}$ part of $E_t$. Similarly, (18) implies that $E_t \{\pi_{H,t+1}\}$ is part of $E_t$. For $E_t \{\pi_{t+1}\}$ one should realize that households base their consumption decision on $E_t \{\pi_{t+1}\}$ as a whole, not on just the $p_t$ part within it. Hence, $i_t$ can only affect $c_t$ via $E_t \{\pi_{t+1}\}$ if the latter as a whole changes, so that $E_t \{\pi_{t+1}\}$ does not contain a contemporaneous channel and is thus part of $E_t$.

The resulting exchange rate function in $(i_t, E_t)$-form is

$$s_t = -wi_t + v'E_t,$$  \hspace{1cm} (26)

where

$$w = \frac{\kappa_{mc}\bar{\omega}_c}{\sigma} + \frac{1 + \kappa_{mc}\bar{\omega}_{\text{tot}}}{\nu}$$  \hspace{1cm} (27)
and

\[ v = \begin{bmatrix} w \\ w \\ 1 \\ \omega \sigma \\ \beta \\ -\kappa_{mc} (\gamma + 1) \end{bmatrix} \quad \text{and} \quad E_t = \begin{bmatrix} i_t^* \\ E_t \pi_{t+1} - E_t \pi_{t+1}^* \\ s_{t-1} - \text{tot}_{t-1} \\ E_t c_{t+1} - E_t c_{t+1}^* \\ E_t \pi_{H,t+1} - E_t \pi_{F,t+1}^* \\ a_t - a_t^* \end{bmatrix}. \] (28)

Equation (26) illustrates exchange rate function (1). It highlights the special role of \( i_t \) resulting from the \((i_t, E_t)\)-separation.

Formula (27) is the model-implied version of (4), so it specifies the effectiveness of the interest rate to counteract depreciation while keeping \( E_t \) constant. We get \( w > 0 \), so an interest rate increase strengthens the home currency. The \( w \) parameter is a function of the structural parameters of the model, so the effectiveness is fully determined by the structure of the economy. Section 4.3 describes the economic mechanisms at work.

Expression (28) for \( E_t \) discloses what else matters for the exchange rate according to the model. Most determinants occur in a simple relative form, an attractive consequence of our streamlined derivation of the \( s \)-function. Because \( s_{t-1} \) has a unit coefficient in \( v \), one could also write (26) in terms of \( \Delta s_t \) and the adjusted \( E_t \) would then consist of stationary variables only. However, that does not imply that \( s_t \) is non-stationary, because to implement an exchange rate regime the interest rate \( i_t \) may depend on the exchange rate level so as to counteract deviations from target, and that can result in a stationary \( s_t \), similar to an error-correction specification.

### 4.3 Interest rate effectiveness

The effectiveness \( w = -\frac{\partial s_t}{\partial i_t} \) is one of the key novelties of our interest rate rule. To analyze the channels underlying \( w \) in (27), we consider asset market equilibrium (19), substitute the CPI relations (24) and (25) (see the derivation of (26) in Web Appendix B for the result) and then differentiate. This yields

\[
\frac{\partial \sigma_{c_t}}{\partial i_t} = \frac{\partial s_t}{\partial i_t} - \left[ \nu \frac{1}{1 + \kappa_{mc} \omega_{\text{tot}}} \kappa_{mc} \left( \omega_c \frac{\partial c_t}{\partial i_t} + \omega_{\text{tot}} \frac{\partial s_t}{\partial i_t} \right) + (1 - \nu) \frac{\partial s_t}{\partial i_t} \right],
\] (29)

where the term in square brackets is \( \frac{\partial p_t}{\partial i_t} \).

We analyze this equation by considering an increase in the interest rate \( i_t \). This enters the economy by making current consumption more expensive relative to future consumption, inducing households to reduce \( c_t \). This is quantified by \( \frac{\partial c_t}{\partial i_t} = -\frac{1}{\sigma} \), based on (16).
The drop in $c_t$ affects the asset market in two ways. One is captured by the derivative on the left of (29). The other is witnessed by $\nu_1 + \kappa_{mc}\varpi_{tot}\kappa_{mc}\varpi_c\frac{\partial c_t}{\partial i_t}$. The intuition of the latter is that a lower $c_t$ reduces wage $w_t$ directly (with coefficient $\sigma$) and indirectly by decreasing output $y_t$ ($\nu$) and thereby labor demand and thus $w_t$ ($\gamma$); the full effect on the product wage $w_t - p_{Ht}$ is $\varpi_c$. This lowers marginal cost $m_{cHt}$ one for one and thus $p_{Ht}$ ($\kappa_{mc}$). This is somewhat weakened by a feedback effect, reflected by $\frac{1}{1+\kappa_{mc}\varpi_{tot}}$.

Overall, $p_{Ht}$ decreases and thus by definition also $p_t$ ($\nu$).

The asset market disturbance by the decreases in $c_t$ and $p_t$ causes $s_t$ to decrease. The latter is by assumption. We have not explicitly modeled how exactly the exchange rate drops, but a possible mechanism would be that, due to the cheaper home bond, households substitute from foreign to home bonds, thereby demanding home currency. The appreciation decreases import prices $p_{Ft}$, and by the two $\frac{\partial s_t}{\partial i_t}$-terms between the square brackets in (29) this reduces $p_t$, so that $s_t$ has to decrease even further to restore equilibrium.

Solving (29) for $\frac{\partial s_t}{\partial i_t}$ gives the value of $w$ in (27). Hence, the above economic channels determine the effectiveness of the interest rate. Changes in the economic structure alter the effectiveness. For example, less price stickiness (lower $\omega$) magnifies the impact of $m_{cHt}$ on $p_{Ht}$ (higher $\kappa_{mc}$) and thereby the asset market disturbance and thus $w$. Hence, increasing price flexibility enhances interest rate effectiveness, facilitating the defense of a peg for given pressure.

4.4 Interest rate rule

We still focus on the weighted fixed-floating exchange rate regime and thus take rule (9). For the first ingredient, the domestically-oriented rate, we take

$$\delta^d_t = \delta + \varphi_{\pi} \pi_{Ht},$$

(30)

though one could also use a Taylor rule with CPI inflation and output gap. The effectiveness $w$ follows from (27), and the degree of exchange rate management $\mu$ and the target rate $s_t$ are both taken as given. The intermedial exchange rate $s^I_t$ follows directly from definition (2) and the model-implied $s$-function (26), so that

$$s^I_t = -wi^d_t + v'E_t.$$

(31)

Note that, given the direct link between $s^I_t$ and $EMP_t$ in definition (6), (31) shows what variables determine $EMP_t$, namely $\pi_{Ht}$ via (30) and $E_t$ in (28). So the model-independent derivation in Section 2 reveals that $EMP$ is the key variable for setting
the interest rate, and a model can then help the central bank to get to its drivers. Put differently, EMP is the vehicle through which those drivers matter for interest rate policy.

The interest rate rule results by substituting (31) in rule (9), giving

\[ i_t = (1 - \mu) i^d_t + \mu \frac{1}{w} (v' E_t - s^t). \]  

(32)

This rule guarantees that the exchange rate regime is implemented at every \( t \). The interest rate is a weighted average of \( i^d_t \) and \( \frac{1}{w} (v' E_t - s^t) \). For example, in a float (\( \mu = 0 \)), the interest rate is simply the domestically-oriented rule \( i^d_t \), as usual.

To implement a fixed rate (\( \mu = 1 \)), the central bank cannot pursue \( i^d_t \) at all, in line with the well-known incompatible trinity. For example, a one percentage point lower \( i^d_t \) by itself motivates an equally lower \( i_t \), but implementing that would cause a \( w \) %-points higher \( s_t \), which would have to be offset by a one %-point higher \( i_t \) to maintain the peg, making \( i^d_t \) on balance irrelevant for the actual interest rate. Instead of looking at \( i^d_t \), the central bank should focus on \( v' E_t - s^t \). If market sentiment in \( v' E_t \) tends to move the exchange rate away from the target, the excess change \( v' E_t - s^t \), converted into interest rate units by dividing by \( w \), pins down the interest rate.

Rule (32) illustrates some of the improvements over the Monacelli (2004) rule (10) analyzed in Section 3: our rule covers the fixed exchange rate regime, accounts for the instrument effectiveness \( w \), and our rule recognizes the separate roles of \( w \) and \( \mu \) instead of using the reduced-form parameter \( \phi_s \). A new improvement is that our rule automatically accounts for the incompatible trinity.

The Benigno et al. (2007) rule (11) concerns the fixed exchange rate, and for that case our rule becomes

\[ i_t = i^*_t + \frac{1}{w} (v_2' E_{2t} - s^t), \]

(33)

where \( v_2 \) and \( E_{2t} \) equal \( v \) and \( E_t \) in (28) with the top element left out. Hence, our rule uses a specific model-driven parameter \( w \) instead of the undetermined \( \varphi_s^{BBG} \), and does not need convertibility assumptions. A common feature is that also in our rule following \( i^*_t \) is important for implementing the peg.

5 Simulations from the model

In Section 3 we have discussed the characteristics of our interest rate rule without imposing a specific model. To illustrate some of these characteristics, we now simulate from the model just developed. The main insights from these simulations are not
specific to the model, parameter values, or draws of the shocks.

5.1 Model calibration, solution, and simulation

One period in the model is one quarter. For the simulations we assume that (the log of) home labor productivity \( a_t \) follows an AR(1) process with autoregressive coefficient \( \rho_a \) and that the i.i.d. shock involved has mean zero and standard deviation \( \sigma_a \). The same holds for foreign productivity \( a^*_t \). The foreign central bank follows the rule

\[
i^*_t = \delta + \varphi_\pi \pi^*_t + \varepsilon^*_t,
\]

where \( \varepsilon^*_t \) is an i.i.d. monetary policy shock with zero mean and standard deviation \( \sigma^*_i \).

We set the target \( s^t = 0 \). All parameter values are based on Justiniano and Preston (2010). Table 1 presents them, and its note provides further motivation.

We solve the model numerically using the Sims (2002) algorithm. The solution can be cast as a reduced-form VAR model of the 20 \times 1-vector with elements \( c_t, E_t c_{t+1}, c^*_t, E_t c^*_{t+1}, \pi_H t, E_t \pi_{H,t+1}, \pi_t, E_t \pi_{t+1}, \pi^*_t, E_t \pi^*_{t+1}, y_t, y^*_t, i_t, i^*_t, s_t, E_t s_{t+1}, tot_t, EMP_t, a_t, a^*_t \). We focus on unique stationary solutions, abstracting thus from sunspot equilibria.

The necessary and sufficient condition for equilibrium determinacy is as follows, given our parameter space and thus the symmetry assumption \( \varphi_\pi = \varphi^*_\pi \). Under the float, the condition follows from Bullard and Mitra (2002), and it here reduces to satisfying the Taylor principle \( \varphi_\pi > 1 \). For the other regimes, from the managed float to the fixed rate, we verify determinacy for a grid of parameter values, using the Sims (2002) algorithm. Also here the condition turns out to be \( \varphi_\pi > 1 \). So the same condition applies whatever the regime. It holds for all parameter values we study.

We set \( s_0 = 0 \) and initialize other variables at their steady-state values. The three shocks are drawn from independent normal distributions. We draw them for 60 periods (15 years), from which we compute the paths of the variables of interest. For ease of comparison we keep the realized shocks the same across the plotted paths.

5.2 Implementing multiple regimes

Our rule (32) implements many exchange rate objectives and does so exactly, as explained in Section 3.2. To obtain a first insight into the performance of the rule, we simulate paths for variables in three different regimes, the float (\( \mu = 0 \)), an intermediate regime (say \( \mu = 0.5 \)), and the peg (\( \mu = 1 \)). A representative set of paths is depicted

\(^{15}\)In this special case of the weighted fixed-floating regime, one can replicate our simulations for \( \mu < 1 \) with the Monacelli (2004) rule by setting \( \varphi^*_s \), based on (13). The problem is that this requires \( w \) (except
in Figure 2. The variables $EMP_t, i_t, i_d^t, i_d^s, s_t$ in the graphs are in percentage terms.

Under the float ($\mu = 0$), the interest rate $i_t$ equals the domestically-oriented rate $i_d^t$, visualized by the horizontal line in the second panel. As $i_d^t$ is driven by inflation, the interest rate does not stabilize the exchange rate $s_t$, which is consistent with the fact that the grey line in the bottom panel does not revert to zero.

Figure 2: Paths implied by our rule (32) in various exchange rate regimes: from float ($\mu = 0$, grey) to intermediate ($\mu = 0.5$, dashed) to fixed ($\mu = 1$, black).

for $\mu = 0$), which is not available in his approach. This makes that simulations from his and our rules generally differ. Likewise, the Benigno et al. (2007) rule replicates our simulations for $\mu = 1$. But that necessitates imposing the convertibility assumptions, which we avoid.
The stronger the exchange rate management, the more the central bank has to account for exchange rate fundamentals when setting the interest rate, making $i_t^d$ less and $E_t$ in our rule more relevant. The dashed line in the bottom panel visualizes that $\mu = 0.5$ here already stabilizes the exchange rate considerably. The line also suggests that the weighted fixed-floating regime can be a practical linear approximation of various other exchange rate policies, such as the peg with band.

If the central bank pursues a fixed exchange rate ($\mu = 1$), then the black line in the middle panel visualizes that $i_t = i_t^*$ in equilibrium, and the bottom panel shows that the exchange rate stays on target continuously, as expected. This corroborates that the model contains UIP, by virtue of (19). Still, the top panel reveals that shocks cause periods of noticeable pressure $EMP_t$ on the peg. There the central bank has to accept an interest rate that differs substantially from the domestically-oriented rate (second panel), which in practice may induce policy makers to give up the peg.

5.3 Implementing the peg even after a structural change

A distinctive feature of our rule compared to the traditional rules is that the economic structure matters, via $w$, as explained in Section 3.4. We now study the impact of a structural change for the fixed exchange rate regime ($\mu = 1$) and the consequences if policy makers fail to account for this. We focus on the degree of price stickiness $\omega$. We reduce it (only in this section) from 0.72 to $\tilde{\omega} = 0.60$, the lower end of the average 90% credible interval reported by Justiniano and Preston (2010), so that home producer prices become more flexible.

The interest rate effectiveness $w$ increases from 1.62 to $\tilde{w} = 2.02$, so an increase in $i_t$ now causes a sharper drop in home producer prices and thereby a larger appreciation. This increased effectiveness affects our rule, as it prescribes a less aggressive $i_t$ to maintain the peg for given pressure. This is a plausible novelty of our rule.

The use of $i_t$ not only depends on its effectiveness, but also on the amount of pressure. For the fixed rate, pressure is $EMP_t = w (i_t^* - i_t^d)$, based on EMP definition (6) with (31), rule (33), and equilibrium outcome $i_t = i_t^*$. More price flexibility affects $EMP_t$ in two ways. First, it increases the exchange rate consequence $\tilde{w}$ of a given wedge $i_t^* - i_t^d$. Second, it makes PPI inflation $\pi_{Ht}$ and thus $i_t^d$ more volatile. In total, comparing the black line in the top panel of Figure 3 to that in Figure 2 demonstrates that here more price flexibility creates more volatile pressure. Such consequences of a structural change for pressure are important for policy makers to be aware of.

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16This implies $\tilde{\kappa}_{mc} = 0.27$ and $\tilde{w} = 2.02$. The other parameters in Table 1 do not change. In particular, $\omega^*$ is unchanged; changing it would have virtually no impact on the results relevant here.
Figure 3: Impact of increased price flexibility (lower $\omega$), depending on whether the fixed exchange rate policy accounts for it (black) or not (grey).

The equilibrium value of $i_t$ remains $i_t^*$, so the higher instrument effectiveness and the increased volatility of pressure cancel out. The black line in the middle panel of Figure 3 thus still equals $i_t^*$, and the bottom panel demonstrates that our automatically accommodating rule keeps on implementing the fixed exchange rate.

The outcome changes if the central bank does not adjust the rule in response to the reduced price stickiness, that is, it uses $w$ instead of $\tilde{w}$ in the rule. As the grey lines in Figure 3 reveal, $EMP_t$ and $i_t$ become more volatile, and the peg is missed. The intuition is as follows. The central bank no longer weakens the interest rate response to a given $EMP_t$. This causes an overreaction of $i_t$. It turns out that $EMP_t$ becomes also more volatile, and this further intensifies the interest rate response. This causes $i_t$ to deviate from $i_t^*$. If $i_t > i_t^*$, the home currency appreciates, so the target is missed.

The policy recommendation is that central bankers should account for the effectiveness of their policy instrument, and thus for the economic structure, when determining its use. This matches economic intuition. The point is that only our rule accounts for it, thus improving on traditional rules.
6 Conclusion

This paper has derived that EMP is a key variable for a central bank when setting the interest rate to implement a given exchange rate objective. This guides policy and, vice versa, actual policy confirms the relevance of EMP. We have formalized this in a new interest rate rule that contains EMP.

We have also derived that the economic structure matters for the EMP coefficient in the rule: the more effective the interest rate, the less it should be used to offset a given pressure. Our rule exactly implements the exchange rate objective, and it does so for many regimes and models. All these features improve on traditional rules.

We have introduced the weighted fixed-floating regime, with weight $\mu$ on the fixed regime. Our rule can be conveniently combined with this regime. This leads to a coefficient of EMP in the rule that discloses two structural parts, namely the model-determined interest rate effectiveness $w$ and the degree of exchange rate management $\mu$ chosen by the policy maker. We have thus disentangled a Taylor-rule type of coefficient into two underlying structural parameters. We have exploited this to show how the de facto exchange rate regime can be estimated in a novel way.

As a by-product, we have extended the EMP literature. That is, we have refined the EMP formalization and computed EMP in a modern sticky-price model, whereas the EMP literature typically relies on some variant of the flexible-price monetary model. We have also formalized how EMP is an ingredient for policy, and how the sticky-price model helps the policy maker to learn the determinants of EMP. All this may stimulate further research on EMP.

The general applicability of our rule and the inherent consistency with the regime and model can facilitate future research. Think of studies on the optimal degree of exchange rate management, further eased by our new structural parameter $\mu$, and research on models with incomplete markets and risk premia. For example, in another paper we apply our idea to analyze foreign exchange interventions by the central bank under capital controls. This could then facilitate studies on emerging markets where central banks use forex intervention to pursue leaning-against-the-wind exchange rate management. This is left for future research.
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