How do governments respond to interest rates?[1]

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Abstract

We explore the optimal and actual responses of fiscal policy to changes in the interest rate on newly-issued public debt (the "marginal interest rate"). We set up a simple theoretical framework with a government aiming to smooth public consumption over time. The distinctive feature is that the government issues debt of different maturities. This introduces a "valuation effect" that has received little attention so far: a rise in the marginal interest rate increases the rate of discounting and, thus, lowers the value of non-maturing debt, which relaxes the budget constraint, thereby inducing a fall in the primary balance. Still, the framework predicts that the total effect of a rise in the marginal interest rate is an increase in the primary balance. Estimates for developed countries suggest that a 1 percentage-point higher marginal interest rate leads, on average, to roughly a 1 percentage-point higher primary balance. These findings are consistent with governments smoothing the impact of changes in the marginal interest rate and exploiting the valuation effect. Finally, estimates suggest a role for the average (or "effective") interest rate on outstanding debt.

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1. Introduction

How does a change in the interest rate on the public debt affect a government´s fiscal policy? Historical experience has demonstrated the importance of this question. For example, in the run-up to and following the start of the Euro, nominal interest rates of some Member States fell substantially, freeing up substantial fiscal space. However, this fiscal windfall was often not used in a countercyclical manner to build up buffers for less benign times, but rather for additional public spending (see e.g., European Fiscal Board, 2020).[5] In the years following the Eurozone debt crisis until the Covid-19 crisis, macroeconomic conditions were again relatively benign and monetary policy was effectively stuck at the zero lower bound, leading to falling interest burdens on outstanding government debt. Again, countries, in particular EU countries with high public debt, failed to use the extra fiscal space to strengthen their public financial buffers. Now, at the time of this writing, circumstances have become very different. Inflation has shut up and, because it was unforeseen, this has helped to reduce the public debt increases caused by the Covid-19 crisis (IMF, 2023b). However, most monetary authorities are on a tightening path by raising interest rates, implying a gradual increase in interest expenditures by governments. In combination with public debt ratios set to rise in major economic blocks, such as the US and the EU, once the downward effect from the unexpected inflation surge has vanished, this confronts their governments with the question how to respond to these developments, so as to avoid crowding out of government services in the longer run. For example, see IMF (2023b), European Commission (2023a, 2023b) and Congressional Budget Office (2023).

This paper explores the responses of governments´ primary budget balances to changes in the interest rates on public debt from both a theoretical and an empirical perspective. We start by setting up a simple intertemporal theoretical framework in which the government aims at smoothing public consumption over time. The distinctive feature of the model is that the government issues debt of different maturities, short- and long-term debt, say. This feature may matter for the response to interest rate changes, because for debt that does not mature this period a change in the interest rate on newly-issued debt (the "marginal interest rate") is irrelevant for interest payments. The feature also adds realism since most of the outstanding debt does not mature soon. For example, the residual maturity of outstanding debt in OECD countries is eight years, on average (OECD, 2022).[6]

[5] Public spending was frequently put to inefficient use. For example, a building spree in Spain resulted in many underused infrastructure objects (NY Times, https://www.nytimes.com/2011/06/25/business/global/25iht-transport25.html). For a more detailed account of the budgetary developments in EU peripheral countries, see Ciżkowicz et al. (2015).

[6] In fact, many countries have lengthened their debt maturities in recent years. The European Commission (2023a, Graph 4.2) shows a trend increase in the average residual maturity of public debt from 2009 through

The theoretical framework predicts that the total effect of a higher marginal interest rate is that the government raises the primary balance. This total effect is the combination of a positive standard wealth effect, a positive substitution effect, a negative income effect and a negative "valuation effect" on the primary balance. This valuation effect, which constitutes a specific type of wealth effect, has received little or no attention in the literature so far.[7] Essentially, it measures the fall in the value of non-maturing debt due to the higher rate of discounting.

The multiple maturities in the theoretical framework also matter for the empirical approach. Even though we are interested in the impact of the marginal interest rate on the primary balance, the valuation effect motivates the inclusion in the regression model of (roughly) the effective interest rate, the weighted average of the interest rates on outstanding debt. Given the high correlation between the marginal and effective rates, including the effective interest rate can help avoid omitted variable bias. We estimate the model focusing on a panel dataset comprised of developed countries. We find that a 1 percentage-point higher marginal interest rate in the long run leads to an about 1 percentage-point higher primary balance on average. The magnitude depends on the level of debt, rising from 0.86 to 1.40 percentage-points if the debt-to-GDP ratio increases from 60 to 100%.

The empirical results are well-behaved and mostly in line with the predictions of the theoretical framework. The estimates deviate when it comes to the impact of the marginal interest rate. First, we find that the marginal interest rate in isolation has a statistically insignificant impact on the primary balance. Second, the difference between the marginal and the effective interest rate on outstanding debt exerts a positive effect on the primary balance. A plausible explanation for the combination of these findings is the presence of a positive forward-looking smoothing effect on the primary balance of an increase in the marginal rate that more than offsets the negative valuation effect. The smoothing effect says that a rise in the marginal rate above the effective one raises future interest bills, inducing the government to save more now. Indeed, a reading of available policy documentation suggests that governments tend to use a constant discount rate for their cost-benefit analyses and respond to current interest rates moving above (below) the effective rate by budgetary contraction (expansion).

From an empirical perspective, that the primary balance only responds to the deviation of the marginal from the effective interest rate has consequences for the public debt trajectory, as the effective rate will gradually adjust to the level of the marginal rate, thereby eroding the positive effect of the

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2022, while Table 4.2 shows that the far majority of the individual countries raised the average residual debt maturity, sometimes by several years. See also the evidence in De Graeve and Mazzolini (2023).

[7] The effect is implicitly present in Barro (1979), who touches upon the "empirically relevant situation where government debt exists with different maturity dates", but it is a side element of his analysis to which he does not pay explicit attention.

marginal rate on the primary balance. Yet, sustainability in our empirical model is assured by the primary balance rising in response to an increase in the sum of debt and interest payments.

There exists an extensive literature that estimates the impact of lagged debt-to-GDP ratios on the primary balance, typically based on Bohn's (1998) regression framework.[8] Mauro et al. (2015) extend his approach by allowing this impact to depend on various indicators of the economic situation, one of which is the marginal interest rate. They demonstrate that the primary balance response is stronger when inflation is low and the marginal interest rate is high. The current paper, instead, studies how governments react to windfall gains and losses associated with changes in interest rates, while controlling for the lagged debt-to-GDP ratio. In addition, as motivated above, we account for the multiple-maturity structure of public debt and find that empirically the role of the marginal interest rate is taken over by its difference with the effective rate on outstanding debt.

The marginal interest rate allows us to account for the forward-looking smoothing effect set out above. In contrast, some studies estimate the impact of the effective interest rate, which summarizes the rates on existing debt and, thus, has a backward-looking orientation. Berti et al. (2016) find that the effect of a higher effective interest rate on the change in the primary balance is often insignificant and, if significant, can go both ways. Debrun and Kinda (2016) find that the effective rate has a positive impact on the primary balance. In contrast to both papers, we show that it is the aforementioned difference between the marginal and effective interest rate that matters. Hence, our results can be interpreted as that, given the marginal interest rate, a higher effective rate has a negative effect on the primary balance.

The current paper also connects to the literature on resource windfalls not coming from interest rate changes. Raveh and Tsur (2020) investigate the response of public debt to oil and gas windfall shocks in the presence of political myopia from re-election-seeking politicians. Such myopia causes a budget deficit bias. The resulting additional debt build-up is worsened by resource windfalls. Finally, somewhat related in conceptual terms, Tornell and Lane (1998) explore how windfalls from temporary terms-of-trade booms affect the current account balance. They show that aggregate spending rises more than the windfall itself, causing it to lead to a deterioration of the current account balance.

The remainder of this paper is organized as follows. Section 2 presents the theoretical framework. Section 3 lays out the empirical approach. Section 4 presents and discusses the empirical results. Finally, Section 5 concludes the main text.

[8] See e.g. Afonso and Jalles (2020) and Afonso et al. (2021).

2. Theoretical framework

In this section, we present a simple theoretical framework that we later take to the data. We allow for short and long public debt. The framework highlights the role of the difference between the marginal interest rate on newly-issued public debt and the effective interest rate on outstanding debt.

2.1 Model for the primary balance

The model starts in period $t = -1$ when a first debt issuance was made, and it ends in period $t = 2$. We take the vantage point of the government in period $t = 1$, which takes debt issuance decisions made in the past as given. It has to decide about the amount of public consumption in period 1 versus period 2, where G_t is deterministic nominal spending on public consumption in periods $t = 1, 2$. For now, we assume that inflation is absent. The government's objective function, which depends on G_1 and G_2, will be presented later. We first focus on the period budget constraints.

Public consumption needs to be financed by raising taxes or by issuing public debt. The government's period budget constraints in nominal terms are given by:

$$G_1 + \left(1 + r_{-1,1}\right)D_{-1,1} + (1 + r_0)D_{0,1} + r_{0,2}D_{0,2} = T_1 + D_{1,2}, \quad (1)$$

$$G_2 + \left(1 + r_{0,2}\right)D_{0,2} + (1 + r_1)D_{1,2} = T_2, \quad (2)$$

where $D_{-1,1}$ is the amount of nominal debt issued in period -1 maturing in (the beginning of) period 1. The nominal (coupon) interest rate in periods -1 and 0 is constant at $r_{-1,1}$, set at the moment of issuance in period -1. Similarly, $D_{0,1}$ is the amount of debt issued in period 0 and maturing in period 1, featuring the coupon interest rate r_0, which is short-hand for $r_{0,1}$; $D_{0,2}$ is the amount of debt issued in period 0 maturing in period 2, with constant coupon interest rate $r_{0,2}$; and $D_{1,2}$ is the amount of debt issued in period 1 and maturing in period 2, with coupon interest rate r_1, which is short-hand for $r_{1,2}$. We take r_1 as given (in the empirics we generalize this). We assume all debt tranches to be positive, in line with reality. Finally, T_1 and T_2 are exogenous nominal tax revenues in periods 1 and 2. Because tax revenues are given, the only decision effectively taken by the government in period $t = 1$ is the choice of $D_{1,2}$, which then fixes the intertemporal allocation of public consumption.

The predetermined parts of debt in (1) and (2) can be combined, after discounting to t=1, into

$$\left(1+r_{-1,1}\right)D_{-1,1}+\left(1+r_0\right)D_{0,1}+r_{0,2}D_{0,2}+\left(\frac{1+r_{0,2}}{1+r_1}\right)D_{0,2}=\left(1+r_0^{eff}-r_1^n\right)D_0^t, \qquad (3)$$

where

$$D_0^t=D_{-1,1}+D_{0,1}+D_{0,2};\ r_0^{eff}=\frac{r_{-1,1}D_{-1,1}+r_0D_{0,1}+r_{0,2}D_{0,2}}{D_{-1,1}+D_{0,1}+D_{0,2}};\ \text{and}\ r_1^n=\left(\frac{r_1-r_{0,2}}{1+r_1}\right)\frac{D_{0,2}}{D_0^t}. \qquad (4)$$

Hence, D_0^t is the total amount of outstanding government debt in period 0 after the new issuance in that period, and r_0^{eff} is the debt-weighted average interest rate paid on this debt, which we refer to as the "effective interest rate". The part $\left(1+r_0^{eff}\right)D_0^t$ is known from the empirical literature. If all debt is short term, r_0^{eff} equals r_0, the rate that typically features in theoretical models.

Our novel term is $-r_1^n$. It comes from two parts. The first is the market value of non-maturing debt, $\left(\frac{1+r_{0,2}}{1+r_1}\right)D_{0,2}$, so the redemption plus interest payment next period, discounted at the interest rate r_1 on new debt. The second part of $-r_1^n$ corrects for the fact that bringing in $\left(1+r_0^{eff}\right)D_0^t$ has added non-maturing debt $D_{0,2}$ to treat it in the same way as maturing debt $D_{-1,1}+D_{0,1}$, that is, by doing as if $D_{0,2}$ is also paid back in period $t=1$. So, the required correction is $-D_{0,2}$. In total, $-r_1^n$ is $\left(\frac{1+r_{0,2}}{1+r_1}\right)D_{0,2}-D_{0,2}$, as a fraction of total inherited debt D_0^t. Next, realize that $-D_{0,2}$ is minus the market value of $D_{0,2}$, if it were issued at rate r_1. Hence, r_1^n captures the impact of r_1 being different from $r_{0,2}$ on the market value of non-maturing debt. We make this more explicit by rewriting r_1^n as $\left(\frac{r_1-r_{0,2}}{1+r_1}\right)\frac{D_{0,2}}{D_0^t}$ in (4). If $r_1>r_{0,2}$, the market value of non-maturing debt $D_{0,2}$ is lower than its nominal value: the interest payment in period 2, based on $r_{0,2}$, does not accommodate to the higher r_1. That relaxes the budget constraint. The gain is proportional to $D_{0,2}/D_0^t$, the fraction of inherited debt that does not mature in period 1. As this fraction is large for most countries (cf. Footnote 6), $r_1-r_{0,2}$ is almost as influential as r_0^{eff} in $\left(1+r_0^{eff}-r_1^n\right)D_0^t$. Hence, $r_1^nD_0^t$ is of an order of magnitude comparable to the interest payment in period 1. Still, both are minor compared to debt itself, represented via the coefficient of 1 on D_0^t in the term $\left(1+r_0^{eff}-r_1^n\right)$.

Combining (1) and (2) and using (3) gives the government's intertemporal budget constraint:

$$G_1+\frac{G_2}{1+r_1}=T_1+\frac{T_2}{1+r_1}-\left(1+r_0^{eff}-r_1^n\right)D_0^t. \qquad (5)$$

The government's objective function is given by

$$u(G_1,G_2)=u(G_1)+\beta u(G_2),\ \text{with}\ u(G_t)=\frac{G_t^{1-1/\sigma}}{1-1/\sigma}, \qquad (6)$$

where $0 < \beta \leq 1$ and u is the period utility function, where parameter $\sigma > 0$ is the elasticity of intertemporal substitution; for $\sigma = 1$ we have log utility in (6). Maximizing $u(G_1, G_2)$ in (6) subject to (5) yields

$$G_2 = \left(\beta(1+r_1)\right)^{\sigma} G_1,$$

which combined with (5), and after rewriting, yields the following expression for the primary balance:

$$T_1 - G_1 = T_1 - f_\sigma^{-1}\left[\left(T_1 + \tfrac{T_2}{1+r_1}\right) - \left(1 + r_0^{eff} - r_1^n\right)D_0^t\right], \tag{7}$$

where

$$f_\sigma = 1 + \frac{[\beta(1+r_1)]^{\sigma}}{1+r_1},$$

where, to keep the notation light, dependence on the period, i.e., period 1, has been suppressed in f_σ. All the elements on the right-hand side of (7) are taken as given by the government in $t = 1$, hence this is the final solution for the primary balance.

2.2 Impact of the marginal interest rate

What is the overall effect of a change in the marginal interest rate r_1 on the primary balance? Totally differentiating $T_1 - G_1$ with respect to r_1 yields:

$$\frac{d(T_1 - G_1)}{dr_1} = \frac{\partial(T_1 - G_1)}{\partial r_1} + \frac{\partial(T_1 - G_1)}{\partial r_1^n}\frac{dr_1^n}{dr_1} + \frac{\partial(T_1 - G_1)}{\partial f_\sigma}\frac{df_\sigma}{dr_1}$$

$$= \underbrace{f_\sigma^{-1}\frac{T_2}{(1+r_1)^2}}_{WE>0} \underbrace{-f_\sigma^{-1}\frac{1+r_{0,2}}{(1+r_1)^2}D_{0,2}}_{VE<0} \underbrace{+f_\sigma^{-1}\frac{G_2}{(1+r_1)^2}\sigma}_{SE>0} \underbrace{-f_\sigma^{-1}\frac{G_2}{(1+r_1)^2}}_{IE<0}$$

$$= f_\sigma^{-1}\frac{(1+r_1)D_{1,2} + G_2\sigma}{(1+r_1)^2} > 0. \tag{8}$$

Note that we have kept tax revenues constant, in line with their exogeneity, so that *overall* refers to the total impact of r_1 in this ceteris paribus sense. In the overall effect, *WE* is the "standard" wealth effect, *SE* the substitution effect,[9] and *IE* the income effect. Further, *VE* is a new effect, which we label the "valuation effect": it is a specific type of wealth effect that arises from the fact that the market value of the non-maturing public debt decreases when the marginal interest rate increases. The four

[9] Formally, this effect is not exactly the substitution effect, as the latter involves the compensated demand function, which we do not have.

separate effects have differing signs. However, taking them together the overall effect of an increase in the marginal interest rate r_1 on the primary balance is positive.

The standard wealth effect dominates the sum of the income and valuation effects. The intuition is as follows. When r_1 increases, G_2 gets cheaper in terms of period 1 goods and the market value of $(1+r_{0,2})D_{0,2}$ drops. Both reduce the need to save, hence lead to lower $(T_1 - G_1)$. However, because T_2 is relatively high, as it needs to cover spending G_2 and the repayments of $(1+r_{0,2})D_{0,2}$ and $(1+r_1)D_{1,2}$, the higher rate of discounting of the revenues T_2 dominates, thereby overall increasing the need to save and pushing up $(T_1 - G_1)$. The final line in (8) combines the standard wealth, valuation and income effects into a single term involving $(1+r_1)D_{1,2}$, which captures that new borrowing has become more expensive, and a term with σ, which captures the substitution effect. This last term we expect to be small, because empirically σ is close to zero. [10]

The results derived in this section can be generalized to an analogous three-period framework with the government trading off public consumption in periods 1, 2 and 3, and with debt with a maturity of up to three periods. The outcomes are reported in the Appendix. The algebra is rather cumbersome, but it can be shown that the effect of a permanent increase in the marginal interest rate on the period-1 primary balance is positive, while there is again a negative valuation effect on the period-1 primary balance resulting from a fall in the market value of the non-maturing debt. Again, the standard wealth effect dominates the sum of the valuation and income effects. Further, while the setting with the world ending in period $T = 2$ does not allow to take into account an effect of r_1 on future effective interest rates, an effect that may be relevant in reality, the setting with the world ending in period $T = 3$ does make this possible.

3. Empirical approach

This section lays out how we take the above solution for the primary balance to the data. We have ignored inflation so far. Moreover, the equation contains future tax revenues, which we do not yet observe, while r_1^n depends on the non-maturing debt and the corresponding interest rate, which we may not observe either. We resolve these issues so as to be able to specify the regression model.

[10] For example, see Best et al. (2020), who estimate the elasticity of intertemporal substitution at 0.1. The meta study of Havranek (2015) gets to an estimate of zero for macro estimates and 0.3 – 0.4 for micro studies.

3.1 Accounting for inflation

The theory above has for simplicity ignored inflation. In the empirical analysis we need to account for inflation, so we first generalize the theoretical model slightly. The objective function (6) now becomes $\tilde{u}(G_1, G_2) = u\big(G_1, G_2/(1+\pi_1)\big)$, where π_1 is the inflation rate between periods 1 and 2. Using $1 + r_1 = (1+\rho_1)(1+\pi_1)$, where ρ_1 is the real interest rate, discounted tax revenues can be written as $T_1 + \frac{T_2}{1+r_1} = T_1 + \frac{T_2/(1+\pi_1)}{1+\rho_1}$. The primary balance solution remains (7),[11] albeit with f_σ substituted by

$$\varphi_\sigma \equiv 1 + \frac{[\beta(1+\rho_1)]^\sigma}{1+\rho_1}. \tag{9}$$

Two determinants of the primary balance depend on inflation. First, an increase in inflation may increase $T_2/(1+\pi_1)$ if taxes are progressive and tax brackets are not adjusted. This would lead to a lower primary balance $T_1 - G_1$. Second, our new term r_1^n brings in a novel impact of inflation. The term can be written as

$$r_1^n = \left(\frac{r_1 - r_{0,2}}{1+r_1}\right)\frac{D_{0,2}}{D_0^t} = \left[1 - \frac{1+r_{0,2}}{(1+\rho_1)(1+\pi_1)}\right]\frac{D_{0,2}}{D_0^t}. \tag{10}$$

An increase in inflation π_1 increases the nominal interest rate r_1 without affecting the coupon rate $r_{0,2}$. This implies a higher r_1^n, representing a stronger valuation effect discussed earlier and, hence, a lower primary balance $T_1 - G_1$.

3.2 Operationalizing the tax gap

Total resources in the right-hand side of the inflation-adjusted version of (7) depend on the difference between T_1 and a scaled version of $T_1 + \frac{T_2/(1+\pi_1)}{1+\rho_1}$. To write this in gap form, define the *permanent level* $\bar{T}_1$ of real tax revenues (i.e., expressed in terms of the period-1 price level) as the hypothetical level of real tax revenues that is constant as of period 1 and has the same present value as that of the actual tax revenue stream.[12] That is,

$$\bar{T}_1 = \left[1 + \frac{1}{1+\rho_1}\right]^{-1}\left[T_1 + \frac{T_2/(1+\pi_1)}{1+\rho_1}\right]. \tag{11}$$

The primary balance solution can now be written as

$$T_1 - G_1 = \frac{1}{\varphi_\sigma}\left(1 + r_0^{eff} - r_1^n\right)D_0^t - \frac{1}{\varphi_\sigma}\varphi_0\bar{T}_1 + T_1$$

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[11] The government's first-order condition becomes $G_2 = (1+\pi_1)\left(\frac{\beta(1+r_1)}{1+\pi_1}\right)^\sigma G_1$.

[12] See, for example, Obstfeld and Rogoff (1996, p.74) for an extensive use of permanent values.

$$= \frac{1}{\varphi_\sigma}\left(1 + r_0^{eff} - r_1^n\right)D_0^t + \frac{\varphi_0}{\varphi_\sigma}(T_1 - \bar{T}_1) + \left(1 - \frac{\varphi_0}{\varphi_\sigma}\right)T_1, \tag{12}$$

where

$$\frac{\varphi_0}{\varphi_\sigma} = \frac{2 + \rho_1}{1 + \rho_1 + [\beta(1 + \rho_1)]^\sigma}.$$

Because in reality σ is fairly close to zero (the income effect dominates the substitution effect) and $\beta(1 + \rho_1)$ is close to 1, the term $[\beta(1 + \rho_1)]^\sigma$ will be close to one for realistic values of β and ρ_1. Hence, $\varphi_0/\varphi_\sigma \approx 1$.

The penultimate term on the right-hand side highlights the role of the tax gap. We assume that it is as follows linked to the output gap:

$$\frac{T_1 - \bar{T}_1}{\bar{T}_1} = \frac{Y_1 - Y_1^p}{Y_1^p}, \tag{13}$$

where Y_1 is nominal GDP and Y_1^p is potential nominal GDP both in period 1.[13] In addition, we assume that $\bar{T}_1/Y_1^p = \gamma$ is constant over time. The literature suggests that its value is around 0.4. This constancy can be motivated as follows. While tax revenues are exogenous in the present model, in Barro (1979) the government minimises the present value of tax-raising costs, leading to identical marginal collection costs in each period. Assuming, in addition, a homogeneous collection cost function, he shows that the government aims at keeping the tax-to-GDP ratio constant. The assumption of $\bar{T}_1/Y_1^p = \gamma$ constant and the tax gap equal to the output gap is easily seen to imply that T_1/Y_1 is also constant at γ. In other words, by following a policy of tax revenues fluctuating around its permanent value in a certain proportion to short-run fluctuations in GDP around its potential, governments minimise fluctuations in the marginal tax rate, which in standard settings would keep the present value of losses from tax distortions to a minimum. Therefore, the assumptions of a constant $\bar{T}_1/Y_1^p$ and equal tax and output gaps come in naturally from a theoretical perspective. Substituting both assumptions gives:

$$T_1 - G_1 = \frac{1}{\varphi_\sigma}\left(1 + r_0^{eff} - r_1^n\right)D_0^t + \frac{\varphi_0}{\varphi_\sigma}\gamma\left(Y_1 - Y_1^p\right) + \left(1 - \frac{\varphi_0}{\varphi_\sigma}\right)T_1. \tag{14}$$

Since $\varphi_0/\varphi_\sigma \approx 1$ the impact of the output gap will be around γ, while T_1 will virtually drop out. The latter would be in line with what we expect to find empirically: we expect the size of the government,

[13] One could allow for a non-unity multiple of the output gap on the right-hand side. That would only multiply the parameter γ, without relevant consequences. We also leave out an intercept on the right-hand side. After all, as the permanent level $\bar{T}_1$ is a weighted average of the tax revenues, the tax gap is expected to be close to zero on average, and that is close to the average output gap of -0.3% we find in our dataset.

as measured by T_1/Y_1, to be uninformative about the primary balance ratio $(T_1 - G_1)/Y_1$. The size of the government is a reflection of political or popular preferences, which are a priori unrelated to issues of debt sustainability.

3.3 Linearization around the social discount rate

In the above expression for the primary balance, the real interest rate enters in particular also through $1/\varphi_\sigma$. To interpret $1/\varphi_\sigma$ further, we exploit that it is an almost linear function of ρ_1, as a simple numerical investigation for plausible parameter values shows. We, thus, take a linear approximation around the point where ρ_1, the market discount rate, equals the social discount rate $\bar{\rho}$, which we assume to be constant:[14]

$$\frac{1}{\varphi_\sigma} \approx \kappa_0 + \kappa_1 \tilde{\rho}_1, \tag{15}$$

where $\tilde{\rho}_1 = \rho_1 - \bar{\rho}$, $\kappa_0 = 1\Big/\left(1 + \frac{(\beta(1+\bar{\rho}))^\sigma}{1+\bar{\rho}}\right)$, $0 < \kappa_0 < 1$, $\kappa_1 = \frac{1-\sigma}{1+\bar{\rho}} \frac{(\beta(1+\bar{\rho}))^\sigma}{1+\bar{\rho}} \Big/ \left(1 + \frac{(\beta(1+\bar{\rho}))^\sigma}{1+\bar{\rho}}\right)^2$. In reality, because the income effect dominates the substitution effect (i.e., $\sigma < 1$), $\kappa_1 > 0$.[15]

Using our approximation,[16] the primary balance equation, now scaled by GDP, is:

$$\begin{aligned} \frac{T_1 - G_1}{Y_1} &\approx (\kappa_0 + \kappa_1 \tilde{\rho}_1)\left(1 + r_0^{eff} - r_1^n\right) \frac{D_0^t}{Y_1} + \gamma \frac{Y_1 - Y_1^p}{Y_1} \\ &= \kappa_0\left(1 + r_0^{eff}\right)\frac{D_0^t}{Y_1} + \kappa_1 \tilde{\rho}_1 \left(1 + r_0^{eff}\right)\frac{D_0^t}{Y_1} - \kappa_0 r_1^n \frac{D_0^t}{Y_1} - \kappa_1 \tilde{\rho}_1 r_1^n \frac{D_0^t}{Y_1} + \gamma \frac{Y_1 - Y_1^p}{Y_1}. \end{aligned} \tag{16}$$

Hence, the primary balance depends on five determinants. The first, $\left(1 + r_0^{eff}\right) D_0^t/Y_1$, is the debt repayment plus today's interest payment as a share of GDP. Its impact κ_0 reflects what part of the inherited debt (including interest) is absorbed through the primary balance in period 1, while the remainder is shifted to the future. As $\left(\beta(1+\bar{\rho})\right)^\sigma$ will be quite close to unity, the value of κ_0 is largely driven by the social discount rate $\bar{\rho}$. Hence, if governments only spread the inherited debt based on constant social discounting, κ_0 will capture this. A higher $\bar{\rho}$ means a higher κ_0, hence the need for a higher primary balance. This is the result of two causes. One is that, for a debtor government, discounted tax revenue falls more than discounted spending – the wealth effect dominates the income

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[14] The social real discount rate, used for public policy in practice, tends to be constant for a number of years. For example, the US government has kept it constant since 2003 (Council of Economic Advisers, 2017).

[15] For example, $\beta = 0.99$, $\bar{\rho} = 0.03$, and $\sigma = 0.20$ give $\kappa_0 = 0.51$ and $\kappa_1 = 0.19$. Note that in settings with more than two periods, the geometric series φ_σ gets one additional power for each extra period, and one can show that that lowers κ_0 and increases κ_1. For example, in the case of 10 periods, they become 0.11 and 0.37, respectively. When time tends to infinity, the numbers become 0.03 and 0.76, respectively.

[16] Note that for σ close to zero and $\beta(1+\rho_1)$ close to 1, $\varphi_0/\varphi_\sigma \approx 1$ and $\varphi_\sigma^{-1} \approx \kappa_0 + \kappa_1 \tilde{\rho}_1$.

effect. The other is that the substitution effect also increases the current primary balance.[17] The expression for κ_0 also provides a theoretical underpinning of the coefficient on inherited debt in typical fiscal reaction regressions.[18] Estimates of that coefficient are in the interval [0.01,0.10] in most studies; see Checherita-Westphal and Žďárek (2017), meaning that in period 1 only a fraction of inherited debt is saved for.

The second determinant, $\tilde{\rho}_1\left(1+r_0^{eff}\right) D_0^t/Y_1$, says that, if the real interest rate increases, then being a debtor makes the government worse off, so it has to increase the primary balance, in line with $\kappa_1 > 0$. In summary, the theoretical framework suggests that the government smoothes the future interest burden towards today as follows: $\bar{\rho}$ via κ_0 captures the "basic" smoothing, while $\tilde{\rho}_1$ via $\kappa_1\tilde{\rho}_1$ brings in additional smoothing if the interest rate ρ_1 exceeds $\bar{\rho}$. In Section 4.1.1 we study whether in reality governments indeed handle time variation in interest rates in this way.

The third and fourth terms capture the valuation effect discussed above. They share a common driver, $r_1^n \frac{D_0^t}{Y_1}$. A positive value due to $r_1 > r_{0,2}$, so a beneficial valuation effect from non-maturing debt, reduces the need to save and, thus, lowers the primary balance. For given r_1^n, a higher discount rate strengthens its impact $\kappa_0 + \kappa_1\tilde{\rho}_1$, as explained before.

The final determinant of the primary balance is the output gap, albeit relative to actual rather than potential GDP. The model predicts an impact γ of around 0.4, as motivated before. Including the output gap in the regression equation is also in line with Bohn (1998), for example, who includes a business cycle indicator in his regression of the primary surplus ratio on the debt ratio.

3.4 Operationalizing r_1^n

Expression (4) shows that r_1^n depends on $D_{0,2}$ and the corresponding interest rate $r_{0,2}$. These quantities concern maturity-specific parts of the total debt, which are often not directly observed, because the specific data are not available. The effective interest rate in period 1, $r_1^{eff} = \frac{r_{0,2}D_{0,2}+r_1D_{1,2}}{D_{0,2}+D_{1,2}}$, also contains both parts and is a weighted average of the interest rates on non-maturing debt and newly-issued debt in period 1, with the weights given by the fractions of these components of the

[17] One can rephrase this notion of discounting in terms of smoothing interest payments, as follows. If the actual real interest rate ρ_1 equals $\bar{\rho}$, a higher $\bar{\rho}$ means larger interest payments in period 2, and that burden is smoothed to period 1 in the form of a higher primary balance.

[18] The first determinant, $\left(1+r_0^{eff}\right) D_0^t/Y_1$, is analogous to the usual term $\left(1+r_0^{eff}-g_1\right) D_0^t/Y_0$ found in the literature. Note that we do not have $-g_1$ explicitly in our equation, but we do account for it implicitly. Economic growth matters via $D_0^t/Y_1 = (D_0^t/Y_0)\cdot(Y_0/Y_1)$. Approximating Y_0/Y_1 by $1-g_1$, multiplying this by $\left(1+r_0^{eff}\right)$, and leaving out the second-order term, would bring in the usual $-g_1$ term. We prefer to avoid both approximations, where possible.

total debt outstanding after period 1 decisions have been made. New debt is issued at the new interest rate, so in the difference between r_1 and r_1^{eff} the rate on the new debt drops out. This is explicit in

$$r_1 - r_1^{eff} = \frac{(r_1 - r_{0,2})D_{0,2} + (r_1 - r_1)D_{1,2}}{D_1^t} = \left(r_1 - r_{0,2}\right)\frac{D_{0,2}}{D_1^t} = r_1^n(1 + r_1)\, D_0^t/D_1^t, \tag{17}$$

so that we can derive r_1^n. Hence, without having observations on maturity-specific interest rates and debt, such as $r_{0,2}$ and $D_{0,2}$, one can use data on r_1^{eff} to measure r_1^n.

3.5 The regression model and method

Equation (16) motivates our empirical framework:

$$\begin{aligned} pbal_{it} = \left(\lambda_0 + \lambda_1 debt_{i,t-1}\right) intrisen_{it} + \lambda_2 debtint_{i,t-1} \\ + \lambda_3 gap_{it} + \lambda_4 growth_{it} + \delta pbal_{i,t-1} + other_{it} + \alpha_i + \theta_t + \varepsilon_{it}. \end{aligned} \tag{18}$$

In Section 4 we will show that the estimates of this baseline regression specification are robust to several extensions, and the current section motivates some simplifications used. Here, we also discuss the variables entering our regression equations, linking them to their theoretical counterparts.

Variable $pbal_{it}$ is the primary balance over GDP in the current period *t*, not cyclically adjusted. Its theoretical counterpart is $\frac{T_1 - G_1}{Y_1}$. Variable $intrisen_{it}$ corresponds to r_1^n in the theory. Its (ceteris paribus) impact on $pbal_{it}$ will be the focus of the paper. In this regard, our baseline regression extends formulations in the pioneering work of Bohn (1998) and others. Because $(1 + r_1)$ is close to one, from the theory we have $r_1 - r_1^{eff} \approx r_1^n\, D_0^t/D_1^t$. Hence, we take[19]

$$intrisen_{it} = (intnew_{it} - inteff_{it})\frac{Debt_{it}}{Debt_{i,t-1}}, \tag{19}$$

where $intnew_{it}$ is the interest rate on new debt issued in *t*, representing the marginal interest rate r_1 in the theory, and $inteff_{it}$ is the effective interest rate on debt at *t*, the counterpart of r_1^{eff} in the theory. It is calculated as $inteff_{it} = Intpayment_{i,t+1}/Debt_{it}$, where $Intpayment_{i,t+1}$ is the next-period *t*+1 nominal interest payment. Further, $Debt_{it}$ is the nominal debt at the end of the current period *t*, as opposed to $debt_{it}$, which is $Debt_{it}$ divided by nominal GDP of the same period. An alternative measure of r_1^n is the interest rate on new debt relative to the interest rate on existing debt:

$$intrise_{it} = intnew_{it} - inteff_{i,t-1}. \tag{20}$$

[19] Dividing the right-hand side by $(1 + intnew_{it})$, as suggested by the theory, yields virtually the same results empirically, so we abstract from this complication.

Both $intrisen_{it}$ and $intrise_{it}$ relate the marginal rate to an effective rate on outstanding debt, the current and lagged effective rate, respectively. We will thus, in short and for lack of better terminology, refer to both $intrisen_{it}$ and $intrise_{it}$ as the "difference between the marginal and effective rate".

The interaction $debt_{i,t-1} \cdot intrisen_{it}$ represents $r_1^n \frac{D_0^t}{Y_1}$ in the expression for the primary balance from the theory. Note that the latter divides by Y_1. Further, we define $debtint_{i,t-1} = debt_{i,t-1} + intpayment_{it}$, which is outstanding debt at the end of period *t-1* plus the period-*t* interest payment, both over GDP of period *t-1*. The corresponding variable in the theory is $\left(1 + r_0^{eff}\right) D_0^t / Y_1$. Dividing by GDP of *t-1* instead of *t*, where the latter is suggested by Y_1 in the theory, has a negligible impact.[20,21] We prefer using lagged GDP to scale lagged debt in the empirical analysis, because that is simpler and follows common practice. In fact, for all ratio variables we use the commonly used timing of the denominator, unless explicitly stated otherwise.

Variable gap_{it} is the output gap (according to IMF), representing $\left(Y_1 - Y_1^p\right)/Y_1^p$ from the theory. We take potential GDP as denominator, as usual. Dividing instead by actual GDP has a negligible impact. Next, variable $growth_{it}$ is the real GDP growth rate from period *t*-1 to period *t*. Even though the theory does not imply a role for growth, we include it together with gap_{it} in the regression, because it is difficult to measure the true output gap, and the combination of gap_{it} and $growth_{it}$ can help to capture it more accurately. An alternative is to leave out gap_{it} and just use $growth_{it}$, but that yields similar results. In addition, even though this is not contained in our theory, governments may actually use growth in their policy decisions on the primary balance. For growth, we simply use the relative change of real GDP. For example, demeaning the growth rate, which would yield a better proxy for the business cycle, will not affect the estimates as we include country- and year-fixed effects.

Finally, $other_{it}$ is a linear combination of other potential primary balance determinants. For example, to be able to empirically account for all terms in our theoretical relationship (16), we need a measure of $\tilde{\rho}_1$, so that we can account for the terms $\tilde{\rho}_1\left(1 + r_0^{eff}\right)\frac{D_0^t}{Y_0}$ and $\tilde{\rho}_1 r_1^n \frac{D_0^t}{Y_0}$ in the theory. Therefore, we

[20] Debt sustainability analyses typically exhibit $1 + r - g$, that is, one plus the interest minus growth rate. In that notation, we only have $1 + r$, where r is our r_0^{eff}. We do not need the $-g$ part in our theory, because we there divide by Y_1 instead of the typical Y_0. In the empirics, however, we divide by GDP of *t-1* instead of *t*, so here we essentially ignore a multiplication by Y_0/Y_1, that is $1 - g$. The impact of this simplification is negligible for our estimates, because the horizon in the regression model is just one year, from $t - 1$ to t, as opposed to the infinite horizon often used in a debt sustainability analysis.

[21] From a purely econometric perspective, in a model with interaction $debt_{i,t-1} \cdot intrisen_{it}$ one would expect also $debt_{i,t-1}$ itself to be included on the right-hand side. However, we already have $debtint_{i,t-1}$ in (18), and adding interest payments as a separate regressor leaves the estimates virtually unchanged, as we will show later in Table 4, Column 6.

define $intreal_{it} = intnew_{it} - expinfl_{it}$ as the real interest rate in period *t*, where $expinfl_{it}$ is the expectation in *t* of inflation between *t* and *t+5*.

As a summary of the above exposition, the second and third columns of Table 1 map the variables in the theory to those used in the regressions.

Table 1: Correspondence between variables in theory and empirics, and data sources

Variable description	Variable in		
	Theory	Regression	Data source
Primary balance over GDP	$(T_1 - G_1)/Y_1$	$pbal_{it}$	*ggxonlb / ngdp*
End-of-period debt over GDP, lag	D_0^t/Y_0	$debt_{i,t-1}$	*ggxwdg-1 / ngdp-1*
Effective interest rate, lag	r_0^{eff}	$inteff_{i,t-1}$	*ggei / ggxwdg-1*
Interest payment over lagged GDP	$r_0^{eff}\, D_0^t/Y_0$	$intpayment_{it}$	---
Debt including interest over GDP, lag	$\left(1 + r_0^{eff}\right) D_0^t/Y_0$	$debtint_{i,t-1}$	---
Interest rate on new debt (marginal rate)	r_1	$intnew_{it}$	IFS: *FIGB_PA/100*
Marginal minus effective rate	$\approx r_1^n$	$intrisen_{it}$	---
Marginal minus lagged effective rate	$\approx r_1^n$	$intrise_{it}$	---
(Expected) inflation	π_1	$expinfl_{it}$	WEOhistorical.xlsx
Real interest rate	ρ_1	$intreal_{it}$	---
Output gap	$\left(Y_1 - Y_1^p\right)/Y_1^p$	gap_{it}	*ngap_npgdp/100*
Real GDP growth	---	$growth_{it}$	change in *log(ngdp_r)*
Government revenue	T_1/Y_1	rev_{it}	*ggr / ngdp*
Average residual maturity, lag	$\frac{D_{-1,1}+D_{0,1}+2D_{0,2}}{D_{-1,1}+D_{0,1}+D_{0,2}}$	$maturity_{i,t-1}$	OECD and ECB

Notes: In the final column, data source variable names refer to the IMF World Economic Outlook (WEO), unless stated otherwise. The "WEOhistorical.xlsx" and "OECD and ECB" entries are discussed in Section 3.6. Finally, "---" in the "Theory" column indicates that we have no income growth variable in our theoretical framework.

Equation (18) contains country-fixed effects α_i to correct for all year-invariant determinants (such as geographical variables) and to capture unobserved heterogeneity across countries, and year-fixed effects θ_t for all country-invariant characteristics to control for global shocks. We assume the error term ε_{it} has expectation zero conditional on past information, which is motivated by the inclusion of the lagged dependent variable $pbal_{i,t-1}$ and the fact that the estimated impacts of higher-order lags of the dependent variable turn out to be insignificant. We emphasize that we allow for a non-zero correlation of ε_{it} with the contemporaneous variables $intrisen_{i,t}$, gap_{it}, and $growth_{it}$, even though this endogeneity is considered hard to handle in the literature (e.g., Mauro et al., 2015). We also allow for heteroscedasticity, but work under the assumption that ε_{it} is cross-sectionally uncorrelated, partly motivated by the year-fixed effects.

We estimate the model by two-stage least squares (2SLS) with the heteroscedasticity-robust (White) covariance matrix, calculated by Stata. Instruments are based on observations prior to year t. More specifically, they are $intrisen_{i,t-1}$, $debt_{i,t-1}intrisen_{i,t-1}$, $gap_{i,t-1}$, $gap_{i,t-2}$, and $growth_{i,t-1}$. We ignore potential correlation between ε_{it} and future values of the instruments, because the number of observations over time in our data will be substantial (25 on average), so that possible estimation bias is expected to be mild. Alternative estimators yield similar results, as we will show in Section 4.2.2.

3.6 Data

The main dataset we use is the IMF World Economic Outlook (WEO) of April 2019, to which we add some data from other sources, as set out below. We take the countries indicated as "advanced" by the IMF and that have observations for all the variables in our baseline regression. That yields a sample of N=25 countries: Australia, Austria, Belgium, Canada, Cyprus, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, Luxembourg, Malta, Netherlands, New Zealand, Portugal, Slovak Republic, Slovenia, Spain, Sweden, United Kingdom, and the United States.

The number of years available varies across the countries. The earliest year with data is 1982, while the shortest time series start in 2003. To avoid using forecasts instead of realizations, the final observation for all countries concerns 2017, though for Greece we exclude the data from 2010 onwards, given the exceptional circumstances there. On average, the number of observations per country is about 25.

The final column of Table 1 lists the original data sources of the variables in our regressions and shows how we measure these variables. For example, the primary balance $pbal_{it}$ is *ggxonlb* divided by *ngdp*, where *ggxonlb* is the WEO-code for the primary balance in nominal terms, and *ngdp* is nominal GDP. We compute $Debt_{it}$ as general government gross debt, *ggxwdg*. Interest payment $intpayment_{it}$ is the general government interest expense, *ggei*, divided by lagged *ngdp*. The output gap gap_{it} is *ngap_npgdp/100*, and $growth_{it}$ is the change in the logarithm of *ngdp_r*, where *ngdp_r* is real GDP. Government revenue rev_{it} is *ggr/ngdp*, where *ggr* is nominal government revenues.

The interest rate on new debt, $intnew_{it}$, is from the IMF International Financial Statistics. It is *FIGB_PA/100* and represents the yield to maturity of government bonds. Expected inflation, $expinfl_{it}$, is based on https://www.imf.org/external/pubs/ft/weo/data/WEOhistorical.xlsx, which collects IMF CPI inflation forecasts from each WEO publication since 1990. Each year has a Spring and a Fall forecast, and we take the average between the two values. The forecasts concern one, two, three, four and five years ahead. We use the average across those five horizons as $expinfl_{it}$. Finally, the average residual maturity in years, $maturity_{it}$, has been built from various sources, mainly the

OECD and ECB.[22] The detailed construction is available upon request from the authors.

4. Empirical results

This section presents and discusses the estimation results for our empirical specification (18) and variants on it. Specifically, in Section 4.1 we present the estimates of our baseline regression equation. Section 4.2 supports the quality of the instruments and shows that the key results are robust to using alternative estimators. As the regression relies on some simplifications compared to the theory-based equation (16), we motivate these simplifications empirically in Section 4.3. Here, we also show the robustness of our results to model extensions that go beyond our theory.

4.1 Estimates of standard specification

4.1.1 The real interest rate

Table 2 reports estimates of variants of regression equation (18). Column (1) estimates a specification that represents the literature. Specifically, by leaving $intrisen_{it}$ out of the regression the implicit assumption is that all debt matures in year t. In addition, we include the real interest rate $intreal_{it}$, as in Mauro et al. (2015). The latter enters directly and via its interaction with $debtint_{i,t-1}$, the lag of debt including the interest payment on it. The real interest rate has a statistically significantly positive impact on the primary balance, and the impact is stronger for more indebted governments. Also, $debtint_{i,t-1}$ enters the regression directly with a highly significant positive coefficient. The standard interpretation would be that governments react to higher debt-servicing costs by raising the primary balance in order to ensure the sustainability of the public finances. As expected, the lagged primary

[22] From the OECD we take the average term to maturity of outstanding marketable debt in OECD countries, as reported in the document "Sovereign Borrowing Outlook for OECD Countries 2021". Although that document only gives the values for a few years, the OECD has been so kind to send us the full underlying series from 2007 onwards. To get earlier observations, we link the series to the OECD series called "average term to maturity for total debt", discontinued as of 2011 and obtained from the OECD website.
From the ECB Statistical Data Warehouse we use the average residual maturity of Maastricht debt, coded as *GFS.A.N.XX.W0.S13.S1.C.L._Z.GD.TT._Z.YR._T.F.V.A1._T*, where *XX* is the country indicator. This series starts in 1995. To get earlier values, we link it to the ECB series called "average residual maturity of general government debt", coded as *GST.A.XX.N.B0X13.MAV.B1300.SA.Y*, and discontinued as of 2014.
To extend the series for some countries, we have used specific additional sources, namely the OECD "Central Government Debt: Statistical Yearbook" of 2000 and 2010; the OECD "Sovereign Borrowing Outlook" of 2013 and 2014; the gross market value weighted average maturity of UK government debt, gilts only, kindly provided by the UK Debt Management Office; and the Swiss Federal Treasury "Activity Report 2020".
We have filled 39 remaining missing values at the beginning of the time series of six countries by back-casting, based on an AR(1) model with a trend for log maturity, motivated by the gradual movements in the variable.

balance enters with a highly significant positive coefficient, while a favourable business cycle has a positive effect on the primary balance. Noticeably, the output gap and growth enter simultaneously with significant positive coefficients. Interpretation of the size of the coefficient estimates will be deferred to the discussion of the baseline estimates in Column (3) of Table 2.

Table 2: Estimation results for primary-balance model (18)

	(1) Only real interest rate	(2) Real interest rate and $intrisen_{it}$	(3) Baseline: only $intrisen_{it}$	(4) $intrise_{it}$ instead of $intrisen_{it}$
$intreal_{it}$	0.25***	-0.20		
	(0.092)	(0.26)		
$—*debtint_{i,t-1}$	0.25***	-0.12		
	(0.081)	(0.13)		
$intrisen_{it}$		0.35**	0.32***	
		(0.18)	(0.071)	
$—*debt_{i,t-1}$		1.06**	0.51***	
		(0.47)	(0.15)	
$intrise_{it}$				0.26***
				(0.056)
$—*debt_{i,t-1}$				0.47***
				(0.14)
$pbal_{i,t-1}$	0.62***	0.60***	0.63***	0.64***
	(0.079)	(0.073)	(0.071)	(0.074)
$debtint_{i,t-1}$	0.024***	0.031***	0.028***	0.026***
	(0.005)	(0.006)	(0.005)	(0.005)
gap_{it}	0.25***	0.25***	0.26***	0.25***
	(0.071)	(0.069)	(0.068)	(0.064)
$growth_{it}$	0.40***	0.34***	0.34***	0.36***
	(0.11)	(0.10)	(0.11)	(0.11)
Observations	586	568	611	632
Hansen J-test p-value	0.19	0.41	0.49	0.40
Kleibergen-Paap Wald test	30.3	15.6	13.1	27.0
$intrisen_{it}$ & $intrisen_{it}debt_{i,t-1}$ p-value	—	0.048**	0.000***	0.000***

Notes: standard errors are in parentheses. They are robust to heteroscedasticity based on the White covariance matrix. *** indicates a p-value<0.01, ** p<0.05, * p<0.10. The estimated model for the primary balance $pbal_{it}$ is given by equation (18). It has country- and year-fixed effects (included in all specifications). All estimates of λ_0 are presented as the impact of $intrisen_{it}$ at 60% debt, which is close to the average value in our sample. The estimate of the coefficient on $debtint_{i,t-1}$ in Columns (1) and (2) gives its impact measured at $intreal_{it} = 0.03$, which is close to the average real interest rate in our sample. We estimate by 2SLS, where the instruments for $intrisen_{it}$, $intrisen_{it}debt_{i,t-1}$, gap_{it}, and $growth_{it}$ are $intrisen_{i,t-1}$, $intrisen_{i,t-1}debt_{i,t-1}$, $gap_{i,t-1}$, $gap_{i,t-2}$, and $growth_{i,t-1}$. That is, all other regressors are treated as predetermined. This includes $intreal_{it}$ and $intrise_{it}$, even though a negative shock to $pbal_{it}$ increases debt, which may bid up the interest rate on new debt. Still, Stata's xtivreg2 endogeneity test, using the difference between two Hansen J-statistics, gives p-values of 0.69 and 0.94, respectively, suggesting that treating both regressors as predetermined is warranted. The non-rejections may result from including $debtint_{i,t-1}$, thus controlling to a large extent for correlation between the level of debt and the interest rate. "Hansen J-test" is Hansen's J-statistic of overidentifying restrictions, robust to heteroscedasticity, to test that the instruments are uncorrelated with the error term at time t. "Kleibergen-Paap Wald test" is the Kleibergen and Paap (2006) Wald statistic, robust to heteroscedasticity, to quantify the strength of the instruments. "$intrisen_{it}$ & $intrisen_{it}debt_{i,t-1}$" denotes the Wald test that both $intrisen_{it}$ and $intrisen_{it}debt_{i,t-1}$ have no impact, though in Column (4) the test concerns the hypothesis that both $intrise_{it}$ and $intrise_{it}debt_{i,t-1}$ have no impact.

In Column (2) we add our new term, $intrisen_{it}$. The results for the lagged primary balance, $debtint_{i,t-1}$, the output gap and growth remain similar. However, the statistical significance of the coefficient of $intreal_{it}$ and its interaction with $debtint_{i,t-1}$ disappears. Interactions of $intreal_{it}$ with other variables motivated by the theory, that is, gap_{it}, $growth_{it}$ and tax revenues, corroborate the insignificance of the real interest rate, while also the nominal interest rate has an insignificant impact (estimates not reported). In contrast, $intrisen_{it}$ has a positive impact on $pbal_{it}$, and its impact increases with $debt_{i,t-1}$. The joint test also rejects, with a p-value of 0.048, even though the test is conservative.[23] Hence, the fact that in reality not all debt matures in year t matters for the interest rate impact on the primary balance. Apparently, the significance of $intreal_{it}$ in Column (1) is spurious; it picks up some relevance of the interest rate that is more accurately represented by $intrisen_{it}$. The direct impact of the current (i.e., marginal) interest rate level is only in deviation from the effective interest rate, while its indirect impact runs via the debt cum interest payments term.

4.1.2 Relationship with the theory: the effective interest rate

On the face of it, our empirical findings seem to deviate in two ways from our simple and standard theoretical framework. The first apparent deviation concerns the impact of the marginal interest rate. The second concerns the role of the valuation effect.

Let us turn first to the impact of the marginal interest rate. In our theory, it is its level that matters, while the estimates reveal that the marginal rate matters in deviation from the effective rate. To find out what exactly drives this difference from the theory, we go step-by-step from Column (1) to (2), where in each step we make just one change to the regression specification. The key step turns out to be the addition of the effective interest rate. More specifically, if we estimate the specification of Column (1), but with the nominal instead of the real interest rate, the coefficient estimate of the nominal interest rate $intnew_{it}$ becomes 0.21 (standard error 0.066) and of its interaction with $debtint_{i,t-1}$ 0.21 (0.067). This is similar to what is found for the real interest rate in Column (1). Keeping $intnew_{it}$ and its interaction with $debtint_{i,t-1}$ in, we then add $inteff_{i,t-1}$ and its interaction with $debtint_{i,t-1}$. We do this in the form of $intnew_{it} - inteff_{i,t-1}$ and $inteff_{i,t-1}$ to reduce multicollinearity. The specification now attains the format in Column (2), the encompassing specification. (Whether we take $inteff_{i,t-1}$ or $inteff_{it}$, as in Column (2), does not matter). The

[23] The reason for the conservativeness is as follows. At $debt_{i,t-1} = 0$, interest rates cannot be relevant. This implies that the impact of $intrisen_{it}$ at some representative debt ratio should have the same sign as the slope λ_1. Our estimates fulfil this requirement, as both $\hat{\lambda}_0$ and $\hat{\lambda}_1$ are positive. However, the usual Wald test we use does not exploit it, making the p-value higher. Recall that $\hat{\lambda}_0$ is presented as the impact at $debt_{i,t-1} = 60\%$.

estimate for $intnew_{it} - inteff_{i,t-1}$ itself becomes 0.17 (0.073) and for the interaction with $debtint_{i,t-1}$ 0.57 (0.21), and for $inteff_{i,t-1}$ itself 0.02 (0.12) and for the interaction 0.092 (0.077). This confirms that the data want a role for the marginal minus effective rate, and that there is no evidence of a separate role for the level of the marginal interest rate. Note that substituting $inteff_{i,t-1}$ in $intnew_{it} - inteff_{i,t-1}$ by the lag of the marginal rate, $intnew_{i,t-1}$, gives no indication that the latter matters, so it is really the effective nature of the interest rate that matters.

The importance of the effective interest rate leads us to return to our theoretical framework to understand the economic consequences of this finding. Consider the smoothing of the future interest burden through today's primary balance, discussed in Section 3.2 and revisited here in nominal terms. Rewrite budget constraint (5) into

$$T_2 - G_2 = (1 + r_1)\left[\left(1 + r_0^{eff} - r_1^n\right)D_0^t - (T_1 - G_1)\right]. \tag{21}$$

That is, the theory says that a high interest rate r_1 implies a high debt-servicing burden next period, which induces the government to save a lot next period (high $T_2 - G_2$), part of which is then smoothed through high savings today (high $T_1 - G_1$). In the step-by-step regression analysis just presented, the $intnew_{it}$ regressor captures this smoothing mechanism. However, we have just shown that not $intnew_{it}$ but $intnew_{it} - inteff_{i,t-1}$ better describes how governments actually save. So the data suggest that the smoothing mechanism that governments actually use is as follows: a high interest rate r_1 *relative to an effective rate such as* r_0^{eff} implies a relatively high debt-servicing burden next period, which induces the government to save much next period (high $T_2 - G_2$), part of which is then smoothed through high savings today (high $T_1 - G_1$). So our theoretical framework and our empirical results are consistent in the sense that both embed the mechanism of smoothing the future interest burden to the current period, but they differ regarding how the government exactly responds: in the theory, it is the level of the new value of r_1 by itself which is relevant, while the estimates suggest that what matters is its level relative to the effective interest rate on outstanding debt.

Expression (16) for the primary balance in our theory provides further insights. It contains

$$\begin{gathered}(\kappa_0 + \kappa_1\tilde{\rho}_1)\left(1 + r_0^{eff}\right)D_0^t = \\ \kappa_0\left(1 + r_0^{eff}\right)D_0^t + \kappa_1\tilde{r}_0^{eff}\left(1 + r_0^{eff}\right)D_0^t + \kappa_2\left(r_1 - r_0^{eff}\right)\left(1 + r_0^{eff}\right)D_0^t,\end{gathered} \tag{22}$$

where we assume zero inflation for simplicity, $\tilde{r}_0^{eff} = r_0^{eff} - \bar{\rho}$, and the theory restricts $\kappa_1 = \kappa_2$. The right-hand side makes explicit that $\kappa_0\left(1 + r_0^{eff}\right)D_0^t$ is the primary balance if the interest gaps $\tilde{r}_0^{eff}$ and $r_1 - r_0^{eff}$ are zero (and the other determinants in (16) are also zero). This level of the primary balance can be viewed as the "base level", which is key for solvency. This way, a high interest rate level r_0^{eff} causes the primary balance to be high, albeit to a limited extent, as $\kappa_0 < 1$ suffices for solvency. The interest gaps add to this base level, as follows. Theory says that the interest rate level features not only via r_0^{eff}, but a second time via $\tilde{r}_0^{eff}$, that is, $r_1 - \bar{\rho}$, accounting for $\kappa_1 = \kappa_2$. However, the empirical estimates provide no evidence of that. Instead, actual government behavior adds the relative term, driven by $r_1 - r_0^{eff}$, which captures that governments save more than the base level if the marginal interest rate exceeds the effective rate. In other words, the difference $r_1 - r_0^{eff}$ matters in addition to the interest rate level r_0^{eff} in the base level defined above, where the difference has impact κ_2 on the primary balance, while the level has impact κ_0.

Policy documentation provides additional support for our empirical findings by documenting the use of a constant discount rate and the smoothing of the budgetary consequences of interest rate changes. First, in the US the discount rate for cost-benefit analysis of Federal policies has been constant since 2003 (see Council of Economic Advisers, 2017). In the notation of Section 3.2, if the government uses a constant discount rate, then it deploys the constant rate $\bar{\rho}$ instead of the time-varying rate ρ_1 and, hence, $\kappa_1 = 0$ (see equation (16)). This is consistent with our insignificant estimate for the impact of $intnew_{it}$.

Second, the Dutch Ministry of Finance (2022, p.73), for example, argues that the risen interest rate leads to higher future interest payments and requires compensation measures to be taken already now, which means improving the current primary balance. The Ministry here relates future interest payments to the current one. The current payment is governed by the effective rate, not some time-invariant constant rate like $\bar{\rho}$. Our $intrisen_{it}$ regressor captures the difference between future and current interest payments. Hence, our finding that a positive $intrisen_{it}$ increases $pbal_{it}$ is in line with the policy advice by the Dutch Ministry of Finance (2022) to increase the current primary balance in response to a rise in the interest rate on new debt relative to the effective rate on existing debt.

In summary, our regression model provides a framework to do justice to the use of a constant social discount factor in benefit-cost analysis, as well as the impact of time variation in interest payments on the primary balance.[24] The findings are also observationally in line with the fact that some countries that saw a decline in the interest rates due to joining the Euro area failed to improve primary balances to an extent sufficient to absorb future shocks: in our regression, $intrisen_{it} < 0$ yields a low value of

[24] The use of a constant $\bar{\rho}$ and time variation in ρ_1 seem to be conflicting. Our regression model may be seen as a compromise superior to sticking to the theory. The constant $\bar{\rho}$ advocated by the Council of Economic Advisers (2017) eliminates time-varying smoothing, but our $intrisen_{it}$ regressor brings smoothing back into the policy framework.

$pbal_{it}$. At the moment of writing, we observe that $intrisen_{it} > 0$, which based on our regression estimates should result into a high $pbal_{it}$.

The second apparent deviation of the estimates from what our theoretical framework predicts concerns the valuation effect. While according to our theory (equation (16)) the primary balance should fall in response to a rise in $intrisen_{it}$, our estimates show that an increase in $intrisen_{it}$ instead raises the primary balance. A potential explanation is that the smoothing effect described above, i.e., governments respond to a rise in expected future interest payments by saving more now, dominates the negative valuation effect on the primary balance, and that the estimated effect of $intrisen_{it}$ captures the positive net impact.

Our research question concerns the *total* impact of the interest rate on the primary balance. To answer this question, having a combined estimate of the positive smoothing effect and the negative valuation effect suffices and enhances the model´s parsimony. Still, we can use our theory to form an idea about the magnitude of the valuation effect. Subsection 3.3 quantifies the latter as $-(\kappa_0+\kappa_1\tilde{\rho}_1)r_1^n\frac{D_0^t}{Y_1}$. Because κ_0 and κ_1 are also the impacts of $\left(1+r_0^{eff}\right)\frac{D_0^t}{Y_1}$ and $\tilde{\rho}_1\left(1+r_0^{eff}\right)\frac{D_0^t}{Y_1}$, respectively, which are in our empirical analysis represented by $debtint_{i,t-1}$ and $intreal_{it} \cdot debtint_{i,t-1}$, respectively, Column (2) gives estimates $\hat{\kappa}_0 = 0.031$ (standard error 0.006) and $\hat{\kappa}_1 = -0.12$ (0.13). Hence, the coefficient $(\kappa_0+\kappa_1\tilde{\rho}_1)$ of the valuation effect that we can calculate from these estimates is much smaller than the net impact 1.06 (0.47) of $intrisen_{it} \cdot debt_{i,t-1}$ and, hence, this net impact is a good approximation of the size of the aforementioned positive smoothing effect itself.

The larger effect of smoothing on the primary balance may explain why this effect receives more attention in policy documentation than the valuation effect. Still, from the Dutch Ministry of Finance (2022, p.73) we can infer that the valuation effect seems to matter implicitly for policy in practice. Specifically, the Ministry calculates the impact of the increase in the marginal interest rate on the interest burdens in the upcoming years. Because the inherited debt not maturing this year will only be gradually rolled over in the upcoming years, the increase in the marginal interest rate only gradually raises the interest payment burdens in the years to come. If all debt were to mature this year, this interest burden would have increased by the full amount already next year. The benefit from the delayed increase in the interest burden caused by the gradual rolling over of debt, constitutes the valuation effect, which policy makers thus account for in their policy decisions.

In summary, our empirical findings seem to differ from the theoretical framework in two ways. Both deviations may, in fact, be driven by the difference between the marginal and effective interest rate. First, governments respond to this difference rather than merely the marginal interest rate by itself in smoothing the upcoming interest burden. Second, the failure to find explicit evidence of the valuation effect may be due to this effect being dominated by the desire to smooth interest payments.

Our analysis may be particularly relevant in view of projected budgetary developments in major economic blocks in the coming decade. Projections by the Congressional Budget Office (2023, Table 1-2) over the coming decade suggest that rising interest rates, and the resulting rising expenses as a share of GDP, coincide with a falling primary balance over the projection period.[25] Projections by the European Commission (2023a) show a qualitatively similar pattern for both the EU and the euro area: after an initial rise of the structural primary balance of the years 2022 – 2024, essentially the result of an unwinding of the Covid-19 measures, the structural primary balance is set to fall in each of the ensuing years, while at the same time interest expenditures are gradually rising.[26]

4.1.3 Baseline estimates

From now on we drop the real interest rate entirely from the regression equation. This yields the estimates reported in Column (3). Substituting $intrisen_{it}$ by the alternative measure $intrise_{it}$ yields similar results, as Column (4) shows. Because $intrisen_{it}$ is closer to the r_1^n underlying the valuation effect in our theory, we view (3) as the baseline result. The coefficient estimates for $intrisen_{it}$ ($\hat{\lambda}_0 = 0.32$) and its interaction with $debt_{i,t-1}$ ($\hat{\lambda}_1 = 0.51$) are again positive and significantly different from zero. Recall that λ_0 is the impact at $debt_{i,t-1} = 60\%$, so the estimated impact at zero debt is 0.01 (= 0.32 – 0.51*0.60). This is close to zero, as expected.

To interpret the magnitude of the estimates, we estimate the long-run effects, such as $\lambda_0/(1-\delta)$. The estimates are 0.86 (0.25) for $intrisen_{it}$, and 1.37 (0.53) for $intrisen_{it} \cdot debt_{i,t-1}$. Now use the definition of $intrisen_{it}$ in (19) and assume for simplicity that outstanding debt is constant, i.e. $Debt_{it} = Debt_{i,t-1}$. Suppose that the debt-to-GDP ratio is 60%, then an increase in the interest rate on new debt by 1 percentage-point induces the government to raise the primary balance by 0.86 percentage-point (keeping the effective rate constant). When the debt-to-GDP ratio is 100%, the

[25] More precisely, the federal primary balance as a fraction of GDP is set to drop in 2024, despite a projected increase in interest expenditures. After that, the projected primary balance will increase until 2028, the reason being that a number of individual income tax provisions of the Tax Act of 2017 expire. In the years after 2028, the primary balance is set to decrease each year with interest expenditures simultaneously rising. Overall, net interest spending rises from 2.4% of GDP in 2023 to 3.6% of GDP in 2033, while the primary balance decreases from -2.9% of GDP in 2023 to -3.2% of GDP in 2033.

[26] See Appendix Table A7.8 for the EU: over the years 2025 – 2033 the structural primary balance drops from -1.2% to -2.0% of GDP and interest expenditures rise from 1.8 % to 2.2% of GDP.

government raises the primary balance by 1.40 percentage-point.[27] Roughly speaking, across the full sample a 1 percentage-point higher interest rate leads to an about 1 percentage-point higher primary balance.

What do these findings say about debt sustainability? If $intnew_{it}$ rises permanently by one percentage point, then over time $inteff_{it}$ will rise by one percentage point as all existing debt gets rolled over and, hence, $intrisen_{it}$ will fall to zero – see (19). By merely considering $intrisen_{it}$, the primary balance would fall to its original level before the interest hike, while at the same time the interest payments on outstanding debt have gone up permanently. This raises the question whether a rise in the marginal interest rate undermines public debt sustainability. This is not the case: the higher interest payments raise $debtint_{i,t-1}$, to which the primary balance responds positively. Hence, with $intrisen_{it}$ having returned to zero, we are back in the empirical standard setting, where the significantly positive estimate of the coefficient on $debtint_{i,t-1}$ ensures sustainability (e.g., Bohn, 1998).

The coefficient estimates of the other variables are in line with our theory. For gap_{it} we again find a significantly positive impact on the primary balance. The long-run estimate 0.71 (0.21) yields a confidence interval that contains the value 0.4 we expected for γ in Section 3.1, providing further credibility to our results. For $growth_{it}$ we also find a significantly positive impact again. The long-run estimate is 0.93 (0.40).

4.2 Motivating the estimation approach

Before showing the robustness of the baseline results by extending the baseline model in Section 4.3, we study the baseline specification in more detail. In the first part of this subsection, we address instrument validity. In the literature, reverse causality is considered hard to handle, so our instrument analysis to deal with reverse causality is potentially helpful also for subsequent research. The second part of this subsection explores how our results stack up to the use of other estimators.

4.2.1 Instrument validity

We first provide several signals supporting that the instruments are uncorrelated with the error ε_{it}. One signal is the insignificant Hansen J-statistic, with a *p*-value of 0.49 for our baseline specification in Column (3) of Table 2.

[27] Calculated as 0.32/(1-0.63)+0.51/(1-0.63)*0.4.

This specification uses $intrisen_{i,t-1}$ as instrument. To further study whether this instrument is uncorrelated with the error at time t (predetermined), we substitute the regressor $intrisen_{it}$ by the alternative measure $intrise_{it}$. Now, the effective interest rate is only included with a lag and the $Debt_{it}/Debt_{i,t-1}$-ratio is no longer present, so there can be no reverse causality via these channels. Indeed, as shown in the note to Table 2, the data support treating $intrise_{it}$ as predetermined. The estimates are reported in Column (4) of Table 2. They are close to the baseline results in Column (3), which supports the way we have handled the potential reverse causality regarding $intrisen_{it}$.

Finally, we add $intrise_{i,t-1}$ and its interaction with $debt_{i,t-1}$ as regressors to the model of Column (4). The coefficient estimates for both (not reported) are near zero and far from being significant. This supports the assumption that $intrise_{i,t-1}$ is uncorrelated with the error in the original regression, the one underlying Column (4). In total, the data support treating both $intrise_{it}$ and $intrise_{i,t-1}$ as predetermined. This supports treating their components $inteff_{i,t-1}$ and $intnew_{i,t-1}$ as predetermined. These components are the main determinants of $intrisen_{i,t-1}$, providing further support for our baseline assumption that $intrisen_{i,t-1}$ is predetermined.

Next, we study instrument strength. A first impression, though superficial, comes from some sample correlations between instruments and regressors, such as $corr(intrisen_{it}, intrisen_{i,t-1}) = 0.88$, $corr(gap_{it}, gap_{i,t-1}) = 0.70$, $corr(gap_{it}, gap_{i,t-2}) = 0.34$, and $corr(growth_{it}, growth_{i,t-1}) = 0.47$. These signal no evident weak-instrument problem. Still, correlations provide an imperfect picture. The rest of this section provides a more accurate analysis to show that our results indeed do not seem to be driven by instrument weakness.

While the Cragg-Donald (1993) statistic of 78.57 exceeds the Stock and Yogo (2005) 5% critical value of 7.03, we have to realize that the Cragg-Donald test is not robust to heteroscedasticity. The Kleibergen and Paap (2006) Wald statistic, which is robust to heteroscedasticity, is 13.12 in Column (3) of Table 2. Comparing this with the critical value of Stock and Yogo (2005) suggests that the instruments are not weak. However, their critical value is calculated under homoscedastic errors. Like Andrews et al. (2019), we are not aware of a test with valid critical values for the case at hand – multiple endogenous regressors and heteroscedasticity. For the case of a single endogenous regressor combined with heteroscedasticity, Andrews et al. (2019) recommend using the effective F-statistic developed by Montiel Olea and Pflueger (2013). To be able to use that statistic, we mimic their setup by temporarily changing our model into a similar, auxiliary model with just one endogenous regressor, as follows. First, we switch to the model with $intrise_{it}$, instead of $intrisen_{it}$, so that only gap_{it} and $growth_{it}$ remain as not being predetermined. The main potential source of reverse causality for both variables is that ε_{it} might correlate with them via GDP of the same period. To exploit this common source of correlation, consider Δgap_{it}, where Δ denotes the first-difference operator. Not surprisingly, Δgap_{it} correlates heavily with $growth_{it}$, with a correlation of 0.77. Thus, only for now, substitute $growth_{it}$ in the model by Δgap_{it}. We thus obtain gap_{it} and Δgap_{it} as regressors. Write this as gap_{it} and $gap_{i,t-1}$ to make explicit that we are left with only one endogenous regressor, gap_{it}. The only instrument is $gap_{i,t-2}$. The estimates, not reported, are similar to those of Column (4) of Table 2, the advantage being that for this auxiliary model, which thus fulfills the conditions of Montiel Olea and Pflueger (2013), we can calculate what they call the "effective F-test". Its value is 58.39, while the upper bound of the 5% critical values is 23.11; see Montiel Olea and Pflueger (2013). We conclude that our results do not seem to be driven by instrument weakness and return to our baseline model.

4.2.2 Alternative estimation methods

Table 3: Sensitivity of results for primary-balance model (18) to alternative estimators

	(1) Baseline	(2) GMM-diff	(3) GMM-sys	(4) LSDV	(5) 2SLS-part
$intrisen_{it}$	0.32***	0.28***	0.21	0.20***	0.21***
	(0.071)	(0.090)	(0.13)	(0.059)	(0.059)
$-{}^{*}debt_{i,t-1}$	0.51***	0.53**	0.63**	0.36***	0.55***
	(0.15)	(0.24)	(0.25)	(0.13)	(0.13)
$pbal_{i,t-1}$	0.63***	0.62***	0.59***	0.67***	0.64***
	(0.071)	(0.077)	(0.093)	(0.077)	(0.072)
$debtint_{i,t-1}$	0.028***	0.032	0.031	0.026***	0.025***
	(0.005)	(0.020)	(0.022)	(0.005)	(0.005)
gap_{it}	0.26***	0.28***	0.31***	0.17***	0.24***
	(0.068)	(0.080)	(0.12)	(0.051)	(0.065)
$growth_{it}$	0.34***	0.17	0.077	0.20***	0.37***
	(0.11)	(0.12)	(0.18)	(0.064)	(0.11)
Observations	611	559	563	635	625
Hansen J-test *p*-value	0.49	1.00	1.00	—	0.36
Arellano-Bond AR(2) in diff *p*-value	—	0.22	0.20	—	—
$intrisen_{it}$ & $intrisen_{it}debt_{i,t-1}$ *p*-value	0.000***	0.000***	0.021**	0.001***	0.000***

Notes: Columns (2) and (3) report GMM estimates, computed by Roodman's (2009) xtabond2 in Stata. Column (4) uses all regressors themselves as instruments, while Column (5) is a combination of Columns (1) and (4) in the sense that it takes part of the baseline set of instruments by instrumenting only gap_{it} and $growth_{it}$, by $gap_{i,t-1}$, $gap_{i,t-2}$, and $growth_{i,t-1}$. Column (2) concerns one-step difference GMM. The instruments for the first-differenced version of (18) are the first and second lags of $intrisen_{i,t-1}$, $intrisen_{i,t-1}debt_{i,t-1}$, $debtint_{i,t-1}$, $gap_{i,t-1}$, $growth_{i,t-1}$, $pbal_{i,t-1}$ and the interest payment in *t* divided by GDP in *t*-1. This yields 49 instruments (including time dummies). Column (3) concerns one-step system GMM. The instruments for the first-differenced version of (18) are the same as for difference GMM. The instruments for the level equation are the first difference and its lag of $intrisen_{i,t-1}$, $gap_{i,t-1}$, and $growth_{i,t-1}$, yielding 6 additional instruments. In Columns (2) and (3), the Hansen test of overidentifying restrictions is weakened by the large number of instruments. "Arellano-Bond AR(2) in diff" denotes the Arellano-Bond test of the absence of first-order serial correlation in ε_{it}, obtained by testing the absence of second-order serial correlation in $\Delta\varepsilon_{it}$. The note to Table 2 provides further details.

The 2SLS estimator, used above, can exhibit bias due to correlation between the error ε_{it} and future values of the instruments. Because of the substantial number of years in our sample, 25 on average, this bias is expected to be small. Still, any such potential bias can be avoided by deploying the difference GMM and system GMM estimators (Arellano and Bond (1991) and Arellano and Bover (1995)). The dimensions of our data set may not be so suitable for applying these GMM estimators, because they are designed for panels with small *T* and large *N*. Still, we show the results for completeness. At the other extreme, we report the results when treating all regressors as strictly exogenous, using the least-squares dummy-variables (LSDV) estimator.

Table 3, Column (2), reports the estimates for difference GMM. Details on the specification are in the note to the table. The estimates for $intrisen_{it}$ and its interaction with debt are close to the baseline estimates, the 2SLS estimates replicated in Column (1) of the table. The large standard error for $debtint_{i,t-1}$ is the usual consequence of the fact that past levels, such as $debtint_{i,t-2}$, are weak predictors of the change of highly persistent variables, such as $\Delta debtint_{i,t-1}$. Fixing the parameter λ_2 for $debtint_{i,t-1}$ at 0.00, 0.02, or 0.04 gives similar results (not reported).

Column (3) of Table 3 presents the system GMM estimates. The impact of $intrisen_{it}$ becomes insignificant, particularly due to an increase in the standard error, although the point estimate does not change much. Still, the joint test shows significant relevance of $intrisen_{it}$ and its interaction with debt, both with again positive impacts. The standard error of $debtint_{i,t-1}$ is again large. This may seem surprising, as the level equation that system GMM adds to difference GMM intends to compensate for the weakness of $debtint_{i,t-2}$ in predicting $\Delta debtint_{i,t-1}$ in difference GMM. In particular, system GMM typically adds $\Delta debtint_{i,t-1}$ as instrument, expecting that its predictive power for $debtint_{i,t-1}$ reduces standard errors. However, $\Delta debtint_{i,t-1}$ is partly driven by $pbal_{i,t-1}$ and thus correlates with the country fixed effect. Hence, we cannot use $\Delta debtint_{i,t-1}$ as instrument, so the similar standard error for $debtint_{i,t-1}$ as in difference GMM should not be surprising. Fixing λ_2 at 0.00, 0.02, or 0.04 gives similar results (not reported), albeit that the estimates for $intrisen_{it}$ become significant again.

Column (4) of Table 3 reports the LSDV estimates. Those for $intrisen_{it}$ and $intrisen_{it}debt_{i,t-1}$ are again positive and highly significant, so our conclusion based on 2SLS is robust even when ignoring potential correlation between the error ε_{it} and regressors altogether. Both LSDV-estimates are below the 2SLS ones. The same holds for gap_{it} and $growth_{it}$. All this is in line with the idea of reverse causality, that is, a positive shock ε_{it} to the primary balance having a negative impact on the business cycle in year t and thereby on the coefficients of the regressors.[28]

Column (5) reports the 2SLS estimates when treating $intrisen_{it}$ as predetermined by only instrumenting gap_{it} and $growth_{it}$. The results are similar to the baseline results, suggesting that potential reverse causality regarding $intrisen_{it}$ is not empirically relevant. We also use this specification to study heterogeneity in the impacts of $intrisen_{it}$ and its interaction with $debt_{i,t-1}$. Allowing these impacts to vary over the decades (1980s, 1990s, 2000s, and 2010s), their estimates (not reported) are positive in all 8 cases, with 4 significant at the 5% level. For country-specific estimates the standard errors are large, as expected. Still, it is reassuring to see that no country dominates the overall estimates. The overall conclusion of the comparison to LSDV as well as both GMM estimators is that 2SLS is the preferable estimator, in line with prior expectations, and that our conclusions are not driven by some specific time period or country.

4.3 Further robustness analyses

4.3.1 Motivating the simplifications relative to the theory

The baseline regression equation (18) imposes some simplifications compared to the theoretical relationship (16) for the primary balance. We have motivated why leaving out the real interest rate is warranted. The results reported in Table 4 motivate some other simplifications.

For convenience we reproduce the baseline estimates in Column (1). Column (2) acknowledges that the theoretical relationship (16) divides lagged debt by current instead of lagged GDP and divides current minus potential GDP by current instead of potential GDP. Using these theory-based denominators and adding instruments to handle potential correlation between the error term and current GDP keeps the estimates virtually unchanged compared to the baseline results.

Column (3) adds tax revenue as a share of GDP, rev_{it}, instrumented by its lag. Recall that in Section 3.1 we derived that $\varphi_0/\varphi_\sigma \approx 1$, so that the tax revenue should be irrelevant for the primary balance. The near-zero estimate of the coefficient of rev_{it} confirms this. To further support that we have set $\varphi_0/\varphi_\sigma = 1$ in our baseline estimation, we have added interactions of $intreal_{it}$ with rev_{it}, gap_{it}, and $growth_{it}$. All of these have insignificant coefficient estimates, in line with the simplification.

[28] The magnitude of these downward shifts in the coefficient estimates may be useful for research on fiscal rules that leave out endogeneity corrections.

Table 4: Sensitivity of results for primary-balance model (18) to changing and adding regressors

	(1) Baseline	(2) Scale by current GDP	(3) Add revenue	(4) Add squared intrisen	(5) Add squared debt	(6) Add interest paid	(7) Add residual maturity	(8) Add inflation
$intrisen_{it}$	0.32***	0.35***	0.33***	0.38***	0.32***	0.31***	0.31***	0.27**
	(0.071)	(0.075)	(0.080)	(0.11)	(0.072)	(0.070)	(0.072)	(0.11)
$-{*}debt_{i,t-1}$	0.51***	0.55***	0.51***	0.55**	0.50***	0.56***	0.46***	0.85***
	(0.15)	(0.17)	(0.16)	(0.25)	(0.16)	(0.18)	(0.17)	(0.28)
$pbal_{i,t-1}$	0.63***	0.63***	0.63***	0.63***	0.63***	0.63***	0.62***	0.60***
	(0.071)	(0.071)	(0.071)	(0.071)	(0.073)	(0.071)	(0.072)	(0.072)
$debtint_{i,t-1}$	0.028***	0.029***	0.027***	0.028***	0.028***	0.032***	0.036***	0.038***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.008)	(0.007)	(0.009)
gap_{it}	0.26***	0.27***	0.26***	0.27***	0.26***	0.27***	0.26***	0.29***
	(0.068)	(0.067)	(0.069)	(0.071)	(0.068)	(0.071)	(0.070)	(0.076)
$growth_{it}$	0.34***	0.36***	0.35***	0.35***	0.34***	0.34***	0.36***	0.36***
	(0.11)	(0.11)	(0.13)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)
rev_{it}			0.015					
			(0.048)					
$intrisen_{it}^2$				-0.65				
				(0.82)				
$-{*}debt_{i,t-1}$				-0.85				
				(2.36)				
$debtint_{i,t-1}^2$					-0.001			
					(0.004)			
$intpayment_{it}$						-0.083		
						(0.093)		
$maturity_{i,t-1}$							-0.036	
							(0.065)	
$-{*}debt_{i,t-1}$							-0.13	
							(0.081)	
$expinfl_{it}$								-0.10
								(0.21)
$-{*}debt_{i,t-1}$								-0.47
								(0.36)
Observations	611	611	610	611	611	611	605	568
Hansen J p-value	0.49	0.49	0.49	0.51	0.48	0.48	0.42	0.31
Kleib.-Paap Wald	13.1	11.0	11.6	9.65	13.2	13.1	12.7	13.9

Notes: $expinfl_{it}$ is expected inflation. Further, in Column (2), the denominators of $debt_{i,t-1}$ and $debtint_{i,t-1}$ are current instead of lagged GDP, and for gap_{it} the denominator is current GDP instead of current potential GDP. This may imply that ε_{it} correlates with these adjusted regressors. To control for this, we use (the unadjusted) $debt_{i,t-1}$ and $debtint_{i,t-1}$ as additional instruments. For all interactions with $debt_{i,t-1}$ the estimate is presented as the impact at $debt_{i,t-1} = 60\%$. In Column (3) we use $rev_{i,t-1}$ as instrument. The note to Table 2 provides further details.

4.3.2 Adding other variables

We now add other variables to the baseline specification, one by one. A natural hypothesis would be that for political motivations governments find it easy to hand out financial windfalls ("squandering"), but difficult to improve the solvency of the public sector after a negative financial shock. In the baseline specification, the marginal effect of a change in $intrisen_{it}$ for given debt is assumed constant, as is the impact of debt on the marginal effect. In Column (4) we allow the marginal effect to linearly depend on $intrisen_{it}$ by adding $intrisen_{it}^2$ and its interaction with $debt_{i,t-1}$. This generalization does not change our main conclusions. We thus find no evidence that governments expand more after a fall in the difference between the marginal interest rate and that on non-maturing debt than they consolidate extra after a hike of the same size in this difference.

The remaining regressions also support the robustness of the baseline results. More specifically, Column (5) adds the square of $debtint_{i,t-1}$ to address possible fiscal-fatigue issues (Ghosh et al., 2013). Column (6) allows the interest-payment part of $debtint_{i,t-1}$, i.e., $intpayment_{it}$, to have a different effect than the lagged-debt part. Column (7) adds the average residual maturity of lagged debt (divided by 100) and allows its impact to vary with the level of the lagged debt. Finally, expected inflation and its interaction with lagged debt, treated as predetermined variables in Column (8), are not significant either. The latter conclusion also holds if we instrument inflation by its lag.

5. Concluding remarks

We explored the optimal responses of governments to changes in public debt interest rates. The importance of the issue is evidenced by the rising interest rates following the COVID-19 and Ukraine crises, and the danger this may pose for countries with very high levels of public debt (IMF, 2023a). To this end, we set up a simple theoretical framework with a government choosing public good provision and issuing debt of different maturities, so as to smooth public consumption over time. The total effect of an increase in the marginal interest rate is positive and is the combination of a standard wealth effect, an income effect, a substitution effect and a valuation effect, a specific type of wealth effect that has received little to no attention in the literature so far.

The empirical analysis confirms the positive impact of a rise in the marginal interest rate (on new debt), albeit that what matters is the marginal rate relative to the effective interest rate. This most likely is the result of governments seeking to smooth the higher future interest payments, an effect that dominates the opposite effect of the fall in the market value of outstanding public debt.

As the effective interest rate over time catches up with the level of the marginal interest rate, this response of the primary balance to a permanent interest rate increase will gradually fall to zero, implying that the specific dampening effect of this rise in the primary balance on the debt dynamics vanishes over time. Yet, we find that sustainability in our empirical model is assured by the primary balance rising in response to an increase in debt plus interest payments together.

Our analysis may have a number of policy implications. First, our theoretical framework as well as our empirical analysis may provide a benchmark against which one can evaluate projected developments of budgetary variables, in particular (structural) primary balances and interest expenditures, and that may provide guidance for potential policy responses to these developments. Second, consider monetary policy to fight inflation. Insofar monetary contraction increases long-term interest rates, the resulting increase in government savings that we find assists central bankers in fighting inflation. Our estimates thus help to quantify the role of fiscal policy in the monetary policy transmission mechanism, which central bankers can then exploit in their decision making. Third, there could be implications for the design of the (EU) budgetary rules, on which the European Commission has recently issued a Communication (European Commission, 2022), followed by a set of legislative proposals aimed at revising the Stability and Growth Pact and the requirements for the budgetary frameworks of the Member States.[29] The 3% reference value for the public budget deficit is based on the headline deficit, which includes debt interest payments. Depending on the maturity structure of the debt, the latter are partly beyond the control of the current government. Even with a short maturity structure, they depend to a substantial extent on the market circumstances when debt needs to be rolled over. This makes headline deficits harder to steer than primary deficits. Yet, interest rate changes do affect debt dynamics and the instrument to deal with these is the primary balance. Indeed, for Member States with public debt exceeding the reference level of 60% of GDP the European Commission's proposes to present a reference adjustment path for net primary expenditure which covers at least four years. This is then converted into a corresponding structural primary balance level to be reached at the end of this period. The four-year period can be prolonged in the case of investment and (structural) reform proposals that will strengthen fiscal sustainability.[30]

[29] For an analysis of the Stability and Growth Pact, see e.g. Beetsma and Uhlig (1999). The proposed revision (European Commission, 2023b) of the Council Directive 2011/85/EU on requirements for budgetary frameworks also foresees new roles for the so-called Independent Fiscal Institutions, official watchdogs scrutinizing the governments' fiscal policies. For an analysis, see Beetsma et al. (2022).
[30] On the link between structural reforms and fiscal sustainability see, e.g., Furceri and Jalles (2020).

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Appendix: more general setting of theoretical model, using three periods

We assume that all new interest rates reset in period 1 and later are set to r_1. That is, we will consider a permanent increase in the marginal interest rate. Hence, $r_1 = r_2 = r_{1,3}$. Here, r_1 is short-hand for $r_{1,2}$ and is the interest rate on new one-period debt $D_{1,2}$, $r_{1,3}$ is the interest rate on new two-period debt $D_{1,3}$, and r_2 is short-hand for $r_{2,3}$ and is the interest rate on new one-period debt issued in period 2. Optimizing (G_1, G_2, G_3) subject to the intertemporal government budget constraint yields:[31]

$$T_1 - G_1 = T_1 - f_\sigma^{-1} \cdot \left\{ \left[T_1 + \frac{T_2}{1+r_1} + \frac{T_3}{(1+r_1)^2} \right] - \left(1 + r_0^{eff} - r_1^n\right) D_0^t \right\},$$

where

$$f_\sigma = 1 + \frac{[\beta(1+r_1)]^\sigma}{1+r_1} + \frac{\left[\beta^2(1+r_1)^2\right]^\sigma}{(1+r_1)^2},$$

$$D_0^t = D_{-2,1} + D_{-1,1} + D_{0,1} + D_{-1,2} + D_{0,2} + D_{0,3},$$

$$r_0^{eff} = \frac{r_{-2,1}D_{-2,1} + r_{-1,1}D_{-1,1} + r_0 D_{0,1} + r_{-1,2}D_{-1,2} + r_{0,2}D_{0,2} + r_{0,3}D_{0,3}}{D_0^t}, \text{ and}$$

$$r_1^n = \left(\frac{r_1 - r_{-1,2}}{1+r_1}\right)\frac{D_{-1,2}}{D_0^t} + \left(\frac{r_1 - r_{0,2}}{1+r_1}\right)\frac{D_{0,2}}{D_0^t} + \left(\frac{r_1 - r_{0,3}}{1+r_1}\right)\frac{D_{0,3}}{D_0^t} + \left(\frac{r_1 - r_{0,3}}{(1+r_1)^2}\right)\frac{D_{0,3}}{D_0^t}.$$

A substantial amount of straightforward algebra can be used to show that:

$$\frac{d(T_1 - G_1)}{dr_1} = f_\sigma^{-1} \cdot \frac{(1+r_1)D_{1,2} + (2+r_1)D_{1,3} + D_{2,3}}{(1+r_1)^2} + f_\sigma^{-1} \cdot \left[\frac{G_2}{(1+r_1)^2} + \frac{2G_3}{(1+r_1)^3}\right]\sigma > 0.$$

[31] Notice that the absence of uncertainty requires that $r_1 = r_2 = r_{13}$, otherwise different debt tranches cannot co-exist: investors would only be prepared to hold the tranche with the highest yield, which would drive this yield down to that of the other tranches. For this reason, we do not optimize over the maturity composition of issued debt, but take this composition as given, subject to the optimal total amount of outstanding debt, which follows from the combination (G_1, G_2, G_3).