

## Solutions

**Part 1:**

Question 1:

- (a)  $x = 0$  or  $x = -1$
- (b)  $x = 0$  or  $x = 1$  or  $x = 3$
- (c)  $x = 0$  or  $x = -6$  or  $x = -5$
- (d)  $x = 0$  or  $x = 2$  or  $x = -2$
- (e)  $x = -\frac{4}{3}$
- (f)  $y = 0$  or  $y = -3 + \frac{\sqrt{97}}{3}$  or  $y = -3 - \frac{\sqrt{97}}{3}$

Question 2:

- (a)  $x = 3(1 + \sqrt{5})$  or  $x = 3(1 - \sqrt{5})$
- (b)  $x = 2\sqrt{7}$  or  $x = -2\sqrt{7}$  or  $x = 2$  or  $x = -2$
- (c)  $x = \frac{16}{9}$
- (d)  $x = 36$

Question 3:

- (a)  $x \leq 18\sqrt{3} + \frac{9}{8}$
- (b)  $2 \leq x \leq 2\sqrt{7}$  or  $-2\sqrt{7} \leq x \leq -2$
- (c)  $0 \leq x \leq 320$
- (d)  $y \geq 0$

Question 4:

(a)

$$\begin{cases} x = 40 \\ y = -16 \end{cases}$$

(b)

$$\begin{cases} x = \frac{21}{40} \\ y = -\frac{51}{20} \end{cases}$$

Question 5:

- (a)  $\frac{7(1-2\sqrt{2})}{8} \leq y \leq \frac{19}{3}$
- (b)  $\frac{99}{128} \leq x \leq 18$
- (c)  $\frac{5}{2} \leq z \leq 24$

**Part 2:**

Question 6:

	Mean	Median	Sample Variance
a	13	9	226.6
b	6.5555..	6	8.02777...
c	15.8755	10	402.6964

Question 7:

(a)

$$P[A \cup B] = \frac{15}{60} + \frac{15}{60} = \frac{1}{2}$$

(b)

$$P[A \cup B] = \frac{4}{60} + \frac{15}{60} - \frac{4}{60} * \frac{15}{60} = \frac{18}{60} = \frac{3}{10}$$

(c)

$$P[A \cap B] = \frac{21}{60}$$

Question 8:

	Project A	Initial investment	Project value at year 1	Pay Off	Probability
(a)	Success	-100	150	50	$\frac{1}{3}$
	Failure	-100	75	-25	$\frac{2}{3}$

$$E(A) = 50 * \frac{1}{3} + (-25) * \frac{2}{3} = 0$$

	Project B	Initial investment	Project value at year 1	Pay Off	Probability
	Success	-100	200	100	$\frac{1}{6}$
	Failure	-100	40	-60	$\frac{5}{6}$

$$E(B) = 100 * \frac{1}{6} + (-60) * \frac{5}{6} = -\frac{100}{3}$$

	Project C	Initial investment	Project value at year 1	Pay Off	Probability
	Success	-100	300	200	$\frac{2}{10}$
	No change	-100	100	0	$\frac{1}{2}$
	Failure	-100	0	-100	$\frac{3}{10}$

$$E(C) = 200 * \frac{2}{10} + (0) * \frac{1}{2} + (-100) * \frac{3}{10} = 10$$

So  $E(C) > E(A) > E(B)$ , it is better to invest in project C.

(b)

$$\begin{aligned} \text{Var}(A) &= (A_s - E(A))^2 * P_s + (A_f - E(A))^2 * P_f \\ &= (50 - 0)^2 * \frac{1}{3} + (-25 - 0)^2 * \frac{2}{3} \\ &= 1250 \end{aligned}$$

$$\begin{aligned} \text{Var}(B) &= (B_s - E(B))^2 * P_s + (B_f - E(B))^2 * P_f \\ &= (100 - (-\frac{100}{3}))^2 * \frac{1}{6} + (-60 - (-\frac{100}{3}))^2 * \frac{5}{6} \\ &= 3555.556 \end{aligned}$$

$$\begin{aligned} \text{Var}(C) &= (C_s - E(C))^2 * P_s + (C_f - E(C))^2 * P_f + (C_n - E(C))^2 * P_n \\ &= (200 - 10)^2 * \frac{2}{10} + (-100 - 10)^2 * \frac{3}{10} + (0 - 10)^2 * \frac{1}{2} \\ &= 10900 \end{aligned}$$