This trial exam consists of two parts. The first part focuses on proficiency in algebra, at the level of Algebra I. The second part includes simple exercises in statistics and probability theory.

More specifically, the test will focus on the following mathematical abilities:

a) Solving linear equations and inequalities with a single variable;

b) Solving and analysing quadratic functions;

c) Solving absolute value equations;

d) Adding, subtracting and multiplying polynomial expressions;

e) Calculating descriptive statistics such as arithmetic mean and variance;

f) Solving simple exercises in probability theory;

g) Calculate future value with interest rate[1].

[1] If there is any question regarding the trial exam, please don't hesitate to contact us by email (x.wang@uva.nl)

Part 1:

1. Solve the following equation. Give your final answer, no need to submit the intermediate steps.
   (a) $x^3 + 4x^2 + x = 2x^2$
   (b) $3x^3 - 12x^2 + 12x = 3x$
   (c) $5x^3 + 55x^2 + 150x = 0$
   (d) $x^4 - 3x^2 = x^2$
   (e) $-\frac{x^2}{4} - \frac{2x}{3} + \frac{2}{9} = \frac{2}{3}$
   (f) $-\frac{y^3}{4} - 2y^2 + \frac{4y}{9} = -\frac{y^2}{2}$
2. Solve the following equation. Give your final answer, no need to submit the intermediate steps.
   (a) $|\frac{x}{3} - 1| = \sqrt{5}$
   (b) $|\frac{x^2}{4} - 4| = \sqrt{9}$
   (c) $|\frac{\sqrt{x}}{4} + \frac{1}{3}| = \frac{4}{6}$
   (d) $|\frac{\sqrt{x}}{2} + 1| = \frac{(\sqrt{x+4})^2}{2} - \frac{(\sqrt{x-4})^2}{2}$
3. Solve the following inequalities. Give your final answer, no need to submit the intermediate steps.
   (a) $\frac{x}{9} - \frac{1}{8} \leq \sqrt{12}$
   (b) $|\frac{x^2}{4} - 4| \leq \sqrt{9}$
   (c) $\frac{\sqrt{2x}}{2} + \sqrt{10} \leq \sqrt{250}$
   (d) $\frac{\sqrt{11y}}{2} + \frac{\sqrt{2}}{\sqrt{12}} \geq \frac{1}{\sqrt{6}}$
4. Solve the following system of equations. Give your final answer, no need to submit the intermediate steps.
   (a)
   $$\begin{cases} x + 2y - 2 = 6 \\ \frac{x}{4} + \frac{y}{4} + 4 = 10 \end{cases}$$
   (b)
   $$\begin{cases} \frac{11x}{4} + \frac{2y}{3} - \frac{y}{4} = \frac{2x}{8} + \frac{1}{4} \\ \frac{2x}{5} + \frac{y}{5} + \frac{8}{10} = \frac{1}{2} \end{cases}$$
5. Solve the following system of inequalities. Give your final answer, no need to submit the intermediate steps.
   (a)
   $$\begin{cases} 3y - 11 \leq \sqrt{64} \\ \frac{4y}{7} + \frac{\sqrt{8}}{2} \geq \frac{1}{2} \end{cases}$$
   (b)
   $$\begin{cases} \sqrt{3x} - \sqrt{6} \leq \sqrt{24} \\ \frac{2x}{3} - \frac{1}{64} \geq \frac{64}{128} \end{cases}$$
   (c)
   $$\begin{cases} |\frac{z}{3} - 1| < \frac{21}{3} \\ \frac{2z}{3} - 2 \geq -\frac{1}{3} \end{cases}$$

Part 2:

6. Calculate the mean, median, sample variance and sample standard deviation of the following set of observations.

   (a) $(10, 2, 3, 6, 18, 9, 3, 23, 54, 9, 6)$

   (b) $(2, 8, 9, 10, 10, 4, 5, 5, 6)$

   (c) $(9, 10, 4, 5, 12, 65, 12, 10)$

7. Using your knowledge of probability theory to answer the following questions.

   Suppose one ball is picked at random from a box containing 60 balls with 15 in red, 15 in blue, 15 in yellow and 15 in green.

   (a) Let A denote the probability of obtaining a "red" ball, and let B denote the probability of obtaining a "blue" ball. Calculate $P[A \bigcup B]$.

   (b) Suppose the balls are marked with numbers ranking from 1 to 20 for each color, namely for each number there are fours balls in red, blue, yellow and green separately. Let A denote the probability of obtaining a ball with a number 20, and let B denote the probability of obtaining a ball in red color. Calculate $P[A \bigcup B]$.

   (c) Suppose the balls are marked with numbers raking from 1 to 60. Let A denote the probability of obtaining a ball with a number no less than 20, and let B denote the probability of obtaining a ball with a number no more than 40. Calculate $P[A \cap B]$.

8. Using your knowledge of probability theory to answer the following questions.

   Consider: You have an investment opportunity right now to invest in three different projects A, B or C. Due to the income constraint it is only possible for you to invest in one of those projects. Suppose that you are a rational person and you will make your investment decision fully based on the expectation of the payoffs of the projects. Each project will turn out to be a "good" one or "bad" one with different probabilities and payoffs after one year (ignore time value in this question). Any of those three projects needs an initial investment of 100 euro today.

   Project A: There is a chance that the project turns out to be a success and this project will worth 150 euro in one year. There is also a chance that the project turns out to be a failure and this project will only worth 75 euro in one year. The probability of success is 1/3 and the probability of failure is 2/3.

   Project B: There is a chance that the project turns out to be a success and this project will be worth 200 euro in one year. There is also a chance that the project turns out to be a failure and the project will be only be worth 40 euro in one year. The probability of success is 1/6 and the probability of failure is 5/6.

   Project C: There is a chance that the project turns out to be a big success and this project will be worth 300 euro in one year. However, there is also a chance that the project turns out to be a big failure and the project will be worthless (0 euro). The probability of success is 2/10 and the probability of failure is 3/10. Besides those scenarios, the project might also keep the value as your initial investment (100 euro) after one year, with the probability of 1/2.

   (a) Calculate the expected payoffs of each project and conclude in which project you would like to invest. (Note: Payoff = Project Value in One year-Initial Investment)

   (b) Calculate the variance of the payoff of different project A, B and C.

9. Calculate the accumulated income in three years given

   (a) A deposit of \$800 every year and an annual interest rate of 5% per year.

   (b) A deposit of $\$1,000$ every year and an annual interest rate of 2.5% per year.

   (c) A deposit of $\$3,000$ only at the beginning of the first year and an annual interest rate of 2.5% per year.

(d) A deposit of $\$1,500$ only at the beginning of the first year and an annual interest rate of 5% per year.

Assume that the annual interest rate stays the same every year.

Solutions

Part 1:

Question 1:

(a) $x = 0$ or $x = -1$

(b) $x = 0$ or $x = 1$ or $x = 3$

(c) $x = 0$ or $x = -6$ or $x = -5$

(d) $x = 0$ or $x = 2$ or $x = -2$

(e) $y = -\frac{4}{3}$

(f) $y = 0$ or $y = -3 + \frac{\sqrt{97}}{3}$ or $y = -3 - \frac{\sqrt{97}}{3}$

Question 2:

(a) $x = 3(1 + \sqrt{5})$ or $x = 3(1 - \sqrt{5})$

(b) $x = -2$ or $x = 2$ or $x = 2\sqrt{7}$ or $x = -2\sqrt{7}$

(c) $x = \frac{16}{9}$

(d) $x = 36$

Question 3:

(a) $x \leq 18\sqrt{3} + \frac{9}{8}$

(b) $2 \leq x \leq 2\sqrt{7}$ or $-2\sqrt{7} \leq x \leq -2$

(c) $0 \leq x \leq 320$

(d) $y \geq 0$

Question 4:

(a)

$$\begin{cases} x = 40 \\ y = -16 \end{cases}$$

(b)

$$\begin{cases} x = \frac{21}{40} \\ y = -\frac{51}{20} \end{cases}$$

Question 5:

(a) $\frac{7(1-2\sqrt{2})}{8} \leq x \leq \frac{19}{3}$

(b) $\frac{99}{128} \leq x \leq 18$

(c) $\frac{5}{2} \leq z \leq 24$

Part 2:

Question 6:

	Mean	Median	Sample Variance	Sample standard deviation
a	13	9	226.6	15.05324
b	6.5555..	6	8.02777...	2.8333...
c	15.875	10	402.6964	20.0673

Question 7:

(a)

$$P[A \cup B] = \frac{15}{60} + \frac{15}{60} = \frac{1}{2}$$

(b)

$$P[A \cup B] = \frac{4}{60} + \frac{15}{60} - \frac{4}{60} * \frac{15}{60} = \frac{18}{60} = \frac{3}{10}$$

(c)

$$P[A \cap B] = \frac{21}{60}$$

Question 8:

(a)

Project A	Initial investment	Project value at year 1	Pay Off	Probability
Success	-100	150	50	$\frac{1}{3}$
Failure	-100	75	-25	$\frac{2}{3}$

$E(A) = 50 * \frac{1}{3} + (-25) * \frac{2}{3} = 0$

Project B	Initial investment	Project value at year 1	Pay Off	Probability
Success	-100	200	100	$\frac{1}{6}$
Failure	-100	40	-60	$\frac{5}{6}$

$E(B) = 100 * \frac{1}{6} + (-60) * \frac{5}{6} = -\frac{100}{3}$

Project C	Initial investment	Project value at year 1	Pay Off	Probability
Success	-100	300	200	$\frac{2}{10}$
No change	-100	100	0	$\frac{1}{2}$
Failure	-100	0	-100	$\frac{3}{10}$

$E(C) = 200 * \frac{2}{10} + (0) * \frac{1}{2} + (-100) * \frac{3}{10} = 10$

So $E(C) > E(A) > E(B)$, it is better to invest in project C.

(b)

$$\begin{aligned} Var(A) &= (A_s - E(A))^2 * P_s + (A_f - E(A))^2 * P_f \\ &= (0-50)^2 * \frac{1}{3} + (-25-0)^2 * \frac{2}{3} \\ &= 1250 \end{aligned}$$

$$\begin{aligned} Var(B) &= (B_s - E(B))^2 * P_s + (B_f - E(B))^2 * P_f \\ &= (100 - (-\frac{100}{3}))^2 * \frac{1}{6} + (-60 - (-\frac{100}{3})^2 * \frac{5}{6} \\ &= 3555.556 \end{aligned}$$

$$\begin{aligned} Var(C) &= (C_s - E(C))^2 * P_s + (C_f - E(C))^2 * P_f + (C_n - E(C))^2 * P_n \\ &= (200-10)^2 * \frac{2}{10} + (-100-10)^2 * \frac{3}{10} + (0-10)^2 * \frac{1}{2} \\ &= 10900 \end{aligned}$$

Question 9:

(a) $800 * (1+5\%)^3 + 800 * (1+5\%)^2 + 800 * (1+5\%)$

(b) $1000 * (1+2.5\%)^3 + 1000 * (1+2.5\%)^2 + 1000 * (1+2.5\%)$

(c) $3000 * (1+2.5\%)^3$

(d) $1500 * (1+5\%)^3$