Solutions

Part 1:

Question 1:

(a) $x = 0$ or $x = -1$

(b) $x = 0$ or $x = 1$ or $x = 3$

(c) $x = 0$ or $x = -6$ or $x = -5$

(d) $x = 0$ or $x = 2$ or $x = -2$

(e) $x = -\frac{4}{3}$

(f) $y = 0$ or $y = -3 + \frac{\sqrt{97}}{3}$ or $y = -3 - \frac{\sqrt{97}}{3}$

Question 2:

(a) $x = 3(1 + \sqrt{5})$ or $x = 3(1 - \sqrt{5})$

(b) $x = 2\sqrt{7}$ or $x = -2\sqrt{7}$ or $x = 2$ or $x = -2$

(c) $x = \frac{16}{9}$

(d) $x = 36$

Question 3:

(a) $x \leq 18\sqrt{3} + \frac{9}{8}$

(b) $2 \leq x \leq 2\sqrt{7}$ or $-2\sqrt{7} \leq x \leq -2$

(c) $0 \leq x \leq 320$

(d) $y \geq 0$

Question 4:

(a)

$$\begin{cases} x = 40 \\ y = -16 \end{cases}$$

(b)

$$\begin{cases} x = \frac{21}{40} \\ y = -\frac{51}{20} \end{cases}$$

Question 5:

(a) $\frac{7(1-2\sqrt{2})}{8} \leq y \leq \frac{19}{3}$

(b) $\frac{99}{128} \leq x \leq 18$

(c) $\frac{5}{2} \leq z \leq 24$

Part 2:

Question 6:

	Mean	Median	Sample Variance
a	13	9	226.6
b	6.5555..	6	8.02777...
c	15.8755	10	402.6964

Question 7:

(a)

$$P[A \cup B] = \frac{15}{60} + \frac{15}{60} = \frac{1}{2}$$

(b)

$$P[A \cup B] = \frac{4}{60} + \frac{15}{60} - \frac{4}{60} * \frac{15}{60} = \frac{18}{60} = \frac{3}{10}$$

(c)

$$P[A \cap B] = \frac{\frac{21}{40}}{\frac{40}{60}} = \frac{21}{60}$$

Question 8:

(a)

Project A	Initial investment	Project value at year 1	Pay Off	Probability
Success	-100	150	50	$\frac{1}{3}$
Failure	-100	75	-25	$\frac{2}{3}$

$E(A) = 50 * \frac{1}{3} + (-25) * \frac{2}{3} = 0$

Project B	Initial investment	Project value at year 1	Pay Off	Probability
Success	-100	200	100	$\frac{1}{6}$
Failure	-100	40	-60	$\frac{5}{6}$

$E(B) = 100 * \frac{1}{6} + (-60) * \frac{5}{6} = -\frac{100}{3}$

Project C	Initial investment	Project value at year 1	Pay Off	Probability
Success	-100	300	200	$\frac{2}{10}$
No change	-100	100	0	$\frac{1}{2}$
Failure	-100	0	-100	$\frac{3}{10}$

$E(C) = 200 * \frac{2}{10} + (0) * \frac{1}{2} + (-100) * \frac{3}{10} = 10$

So $E(C) > E(A) > E(B)$, it is better to invest in project C.

(b)

$$\begin{aligned} Var(A) &= (A_s - E(A))^2 * P_s + (A_f - E(A))^2 * P_f \\ &= (50-0)^2 * \frac{1}{3} + (-25-0)^2 * \frac{2}{3} \\ &= 1250 \end{aligned}$$

$$\begin{aligned} Var(B) &= (B_s - E(B))^2 * P_s + (B_f - E(B))^2 * P_f \\ &= (100 - (-\frac{100}{3}))^2 * \frac{1}{6} + (-60 - (-\frac{100}{3}))^2 * \frac{5}{6} \\ &= 3555.556 \end{aligned}$$

$$\begin{aligned} Var(C) &= (C_s - E(C))^2 * P_s + (C_f - E(C))^2 * P_f + (C_n - E(C))^2 * P_n \\ &= (200-10)^2 * \frac{2}{10} + (-100-10)^2 * \frac{3}{10} + (0-10)^2 * \frac{1}{2} \\ &= 10900 \end{aligned}$$