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6. *Summary*

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In this dissertation we contribute to the development and understanding of the EWMA control chart based on estimated parameters. The presented research is based on Zwetsloot, Schoonhoven, and Does (2014, 2015a, 2015b), Saleh, Mahmoud, Jones-Farmer, Zwetsloot, and Woodall (2015a), and Zwetsloot (2015). In this concluding chapter, we summarize our findings.

6.1 EWMA control charts

All processes show variation. Part of this variation is *inherent* to the process when it functions in a predictable manner. However, sometimes variation may occur because of *special causes* of variation. Deciding which observation occurs because of special causes is not always easy. As Box, Hunter and Hunter (2005, page 566) pointed out "nothing looks so unrandom as a series of random numbers". Control charts are used to operationalize 'common' and 'special' causes of variability.

The original control chart was developed by W. A. Shewhart in a memorandum issued on May 16, 1924. The exponentially weighted moving average (EWMA) control chart was introduced by S. W. Roberts in 1959. The EWMA statistic is a weighted average of measurements, giving heaviest weights to the most recent observations. This provides the chart the advantage of being sensitive to small- and moderate-sized sustained shifts in the process parameters.

It is generally accepted that a control chart is implemented in two phases: Phase I, to define the stable state of the process characteristic and to estimate the process parameters; and Phase II to monitor the process. For an overview of Phase I methods, see Chakraborti et al. (2009) and Jones-Farmer et al. (2014b).

In this dissertation, we study the EWMA chart based on estimated parameters. We consider monitoring both the location as well as the dispersion. We assume that the process characteristic can be modelled as an independent and normally distributed random variable.

6.2 Motivation

We consider the problem of *estimating process parameters from data which can contain contaminated observations*. In practice, data sets often contain outliers, step shifts, recording errors, and other data quality issues (Jones-Farmer et al., 2014b; Vining et al., 2015). Al these types of 'contaminations' are problematic as they can influence the parameter estimates, resulting in control charts with less predictable statistical performance. The possible presence of contaminations in Phase I, provides the motivation for Chapters 2 and 3.

Moreover, we consider *the effect of sampling variability on the monitoring performance* of the EWMA chart. It is well known that the performance of control charts is influenced by Phase I estimation (Jones et al., 2001; Jensen et al., 2006; Psarakis et al., 2014). Generally, control charts based on estimated parameters are evaluated by studying the performance averaged over all possible Phase I estimates. However, the performance of a control chart will depend on the 'actual' estimate. This conditional performance can be very different from the 'average' performance. It is this variability in conditional performance that motivates Chapters 4 and 5.

6.3 Methods

We study *robust estimation methods*, to ensure accurate estimates of the in-control process parameters if contaminations are present. Using a simulation study, we compare the performance of the considered estimation methods for in-control data and for various contamination scenarios. The effectiveness of the estimation methods is evaluated in terms of the accuracy of the resulting estimates and the proportion of successfully identified contaminated observations. Furthermore, we propose new estimation methods for the location and the dispersion, based on EWMA charts.

To evaluate the effect of sampling variability on the EWMA chart's performance, we *study the variability of the conditional average run lengths*. Furthermore, in Chapter 4, we consider an alternative procedure for controlling the EWMA chart's performance proposed by Jones and Steiner (2012) and Gandy and Kvaløy (2013). This procedure is based on bootstrapping to guarantee, with a specified probability, a certain conditional performance for each chart. The main objective of this approach is to limit the proportion of EWMA charts with a low actual in-control ARL.

Furthermore, in Chapter 5, we compare three designs for the EWMA chart for dispersion. *We use the effect of sampling variation on the performance as a comparison metric*.

The main objective of this comparison is to give a recommendation on which EWMA chart for dispersion to use if parameters need to be estimated.

6.4 Results

The results show that data contaminations can have a huge impact on the accuracy of the estimates obtained in Phase I. Existing estimation methods provide accurate estimates for *specific* patterns of contaminations. The new method, based on EWMA charting, provides accurate estimates for *any* of the considered contamination scenarios.

Furthermore, we show that to set up EWMA charts, more Phase I data are required than previously recommended, in order to decrease the sampling variability to a reasonable level. Moreover, we found that sampling variation has a larger effect on the EWMA chart for dispersion than on the EWMA chart for location.

Also, our results show that using a bootstrap-based design approach, results in highly skewed in-control ARL distributions. However, such increases of the in-control ARL did not have much of an effect on the out-of-control performance.

Finally, we found that the EWMA chart based on the sample variance (S^2) has the most predictable performance, compared to the charts based on the sample standard deviation (S), or on the logarithm of the sample variance ($\ln S^2$).

6.5 Discussion and recommendations

We recommend the **use of estimation methods based on screening** if contaminations may be present in Phase I. These methods are efficient under stability and robust to contaminations. We recommend to implement the robust estimation as follows: (1) first screen the initial data to delete the contaminated observations and 'learn from the data'; (2) perform the screening with an EWMA chart designed with a smoothing constant around 0.5. This balances the ability to detect outliers and sustained shifts; (3) use a two-step procedure, namely a robust estimator to construct the initial chart, and an efficient estimator for post-screening estimation.

Automatic implementation of these methods would be unwise. As noted by Jones-Farmer et al. (2014b) careful consideration prior to eliminating process observations in a Phase I analysis is important. The method can be used to signalling those samples which need to be investigated. Furthermore, it ensures that - even if we oversee a contaminated observation - the resulting estimates are near to the target.

A limitation of the proposed method for location is its inaccuracy if single scattered outliers (diffuse disturbances) are present. Furthermore, we have only compared the estimation methods for independent and normally distributed in-control data.

We recommend to **take the effect of sampling variability into account** when choosing and designing EWMA control charts, as parameter estimation has a large impact on performance. Furthermore, we **support the use of the bootstrap-based design** approach of Jones and Steiner (2012) and Gandy and Kvaløy (2013), which was recently proposed for controlling the probability of the in-control ARL being at least a specified value. Charts based on bootstrap show less frequent false alarms and are still able to detect sustained shifts quickly.

A limitation of the bootstrap-based design is its relative complexity; it requires advanced statistical knowledge to implement the design. Another limitation is that we have only considered the parametric bootstrap method for independent and normally distributed data. Gandy and Kvaløy (2013) also proposed a non-parametric version of the bootstrap algorithm. The implementation of this method for the EWMA chart is left for future study.

We recommend the **use of the EWMA chart based on the sample variance** (S^2). Because, it has the lowest variability in conditional ARL performance compared to the other considered EWMA charts for dispersion.

Throughout this dissertation, we primarily considered data collected in samples of size $n = 5$. The methods to estimate and monitor the dispersion, as presented in Chapter 3 and 5, are not easily implemented for $n = 1$. As the standard deviation estimators are based on the assumption that $n > 1$. However, we feel confident that the results will hold for other sample sizes $n > 1$ and can be modified to $n = 1$.