Next-to-Soft Factorization and Unitarity in Drell-Yan Processes

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Summary

In this summary I aim to give a description of the themes of this thesis without discussing technical details. In particular, I hope that it offers the reader a more intuitive insight at least into the meaning of three keywords that compose the title of this thesis: the Drell-Yan process, (next-to-soft) factorization and unitarity.

A different way of seeing

The field of particle physics, to some extent, can be seen as a branch of microscopy, in the sense that it is aimed at viewing things that cannot be seen with the naked eye. In general, the process of observing requires different instruments according to the size of the object we want to study. Normally, in everyday life, we simply use our eyes. Already when we wish to observe human cells we need an artificial instrument like an optical microscope. For nanostructures we have to leave the idea of using light in the common sense and we need more powerful instruments like an atomic-force microscope. For elementary particles we need to go even further and build a particle accelerator. And the smaller the size we want to test, the larger the accelator (and thereby its energy) should be.

All these instrument seems very different from one another. However, they are all based on a scattering process. The idea is quite straightforward and can be intuitively understood thinking about a mysterious object, of which we want to reconstruct the shape. This can be reconstructed throwing another object against it and seeing how it bounces. This we can repeat again and again until, from the pattern of the scattering, we can reconstruct the shape. Even if might be odd, our sight is based on the same principle. When we look at an object illuminated by the sunshine or the light from a bulb, our eye is detecting photons scattered by the surface of this object. Then according to the intensity, the direction and the color (i.e. the frequency) our brain reconstructs from the detected image the shape of the object. High energy experiments are based on exactly the same idea.

Clearly, for actual proton collisions like those that happen at the CERN Large Hadron Collider (LHC) we have to modify a little bit the picture described above. The entangled effect of special relativity and quantum mechanics makes it possible that new particles are created during these scattering processes, and the role of particle physicists is to understand the mechanism that governs the interactions among these particles. In this regard, we can distinguish experimentalists, who take part in the actual measurements of cross-sections (i.e. how these particles are scattered), and theorists, who predict these cross-sections based on a mathematical well-defined framework. Among theorists efforts we can discriminate those that predict the cross-sections in presence of new particles still to be discovered, and those that predict the so-called background, i.e. the cross-section for known effects due to pure Quantum Chromodynamical effects (the theory that governs the nuclear interactions, shortly indicated as QCD). The work presented in this thesis is set in the last group, as it computes QCD corrections to one particular cross-section known as Drell-Yan process.

Feynman diagrams and the Drell-Yan process

In a particle accelerator like the LHC at CERN protons collide and many particles are created and detected as final states. The Drell-Yan process describes a particular outcome for such collisions when among the final particles there is a lepton-antilepton pair. These are two particles that are insensitive to the nuclear force like muons and electrons. This production mechanism can be explained in the context of the parton model. This assumes that protons are made of elementary particles called quarks and gluons collectively called partons. In the Drell-Yan process, when two protons collide at very high energy, a parton from one proton annihilates with another parton in the other proton, creating a photon. However, this photon is not a physical one (whose mass is zero) but has a virtual mass less or equal to the incoming energy of the two partons. This might sounds a bit strange but, thanks to the magic of quantum mechanics, in Feynman diagrams particles can violate some constraint typical of physical particles, provided they do so for the briefest of moments (technically they are called off-shell particles). Therefore, this unphysical photon is an unstable particle and it decays in the lepton pair described above.

To predict the occurrence rate for this process one makes use of the so-called Feynman diagrams, like the one shown on the cover of this thesis. Apart from getting an intuitive picture of how particles interact, we can associate to each of these diagrams a number, which represents the probability that the process will happen through this specified diagram. Computing this number is the goal of a particle theorists because this number can be compared with an experimental measurement. However this task might be very complicated. In particular, the higher the number of particles in a diagram, the more complicated the calculation will be.

The reason why it is convenient to compute cross-sections with Feynman diagrams is the fact that partons are almost free at high energy, and therefore diagrams with many interactions are unlikely and can be neglected. Therefore, we can approximate the probability for the entire cross-section with a subset of easier-to-compute diagrams. Mathematically speaking, we are using perturbation theory and we are approximating the exact result with the first terms of the series. Experiments demand very precise predictions, hence sometimes cross sections have been calculated with many of these Feynman diagrams.

In this thesis we have investigated these computations for the Drell-Yan process both when it is possible to consider only a *few* diagrams (by means of unitarity methods) and when one is forced to take into account a specific effect from *all* diagrams (by means of factorization methods). Let us now discuss them in turn.

Next-to-soft factorization

In a collider experiment the energy of the final particles will clearly depend on the energy of the particles we want to collide. In particular, the incoming energy should be greater than the mass of the particle we want to create. When we accelerate protons we typically control the energy of the protons, but we cannot have control of the energy of its components (quarks and gluons). Hence, we have to take into account the possibility that the energy of these might be very close to the threshold energy required to produce the final particles. For the Drell-Yan case, this corresponds to the situation when the energy is sufficient to produce the lepton-antilepton pair, and all other particles produced are soft (particles with little energy are called soft, while very energetic ones are called hard).

In this limiting situation, the approximation of taking only a few diagrams is not valid anymore and one has to take into account all diagrams with many soft gluons. Mathematically speaking, when we are close to the threshold limit the results of the computations are plagued by logarithms that need to be resummed to all orders. This process can be achieved through a so-called factorization. Indeed, after factorizing the effects of soft gluons, the soft part of the process becomes independent from the hard part and can be organized such that it gives predictions for more complicated diagrams (higher order terms). This well-known procedure called soft-gluon resummation is a well-established field and many cross sections have been computed including these effects.

In this thesis we have extended this factorization procedure to a considerably more precise level, known as next-to-soft. In this case the emitted gluons are less soft than the purely soft case, and the hard particle recoils somewhat after the emission. Specifically, we have studied these next-to-soft effects with three different approaches (a diagrammatic one, the method of regions, and the soft-collinear factorization formula) and we have shown that it is possible to organize the logarithms due to next-to-soft effects to all orders. With this preliminary work the door is now open to more research that can be done, as eventually we would like to use this factorization to compute a full resummed result.

Unitarity

In the last chapter of this thesis we have investigated how to develop new methods for computing Feynman diagrams of the Drell-Yan process. Some of these methods made use of a property called unitarity. The name of this feature comes from the fact that, in a consistent theory, the sum of all probabilities (which as we discussed are related to Feynman diagrams) should be equal to one. With more mathematical precision, for some processes this is defined through a theorem, called the optical theorem.

For the Drell-Yan process this theorem cannot be used at face value and thus we might think that these types of methods cannot be used. However, in this thesis we have shown that it is indeed possible, after moving from the optical theorem to the more general use of cuts of a diagram. This consists in literally cutting a diagrams in two parts with a separator line, and making every particle cut by this line physical (technically, putting it on-shell). The virtue of this perhaps somewhat mysterious procedure is that upon summing over all possible cuts of a diagram, it is possible to reconstruct the full cross-section. For some processes, this is intimately connected to the optical theorem, and hence these cuts are called unitarity cuts. With this method we have shown that it is possible to recompute the Drell-Yan cross sections (and thereby related cross sections such as for Higgs boson production), thus providing a new method for fixed order-computation.

This concludes the summary, where I hope I gave a glimpse of what has been presented in this thesis, omitting technical details. For a more rigorous discussion, the interested reader can simply begin reading the introduction of this thesis.