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Unitarity methods and On-shell Particles in Scattering Amplitudes

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SUMMARY

This thesis is about the world of the smallest particles and describes my research on calculating their behaviour. Here I present a brief and somewhat simplified explanation of that research. It starts with a history of looking at very small things.

The seventeenth-century textile trader Antoni van Leeuwenhoek was a pioneer in the area of microscopy. In order to inspect the quality of his textiles he manufactured strong magnifying glasses. He became such a skilled craftsman that he was able to make the best in the world by far, capable of hundredfold magnification. Out of curiosity he started to point his microscopes at other objects besides textiles. This led him to discover a world of organisms that cannot be seen with the naked eye.

If we imagine to continue magnifying another one thousand times, we would see human DNA in close-up and we would start to distinguish individual water molecules (H_2O). At this scale, one millionth of a millimetre, the electromagnetic force plays a very important role. This force is responsible for holding together molecules, which are built up of atoms. Water, for instance, consists of two hydrogen atoms (H) and one oxygen atom (O). Over a hundred types of atoms like these are known to exist.

Atoms consist in turn of a nucleus made up of protons and neutrons, and far away around it a cloud of electrons. For the electrons the reduction stops here, they are *elementary particles*. The protons and neutrons in the atomic nucleus do however have a substructure, made of quarks bound together by the strong nuclear force. Moreover, there is a second force that plays a role in the nucleus: the weak nuclear force. All phenomena related to radioactivity are caused by this force. Such processes create neutrinos, elementary particles which hardly undergo any interactions; they easily travel through the entire earth. In summary we can therefore say that quarks, electrons and neutrinos together form all visible matter.

Yet more elementary particles have been discovered during the last century, mostly in particle accelerator experiments. We have found three 'families' of the matter particles (quarks, electron and neutrino), which differ from each other by their mass. The heavier families are unstable and short-lived, which explains why they do not make up ordinary matter. A

second class of elementary particles consists of force carriers: the photon for the electromagnetic force, the gluons for the strong nuclear force and the W and Z bosons for the weak nuclear force. Matter particles experience forces by interchanging these force carriers. The Higgs boson, a particle associated to the mechanism that generates mass, forms a separate category. Its existence was spectacularly experimentally established in 2012. These are all known elementary particles.

This world of elementary particles is accurately described by a certain quantum field theory, known as the Standard Model of particle physics. In order to test this theory, either to verify or falsify its correctness, predictions are made for measurable quantities (such as cross sections) in collisions at particle accelerators. An important step in this process is the calculation of a *scattering amplitude*, a mathematical function which describes all possible interactions during a collision process. For this task we make use of *perturbation theory*, an approximation method which expresses the scattering amplitude as a first estimate plus a series of increasingly smaller corrections. Including ever more of these corrections makes the outcome ever more accurate, but at the same time the calculation becomes tremendously more complicated. So it is a real challenge to compute higher-order corrections in perturbation theory.

My research is concerned with such corrections to scattering amplitudes. The specific interactions in these contributions are drawn by means of Feynman diagrams. These diagrams contain internal and external lines which represent particles. The external particles are physical (measurable) and must satisfy Einstein's relationship between energy, momentum and mass. We say that the momentum of a physical particle is on the mass-shell, or *on-shell* for short. Internal particles, on the other hand, do not need to satisfy this relationship and are generally off-shell. In fact, quantum mechanics prescribes that all possible configurations of internal particles are to be added up appropriately. In this way, the internal particles contribute much to the complexity of scattering amplitudes.

In some cases it is therefore useful to come up with a simplification for these scattering amplitudes, for example by imposing a certain restriction. The purpose of this can be to obtain a reasonable approximation of the answer or to extract some specific information from the scattering amplitude. An obvious restriction could be to place (some of the) internal particles on their mass shell, so that they become physical particles. This limits the degrees of freedom of the internal particles, so that their contribution to a scattering amplitude can be more readily determined. This restriction is, in fact, the common thread in this thesis. It is used in various ways in the different chapters, as I will now clarify.

As mentioned, many elementary particles are unstable and decay rapidly into lighter particles. This occurs frequently in high-energy particle collisions. Unstable particles have the freedom to be off their mass shell by an amount characterized by the decay width. An unstable particle whose decay width is small (in comparison to its mass), will typically be close to the mass shell. We call such particles *narrow resonances*. A number of very interesting particles, including the Higgs boson, are such narrow resonances. By assuming in a calculation that narrow resonances are exactly on-shell, they are effectively treated as stable particles. This *narrow-width approximation* splits a scattering process into two subprocesses, production and decay of a narrow resonance, each of which can be computed more easily than the original process featuring both.

The third chapter of this thesis describes an implementation of the narrow-width approximation in the computerprogram MadSpin for simulating the decay of narrow resonances. The name of this program points to two of its features. "Mad" is adopted from MadGraph, the original name of the framework into which the program is embedded. A key feature of this framework is its *general applicability* to a large class of scattering processes. Likewise, MadSpin was created in such a way that it generally applies to all narrow resonances. Secondly, "Spin" refers to *spin correlations,* which cause entanglement among particles that emerge from particle collisions. This measurable phenomenon should be accounted for in theoretical predictions. The program MadSpin combines the narrow-width approximation with spin correlation information from the previous term in the perturbative series. This yields an excellent description of spin correlations, as I demonstrated with an example.

In a certain sense this program forms a concluding piece in the set of theoretical predictions that include first-order corrections in perturbation theory. This level of precision may however not be sufficient in the presence of increasingly accurate experimental measurements. The remainder of this thesis ventures therefore into the territory of higher-order corrections, where much is still to be done.

One aspect of higher-order corrections that I have investigated is *infrared radiation*. This radiation of photons or gluons with extremely large wavelength can have the consequence that Feynman diagrams yield the outcome 'infinity'. In a correct sum of diagrams the infinities cancel, but a trace of the infrared radiation is still left behind. This effect can be studied with the help of simplified Feynman diagrams, so-called *eikonal diagrams*. The simplest type of eikonal diagram describes the exchange of infrared radiation between two energetic particles. In a certain limit, the interaction potential for those two particles is given by the imaginary part of the eikonal diagram. I have developed a method of calculation that imposes the on-shell restriction on the infrared radiation, in order to extract this information from eikonal diagrams in a very direct way. Subsequently I successfully applied this method to highly complex diagrams from second- and third-order corrections in perturbation theory.

The last part of this thesis focuses on the calculation of specific second-order corrections in perturbation theory. For this I made use of *unitarity*, the statement that the probabilities for all possible outcomes of a particle collision must add up to one. This sensible requirement is built into the theory in the form of a unitary scattering matrix. A good question is: can this property be exploited to simplify calculations? For some processes this turns out to be the case, especially if the outcome of the process is not very specific. This can be explained with the following analogy. Roll two dice. What is the probability of obtaining two different numbers? This can happen in many ways: ⚀⚁, ⚀⚂, ⚀⚃, ⚀⚄, ⚀⚅, ⚁⚂, and so on. It is actually easier in this case to consider the complementary event in which both dice display the same numbers, ⚀⚀, ⚁⚁ up to ⚅⚅, and to subtract the probability of this event from one. A classical example of this idea is found in particle physics in the case of deep-inelastic scattering of an electron off a proton. The probability of this process to occur can be extracted from the 'complementary' process in which the colliding particles emerge undeflected from the collision. This is described mathematically in the *optical theorem*, which has been used in the past to determine the total cross section of deep-inelastic scattering in an efficient way.

In chapter 5, I study a different type of process: a collision between two protons that produces a lepton-antilepton particle pair (the Drell-Yan process). This process is interesting because it occurs frequently in current particle accelerators. But there is a problem if we wish to use unitarity here: in this reaction the optical theorem is not valid. The reason is that the final state of the process is quite specific (other final states are also possible), which means that the 'complementary' process is not simple either. By developing general techniques, I have nevertheless identified which contributions are to be subtracted from one. Thereby I have demonstrated that it is in fact possible to determine the cross section of the Drell-Yan process via unitarity; this promises that even more complicated corrections may become calculable.

Like Antoni van Leeuwenhoek discovered a world of invisibly small things through his microscope, I have focused on these selected topics to uncover yet another bit of knowledge in the field of elementary particle physics.