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Towards a Class number Formula for Drinfeld Modules

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Abstract

Let F be the function field of an irreducible, smooth, projective curve over a finite field. Let A be the ring of functions on the curve which are regular away from a fixed closed point ∞. Let F_∞ be the completion of F at ∞.

Consider an integral model φ of a Drinfeld A–module over a finite extension of F. We associate to such a model an element (the L–value of φ) of F_∞, mimicking the residue at 1 of the Dedekind zeta function of a number field, or the top coefficient at 1 of the L–function of an elliptic curve over $\mathbb{Q}$. The class module of φ is an A–module of finite cardinality, which serves as an analogue of the class group of a number field, or the Tate–Shafarevich group of an elliptic curve. The regulator of φ is an invertible sub–A–module of F_∞ and looks like the regulators of number fields and elliptic curves. The aim of this thesis is to present a conjectural formula which relates the L–value, the class module and the regulator of φ. It could be regarded as a function field analogue of the class number formula for number fields or the BSD conjecture for elliptic curves; hence the name "a class number formula for Drinfeld modules."

Our conjecture is a direct generalisation of a theorem by Taelman which states that the formula holds if $A = \mathbb{F}_q[t]$. The current manuscript generalises his work, by introducing the analogues of his L–values, class modules and regulators for general A, by stating a conjectural formula, and by proving it for all A which are principal ideal domains. We also show how our techniques can be used to prove the class number formula in specific cases, for example, when the curve has genus zero.