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Holography with broken Poincaré symmetry

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Since Newton's times we appreciate the fact that the behaviour of any physical system is governed by the simple and beautiful laws. More specifically, once the relevant interactions have been specified we can 'predict' the evolution of the system. On the most basic level we distinguish between four fundamental interactions: electromagnetism, weak force, strong force and gravity. The first three of them are described using the so-called gauge theories. Physicists know how to quantize such theories (at least perturbatively). Quantum versions of these three interactions are united in the so-called {\it standard model} of elementary particles. From the experimental point of view this is arguably the most successful scientific theory created by human beings by now.

Modern situation with the fourth fundamental force - gravity - is different. Classically the Einstein's theory of gravity works remarkably well. So well that it is extremely hard to modify it in any way which would be consistent with observations. However, construction of a quantum theory of gravity is an unsolved problem. Gravity is just very different from other fundamental forces. Many attempts to marry gravity with quantum physics have led to unresolved conceptual puzzles, which are keeping several generations of theoretical physicists baffled.

String theory is one of the frameworks where we can probe some quantum aspects of gravity. On the one hand string theory is a candidate for a quantum theory of gravity on its own. On the other hand it gave rise to a concrete realisation of the so-called {\it holographic principle}, according to which some (quantum) theories of gravity secretly are lower dimensional gauge theories. In other words, gravity is equivalent, or dual, to lower dimensional scale-invariant quantum field theory (QFT). Holographic principle is believed to hold both at the classical and quantum level.

Holographic duality to some extent may be viewed as an alternative definition of gravity. With the precise holographic dictionary in hand we can translate any question about gravity into a question about quantum field theory. Possible difficulties in answering corresponding question on the field theory side may well be more of a technical rather then conceptual character.

Having this new perspective on gravity has been very useful when applied in another direction, i.e. for studying quantum field theories with the help of gravity. Usually holography connects a strongly coupled QFT to a weakly coupled classical gravity, which a priori is a much simpler theory. Understanding physical phenomena at strong coupling rarely can be done using standard QFT techniques analytically. Even computers are not useful so far for some types of problems. Using holography such hard problems sometimes can be reformulated in the dual gravitational language where they become much more tractable. For instance, universal benchmarking results have been obtained in this way for certain transport properties of strongly coupled (QCD-like) quark-gluon plasmas.

Usually the holographic duality connects gravity on the anti-de Sitter (AdS) space to a scale invariant relativistic field theory, often referred to as conformal field theory (CFT). AdS is a particular maximally symmetric solution to Einstein equations with negative cosmological constant. Its isometries coincide with the conformal symmetry group. In the AdS/CFT context the holographic dictionary has been established to great precision and detail. It is natural to ask, if one could extend the AdS/CFT correspondence to the spacetimes which are not asymptotically AdS. It is widely believed that the holographic principle holds for a much broader classes of spacetime asymptotics. However at present only the very special extensions of the standard AdS/CFT correspondence are understood. Often these are obtained as some kinds of analytic continuation or continuous deformation on the either side of the correspondence. One way to modify the AdS asymptotics is to introduce irrelevant deformations on the field theory side. Then if one can keep the theory under computational control one will find a new field theory at high energies and a new non-AdS spacetime on the gravity side. In such cases the original holographic dictionary often gives rise to a natural dictionary in the new duality. This philosophy is followed in a major portion of this thesis, where we study holography for the so-called Lifshitz spacetimes.

Condensed matter physics is a rich source of strongly coupled scale-invariant systems. Many phase transitions (like in the case of the high-T_c superconductivity) are so interesting and hard due to the strong coupling physics involved. When the standard methods do not work one is tempted to model corresponding systems using holography. Holographic approach does not directly identify the theory explaining the corresponding real world behaviour. Nevertheless it may well serve as an inspiration for the new classes of theories which are more likely to describe real materials and for some reason were overlooked before. One should keep in mind, however, that a priori there is little reason to believe that holographic approach should provide appropriate description of any given real-world system. Usually one hopes that holography captures at least some universal features of the system in question.

One general obstacle for applying holographic duality directly to condensed matter problems is the non-relativistic nature of the most real-world systems. At the very least the time and space usually behave very differently. Holography, on the other hand, is best understood in the relativistic (i.e. Poincar{\'e} invariant) context. This thesis deals with some generalisations of the holographic principle to the cases with broken Poincar{\'e} invariance.

The first part of this thesis deals with the holographic approach to the study of the so-called Lifshitz invariant field theories. These are theories with anisotropic scale invariance, where time and space scale differently at the critical point. This anisotropy must be reflected in the dual gravitational description, i.e. the dual spacetime should enjoy these non-relativistic symmetries. Such spacetimes have been constructed as solutions of some simple gravitational theories. However, usually it is not clear how such spacetimes should be interpreted from the dual QFT point of view. Hence it is also not known to which extent such constructions can be connected to the real world phenomena. The spacetime with anisotropic asymptotics cannot correspond to relativistic theory. In this thesis we manage to clarify the field theoretical interpretation of Lishitz invariant spacetimes in the context of a particular gravitational theory, known as the Einstein-Proca model. It turns out that in this case one can view the dual field theory as a particular irrelevant deformation of underlying relativistic QFT. This deformation breaks relativistic invariance and changes the nature of the critical point making it Lifshitz invariant. As expected from the QFT perspective, irrelevant deformations are hard to keep under analytical control in general. However one can proceed systematically using a perturbative expansion in the deformation. This approach allowed us in this thesis to study such deformations both from the gravitational and field theoretical point of view. The resulting picture is very coherent. It provided new understanding of Lifshitz holography and helped to uncover a whole new universality class of Lifshitz invariant quantum field theories. Moreover, as it turned out this new class of theories is not completely disconnected from reality. A representative of this type of Lagrangians has appeared few years ago in the theoretical models explaining 'zig-zag' phase transitions in one-dimensional electron chains! Some results obtained in those studies follow directly from our general analysis.

In the second part of the thesis we consider the so-called conformal defects. Such defects break translational invariance in ambient space, but scale invariance on the defect is still preserved. Such defects are ubiquitous in condensed matter physics. Again it is of interest to see what holographic dualities can teach us about the physics of such systems. However in this case, before one could apply holographic approach to study such defects one is faced with another problem. One has to construct spacetimes which would reflect the symmetry group of the conformal defects. It turns out that it is not easy to find gravitational theories which would allow simple solutions with required properties. In this thesis we have developed a formalism which simplifies this problem significantly. This formalism is similar in spirit to the so-called 'superpotential' formalism for constructing Poincar{\'e} invariant asymptotically-AdS domain wall solutions. In particular it reduces a system of the second order coupled ordinary differential equations to a set of the first order decoupled equations. This method allowed us to construct new simple families of theories and their classical solutions corresponding to conformal boundaries and interfaces. We used constructed solutions to perform holographic computation of some basic observables characterising the nature of such defects. We hope that our method will find its applications in the active field of research on conformal defects.