UNIVERSITY OF AMSTERDAM

Computational Techniques in Queueing and Fluctuation Theory

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Summary

Thesis title: Computational techniques in queueing and fluctuation theory

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Lévy processes, i.e., processes with stationary and independent increments, play an important role in applied probability. They have widespread applications, ranging from insurance and financial mathematics to operation research and even biology. One of the main branches of research on Lévy processes concentrates on analyzing the probabilistic properties of the supremum (or infimum) attained by the process over a given period of time, usually referred to as the running maximum (or minimum). This topic is commonly known as *fluctuation theory.*

The main objective of this thesis is to develop numerical techniques to calculate the probability distribution of the running maximum of Lévy processes, and consider a number of specific financial applications. The other objective is to propose a numerical method to optimize the energy consumption of servers handling traffic in a communication network. The traffic itself is modeled by a random process, usually an on-off process with random on- and off-times.

In the second chapter of this thesis, a numerical technique based on the Wiener-Hopf factorization is developed to evaluate the probability distribution of the running maximum (or minimum) of a general Lévy process (i.e., a Lévy process with possibly two-sided jumps). Thanks to the *numerical Laplace and Fourier inversion* technique developed by den Iseger, we are able to numerically compute the probability with almost machine precision. This approach has a variety of potential applications.

In Chapter 3, we primarily focus on applying the technique developed in Chapter 2 to price specific exotic options, viz. the so-called *lookback option.* However, the method can be employed for pricing many other options which depend on the maximum and/or minimum attained by the underlying Lévy process, for instance the barrier option.

The second technique which is presented in this book is *importance sampling*; see Chapter 4. This technique is essentially used to reduce the variance of the simulation-based estimator. Straightforward simulation for estimating rare event probabilities being inefficient and inaccurate, the idea of importance sampling is to generate simulation paths under an alternative measure such that the event is not rare anymore. Obviously the simulation results have to be corrected by an appropriate likelihood ratio to obtain an unbiased estimate. The main challenge of this method is to find the appropriate alternative measure and corresponding likelihood ratio, which we succeed to find in our Lévy setting.

Energy-aware processors are intended to operate efficiently by adapting the speed of the server CPU to the processing load and the service level requirement. In Chapter 5, we consider a performance objective which is a linear combination of energy usage, queuing cost (reflected by delay) and speed switching cost for a multi-core processor. Our analysis captures the static power as well as the dynamic power of the processor. Our main contribution is that we propose a stochastic fluid model for the analysis and optimization of such multi-core processing systems. We discuss several schemes that lead to energy consumption reduction. We show that the optimal strategies are robust under perturbations of the system parameters and statistical properties of the traffic. This robustness property makes the use of the optimal strategies highly attractive in practical situations.