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The Quench Action Approach to Out-of-Equilibrium Quantum Integrable Models

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Summary

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TO OUT-OF-EQUILIBRIUM
QUANTUM INTEGRABLE MODELS

In part motivated by the spectacular advances in realizing cold-atom experiments, the study of relaxation in isolated many-body quantum systems has attracted much theoretical interest during the past decade. The two basic questions concern the dynamics and equilibration after a system is brought far out of equilibrium. What are efficient ways of describing the time evolution of physical observables and what does this tell us about the underlying physics? Do isolated out-of-equilibrium quantum systems evolve towards an equilibrium and, if so, what determines this equilibrium?

This thesis addresses these questions from the perspective of quantum integrable systems in one dimension. In particular, it aims to describe a new, powerful method, coined the quench action (QA) approach, that purports to give an exact description of the time evolution and the equilibrium of local observables in infinitely large systems after a so-called quantum quench. We present a detailed introduction to the method, its first successful implementations for interacting models and the interesting fundamental questions their results have initiated.

For more than half a century the subject of quantum integrable systems has been a very active field of research, in particular by means of the Bethe Ansatz. The exact knowledge of the full spectrum and Slavnov's formula for overlaps makes it a very powerful theoretical tool. Furthermore, current experimental realizations are in good approximation described by quantum integrable models. Important topics were (and still are) the properties of the spectra of integrable models, the structure of the ground state and low-energy excitations, the computation of matrix elements of physical observables and the limit of large system size. Examples of integrable models are the Lieb-Liniger model, the (an)isotropic Heisenberg spin chain, the Hubbard model, the Kondo and Anderson impurity models and the class of Richardson-Gaudin models.

Triggered by a number of seminal experimental and theoretical findings, during the past decade the study of these models has somewhat shifted towards their out-of-equilibrium properties. In the renowned experiment of the quantum Newton's cradle an integrability-broken one-dimensional Bose gas out of equilibrium is found not to thermalize. This surprising phenomenon is often attributed to integrability, though a full theoretical understanding is still lacking. Not unrelated is the theoretical concept of a generalized Gibbs ensemble (GGE). The extra local conserved charges in a quantum integrable model are thought to deform the thermal Gibbs ensemble and thereby alter the equilibrium expectation values of local observables. There is ample evidence that the GGE works for free theories and recently a first experimental observation of the GGE came from the Schmiedmayer group.

The important concept of a global quantum quench, where a quantum system in a Hamiltonian eigenstate is brought out of equilibrium by suddenly changing one or more parameters of the model, has proved to be the ideal vehicle to study out-of-equilibrium phenomena in quantum integrable models. However, problems involving

the postquench time evolution of observables generally remain hard nuts to crack. The main reason is that brute-force computations encounter a double sum over the Hilbert space, whose number of terms grows exponentially with system size. Furthermore, until recently applications of the GGE to truly interacting systems had been limited, rendering stringent tests of its conjecture absent.

The logic of the QA approach is completely different from conventional numerical or GGE methods. Contrary to for example the GGE, it is a method based on first principles. It predicts the exact postquench expectation values of typical physical operators at any time after a global quantum quench to a Bethe Ansatz solvable model in the strict thermodynamic limit. The problem of the exponentially large number of terms in the Hilbert space sums is overcome by means of a saddle-point approximation. The basic input of the method are the overlaps between the initial state and the eigenstates of the postquench Hamiltonian. Due to an interplay between overlaps and entropy, a single eigenstate effectively dominates the postquench equilibrium of typical physical operators while all other states are suppressed exponentially in system size. Expectation values long after the quench are computed on this representative state, while the full time evolution is recovered via summation solely over excitations in the vicinity of the representative state. Note that this logic is fundamentally different from the usual description in terms of a statistical ensemble, where a mixed state specified by a density matrix predicts all macroscopic properties of the postquench equilibrium.

In this thesis we derived the QA approach from first principles and specified validity conditions for observables and overlaps. Interaction quenches from the free boson gas to the repulsive Lieb-Liniger model and from the Néel state to the spin-1/2 XXZ chain were solved exactly in the thermodynamic limit. Without the QA approach, strict thermodynamic and infinite-time limits for quenches to truly interacting systems generally remain unattainable. We also used the QA method to model the Bragg pulse on hard-core bosons.

The main achievements of the results presented in this thesis are as follows. Through our work we have established the broad applicability of the QA approach to quench problems in quantum integrable models. This mainly depends on the availability of overlaps and of their scaling towards the thermodynamic limit. Considering the universal features of the overlaps presented in this thesis, there is good hope that more general classes of quenches can be studied in the near future. The extension of the QA approach to the time evolution of simple correlators in spin chains is another pressing open problem. Second, we found a failure of the GGE based on all known local charges for the Néel-to-XXZ quench. Even the simplest nearest-neighbor correlators are predicted wrongly by the GGE and depending on the initial state the error can become of order one. A deeper understanding is still lacking, but will likely give us more insights into the physics of relaxation in quantum integrable systems. Third, we have seen a separation of relaxation timescales in the quantum Newton's cradle for a gas of hard-core bosons. After a Bragg pulse there is a short phase of rapid relaxation towards a temporary steady state that is governed by the GGE, followed by the oscillation in the trap at much longer timescales. Extensions of our methods to longer pulses and finite interaction parameter could provide key insights in the underlying physics of the absence of thermalization in the quantum cradle experiment.