



*The Logic of Kant's Temporal Continuum*

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In this thesis I provide an account of the philosophical foundations and mathematical structure of Kant's temporal continuum. I mainly focus on the development of a formalization of Kant's temporal continuum as it appears in the Critique of Pure Reason and in other works of Kant's critical period; most of my results, however, are generally relevant for the problem of developing mathematically rigorous foundations for a phenomenological concept of the continuum. In particular, I argue that the general topological form of the Kantian continuum is that of the Alexandroff COTS: a totally ordered topological space that is the space of Kantian instants on the limit of all finite models of a first-order theory that formalizes Kant's notion of the temporal form of an experience.

The formal apparatus of the thesis is also applied to the elucidation of the elusive distinction, at B161n of the Critique of Pure Reason, between space and time as "forms of intuition" and as "formal intuitions". In particular, I argue that the formal intuition is produced by the action of what Kant terms the figurative synthesis or "synthesis speciosa", which consists in the self-affection of the subject in the description of spaces in outer sense in agreement with the categories. Thus, I propose a conceptualist reading of the distinction at B161n which holds that space and time as formal intuitions are produced in the act of description of particular spaces or times by the figurative synthesis, even though no particular such description is space or time as formal intuition. Moreover, while I argue that the formal intuition is nothing over and above the form of intuition of the TA, I also maintain that there exists in the Critique a distinction between the formal intuition and a purely passive notion of form of intuition that cannot be ignored. This distinction is, however, not sharp but graded, in that different levels of "formality" can be identified. This interpretation of the distinction between the form of intuition and the formal intuition is supported by the formalization.

Finally, I also show that my analysis of the Kantian continuum subsumes and extends Russell's and Walker's constructions of instants from events and that it is closely related to point-free topology in the predicative and constructive tradition of formal topology. This paves the way for a constructive and predicative treatment of bitopological spaces in formal topology and for reviving the Russell-Walker-Whitehead project of constructing relativistic spacetimes from events; I argue that this would shed further light on the foundations of relativity and on the viability of the causal set approach to quantum gravity.

The thesis is structured as a progression from informal philosophical

analysis in the first part to technical mathematical treatment in the second part. In particular, the first two chapters are philosophical: chapter 2 provides a justification for the use of mathematical methods in the exegesis of systematic philosophy, while chapter 3 fleshes out my interpretation of the distinction between the form of intuition and the formal intuition. Chapter 4 is in part philosophical and in part technical, since I provide an overview of the basics of the formal theory, but without proofs and focusing on its philosophical import in relation to Kant's theory of the continuum and the distinction between form and formal intuition. Chapter 5 is technical and is where the main mathematical results of the thesis, and in particular the topological construction of the Kantian continuum, are given. Finally, chapter 6 relates the mathematical framework of the thesis to the constructions of time from events proposed by Russell and Walker and to point-free topology. I show that Russell's and Walker's constructions are special cases of the construction of Kantian instants, so that the two constructions can be unified and given a clear topological interpretation, which sheds light on the debate regarding the most satisfactory construction of instants of time from events. I also show that my account of Kant's continuum is closely related to point-free topology in the constructive and predicative tradition of formal topology, and argue that reviving the Russell-Walker-Whitehead project of constructing relativistic spacetimes on these grounds promises to deliver useful insights in the foundations of relativity.