



*Multivariate Density Forecast Evaluation and Nonparametric Granger Causality Testing*  
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# Summary

This dissertation, titled “**Multivariate Density Forecast Evaluation and Nonparametric Granger Causality Testing**”, is devoted to the study of the evaluation of density forecasts and conditional dependence structure for multivariate time series. Specifically, this dissertation tries to answer the following questions: (1) For a given portfolio of financial assets, what is the most appropriate distribution to assume, supposing there are several candidate distributions, and what is the most relevant modeling method, univariate or multivariate, for the purpose of risk management? (2) How to use simulation-based methodologies to quantify the transfer entropy, and measure conditional dependence between two time-series, if the asymptotic theory of a formal statistical test for conditional independence, i.e. Granger non-causality is missing? (3) Is it possible to develop a robust statistical test for nonparametric Granger non-causality by measuring the transfer entropy; and if so, how does it perform compared to existing tests?

In the first chapter, the motivation of this dissertation is briefly summarized. In particular, time series econometrics has been greatly reshaped in recent decades; the literature has gradually shifted its focus from linear modeling and point forecast to nonparametric modeling and density forecast in many dimensions.

Chapter 2 investigates the first question by comparing multivariate and univariate approaches to assessing the accuracy of competing density forecasts of a portfolio return in the downside part of the support. The common practice to perform multivariate forecast comparisons can be problematic in the context of assessing portfolio risk, since better multivariate forecasts do not necessarily correspond to better aggregate portfolio return forecasts. This is illustrated by examples involving (skew) elliptical distributions and an application to daily returns of a number of US stock prices. Additionally, time-varying test statistics and Value-at-Risk forecasts provide empirical evidence for regime changes over the last decades.

Chapter 3 provides numerical comparisons for simulation procedure-based tests to gain some insights into the statistical behavior of an information theoretic measure — transfer entropy — in the context of detecting conditional independence, i.e. Granger non-causality in a more general sense. Namely, time-shifted surrogates algorithms, the smoothed bootstrap and the stationary bootstrap procedures are presented and compared. By using those proposed resampling techniques, a financial application is provided to illustrate how to detect pairwise

Granger causalities nonparametrically.

Chapter 4 introduces a novel nonparametric test based on a first order Taylor expansion of the transfer entropy. The new test statistic is shown to have an information-theoretical interpretation in terms of Granger non-causality. The proposed test avoids the lack of power of the frequently-used test proposed by Diks and Panchenko (2006), which is not consistent against specific fixed alternatives. Attributed to the U-statistic representation, asymptotic normality of the proposed test statistic is achieved when all densities are estimated with the appropriate sample-size dependent bandwidth. Simulation results confirm the theoretical size and power properties of this test. Finally, two applications to financial data at daily and intra-day frequency conclude this chapter.

Concerning the future work, there are several directions for potential extensions. The method discussed Chapter 2 can be generalized to compare multi-step ahead density forecasts, or to compare nonparametrically estimated densities. Chapter 3 leaves an open question of exploiting entropy-based statistics in testing conditional dependence when there exists a so-called common factor, i.e., looking at the multivariate system containing more than two variables. One potential candidate for this type of test is the partial transfer entropy, coined by Vakorin, Krakovska, and McIntosh (2009). On the theoretical side, it would be practically meaningful to consider causal linkage detection incorporating more lags in a multivariate setting, which requires new asymptotic theory.