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A Unit-Aware Matrix Language and its Application in Control and Auditing

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Support for units of measurement is the main topic of the research project described in this thesis. In this interdisciplinary project, knowledge from the fields of computing and auditing is combined to investigate the possibilities of unit-aware organization modeling. Such modeling also requires computations in proper units of measurement and thus a unit-aware programming language. The first part of this thesis is about unit-aware matrix programming. It describes an investigation into the possibilities of a matrix type based on unit inference with dimensioned vectors spaces. In the second part these ideas are applied to construct unit-aware value cycle models from the field of control and auditing.

Units of measurements are an indispensable part of numerical data and provide obvious opportunities to increase the safety of software, yet programming language support for units is not common. Most programming languages are well equipped to handle numbers, but support for units is mostly absent because it would create enormous overhead. A new technique that eliminates the overhead was introduced by Andrew Kennedy. His inference for units of measurement is part of a static type system and guarantees unit correctness at compile time. This eliminates the overhead, but because it is based on a parametric type it is unfortunately limited to data structures in which all numbers have the same unit. This restriction to homogeneous units makes it unsuitable for compound data structures like vectors or matrices where the units may vary.

The question addressed in the first part of the research is how to apply unit inference to matrix programming. If unit inference is lifted from single numbers to vectors and matrices then the restriction of homogeneous units can be eliminated. The matrix type developed during the research extends unit inference from single numbers to vectors and matrices. The structure of the matrix type is based on Hart's dimensioned vector spaces. In such a dimensioned vector space a unit of measurement is associated with each vector entry. The matrix type infers the dimensioned vector space for any linear algebra expression. This generalizes unit inference from dimensioned numbers to dimensioned matrices resulting in complete type inference for dimensioned linear algebra.

Researching support for units of measurement is not just motivated by safety concerns, but also by the fundamental issue of valuation that underlies financial information. Financial information has the advantage that it is in a single currency. This means that any two numbers are always comparable and can be summed or subtracted without a problem. The caveat is however that information has been lost during the conversion of data to the single currency. Financial data always involves a valuation that hides the underlying non-financial facts. The second part of the thesis studies the feasibility of reformulating the value cycle models from the fields of auditing and control in non-financial units of measurement. It uses the results from the first part to generalize the value cycle and make it available for other applications.

The value cycle is a key concept in Computational Auditing, an ongoing research effort to formalize the foundations of auditing. The value cycle gives a top-level view of the flow of values in an organization's core processes with a focus on the causality and proportionality between the different process steps. This top-level view helps auditors reason about normal behavior in an organization, but since such analysis is done with financial information it is limited due to the unavoidable loss of information. Value cycle models in non-financial units of measurement would allow more accurate modeling and increase the value cycle's applicability.

The question addressed in the second part of this research is how the analytical features based on the value cycle's causality and proportionality can be generalized from auditing to other applications. The concept of normal behavior that the value cycle provides is useful in areas besides auditing.

To generalize the value cycle concept the value net formalism was developed. Value nets are a generalization of the audit net formalism from Computational Auditing. It is a formalization of the value cycle concept, but in non-financial units of measurement. Value nets provide a valuation-free characterization of normal behavior in organizational processes. The use of non-financial units in value nets allows it to formulate normal behavior as a solution to a steady state equation. Each of these solutions forms a tour, an instantiation of the value cycle's cyclic structure. Together, the tours form a tour space that completely describes an organization's normal behavior. Since it is in non-financial units of measurement it is more accurate and free of valuation, which increase the potential applications.

As a proof of concept the value net formalism was implemented using the matrix type. The implementation is fully unit-aware and gives a more accurate foundation for the analysis of organizational processes.

To validate our approach we have modeled and analyzed three small cases: a miller, a bakery, and a wine bottler. Together with experts from the auditing domain we have applied it to a larger case regarding revenue models and dispute resolution in public transport services.