Essays on Mathematical and Computational Finance. With a View towards Applied Probability

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Summary

The primary goal of this monograph is to contribute methodologically to the pricing of options, and their numerical evaluation. To price an exotic option, one needs to:

1. choose a model that is both economically plausible as analytically tractable;
2. calibrate the model to market prices of plain vanilla options;
3. price the exotic option of the calibrated model using appropriate numerical techniques.

This monograph deals with all three steps. The tools used in this thesis to compute prices and sensitivities of complex options are rooted in probability theory, which explains the subtitle *"a view towards applied probability"*. Market fundamentals are modeled using stochastic differential equations (SDEs) and probabilistic techniques are used to compute conditional expectations. There is an important alternative to our approach, namely via partial differential equations (PDEs).

This thesis is written in 4 chapters. The structure is as follows;

Chapter 2 focuses on numerical evaluation techniques related to fluctuation theory for Lévy processes. They can be applied in various domains, e.g., in finance in the pricing of so-called barrier options. The starting point is the so-called *Wiener-Hopf factorization*, which yields an expression for the *Laplace transform* of the running maximum at an exponential epoch. It is shown how to use Laplace inversion techniques to numerically evaluate the density of the running maximum. In our experiments we rely on the efficient and accurate algorithm developed by den Iseger. We illustrate the performance of the algorithm with various examples: Brownian motion (with drift), a compound Poisson process, and a jump diffusion process. In models with jumps, we are also able to compute the density of the first time a specific threshold is exceeded, jointly with the corresponding overshoot. The paper is concluded by pointing out how our algorithm can be used in order to analyze the Lévy process' concave majorant.

In Chapter 3 we propose an intuitive, practical, Lévy-based, dynamic default correlation model, with applications to CDO pricing. In developing this model, we first capture the marginal default probabilities relying on a dynamic structural *Variance Gamma (VG)* process. Then we impose a dynamic correlation structure on the individual obligors, of which the marginals are in line the VG-based model for the single names. This correlation model is constructed by decomposing the VG processes into two components: an individual component and a common component; the former affects just one obligor, whereas the latter has impact on the entire market. The key advantages of our correlation structure are that it is intuitive, dynamic, and allows for easy calibration to the market since the underlyings are market observables, for which there is ample data available. In case of a homogeneous basket of obligors, the evaluation of our model is as easy as a copula-based approach. In case of a non-homogeneous basket, the computations can still be done in a fast and accurate way. An important role is played by recently developed techniques for numerical Laplace transform inversion developed by den Iseger. Our approach is backed up by various numerical experiments.

In Chapter 4 we present the Drift Adjustment Method Laplace Algorithm (DAMLA) for efficiently and accurately calculating conditional expectations of *Ornstein-Uhlenbeck (OU)*

type processes. It is a novel approach in the sense that we can efficiently and accurately calculate conditional expectations, in high dimensions, at each time point in each scenario. Broadly speaking, the method we present can be used in Monte Carlo simulations as well as in quadrature methods. Applications are numerous. We use DAMLA for the simultaneous hedging of the interest and equity risk of a pension fund. More specifically, we price and determine the Greeks of an equity-linked swaption, commonly called hybrid option, to hedge pension fund risk. We find that, by combining the Drift Adjustment Method (DAM) with numerical Laplace transform inversion, i.e. the Drift Adjustment Method Laplace Algorithm (DAMLA), computational speed is greatly enhanced as compared to direct calculation (Monte Carlo). We can price and hedge a portfolio of 100,000 equity-linked swaptions in under 5 minutes with errors of the order 10^{-12}. Finally, we will extend DAMLA to models driven by arbitrary Lévy processes, and show we can exactly proceed as in the diffusion case.

The final chapter, Chapter 5, deals with a correlated overflow model. We present a multi-dimensional overflow problem and due to the ordering of the components, explicit results are obtained. In our setting, each component behaves as a compound Poisson process with unit-sized upward jumps, decreased by a linear drift. The approach relies on a *Beneš-type* argumentation. That is, the idea of partitioning the overflow event with respect to the last "exceedance epoch". It is pointed out how the results can be used in credit risk modeling.