Theory of Interpretation (Day One)

Exclusive Deduction

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There is no paradox of material implication
if there is no material implication

There is no paradox of material implication
if there is no material implication

"Theory of Interpretation by Translation"

• What is it supposed to mean?
• What is it good for?
• How do we do it?

When interpretation is our aim, a method of translation deals with a wrong topic, a relation between two languages, where what is wanted is an interpretation of one (in another, of course, but that goes without saying since any theory is in some language).


So it is the right topic, if properly understood.
Day One: Exclusive Deduction  
Theory of Interpretation by Translation

Theory of Re-Presentation

Present in one’s own words what has been said.

Serve Common Understanding and Formal Reasoning

What does “So_und_So.” mean?
It means “Such_and_Such.”

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A Mathematician’s Credo


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The General Endeavour

A Piece of Cat.  A Piece of Language.

• We easily recognize both.
• The signs signal something.
• We do and we do not see the sign mean something.

To say what a piece of language means.

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Making Sense of Real Language.

• Contextualist.

Pieces of language have no meanings in themselves.

• Actualist.

Meanings, if anything, are types of interpretation of occurrences of types of expressions.

• Intentional.

The assumption that a piece of language signals something, stands for something, presents something, is an idealist assumption.

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Regimentation of the Language

- A theory of interpretation requires a language—under construction—, in which to present the interpretations.
  - It should be well-defined and enable us to agree.
    - Artificial, formal, and recursive.
  - Ontologically non-committal.
    - What there is, remains to be seen, or decided upon.
    - Consistence with any empirical theory.
  - Based on transcendental assumptions only.
    - Exclude the exclusion of the possibility of meaning.
    - Consistence with any theory of meaning.

Foundational Assumptions (Unnegotiable, This Week)

To understand a language (what it is, what it does) one must understand a form of life, that is, a world in which it figures and in order to understand the life-world, one must understand its language, which is used to configure it.

I embrace the idea that there is one and same world to all of us. This is kind of inevitable, and something which I don’t have any ambition or pretense, to escape from or to explain.

I also choose to assume that you understand my language to some extent as being in and about a shared world, and that I understand yours, and that you do the same in return.

I start this course from this very assumption, and I will also explicitly use it at times. I also assume that you do likewise, and that we all assume we agree on the meanings of the expressions employed in this class.

Picturing Our Situation

Our situation can be pictured, significantly, as follows:

\[
\begin{align*}
WE, i, y, \ldots \\
i \in WE \\
y \in WE \\
\vdots 
\end{align*}
\]

in the University of Ireland, Galway, August, 2022, LoI: English.

Outline of the Course

Theory of Interpretation by Translation

- Logic
- Re-Presentation
- Intentionality

- Minimal
- Indexical
- Names
- Predicates
- Attitudes
- Non-Existence

Monday Tuesday Wednesday Thursday Friday
Excluding the Sheffer Stroke

- We can do propositional logic with one connective only.
  [Charles Sanders Peirce, Ludwig Wittgenstein]
  
- Preferably Henry Maurice Sheffer’s Stroke: ‘|’.
  ‘(p | q)’ reads: ‘not both p and q’.
  
- We can do better than that.
  (r | s) can be rendered as (rs).
  (rr) can be rendered as (r).
  Generally, we may write (s).
  * for any finite string s of propositions.
  
- And that is what we will do.

Translation

Define translations \( \bigcirc \) from CPL to EPL and back \( \bigodot \) as follows.

\[
\begin{align*}
\bigcirc p &= p \\
\bigcirc \bot &= ( )
\end{align*}
\]

\[
\begin{align*}
\bigodot \phi \land \psi &= \bigodot \phi \bigodot \psi \\
\bigodot \phi \lor \psi &= (\bigodot \phi (\bigodot \psi)) \\
\bigodot \phi \rightarrow \psi &= [\bigodot \phi \rightarrow (\bigodot \psi)]
\end{align*}
\]

Note that \( \top = \bigodot \) the empty string: “Whatever,”
and that \( \bot = ( ) \): “Exclude anything.”

- Let \( P \) be some set of atomic propositions \( q, \ldots \).

Definition (EPL Syntax)

\( \mathcal{L}_{EPL} \) consists of propositions \( p \) and strings \( s \) such that, for \( q \in P \):

\[
\begin{align*}
p &:= q \mid (s) \\
s &:= p^*
\end{align*}
\]

- A proposition is either an atomic proposition \( q \in P \)
or the exclusion \( (s) \) of a string \( s \).
- A propositional string is a (finite) sequence \( s \) of propositions.
  
  ‘(s)’ reads: ‘\( s \) is excluded’.
  ‘(rst)’ also reads: ‘\( r \) excludes \( st \), or \( rs \) excludes \( t \), etc.

[Note: \( r, s \) and \( t \) may be empty.] [Not unimportant.]

Definition (EPL Semantics)

Satisfaction \( \models \) of \( \mathcal{L}_{EPL} \)-expressions relative to \( V \) is defined so that:

\[
\begin{align*}
V \models q \text{ iff } V(q) = 1; \\
V \models (s) \text{ iff } NOT \ V \models s; \\
V \models s_1 \ldots s_j \text{ iff } AND_{1 \leq i \leq j} V \models s_i.
\end{align*}
\]

A sequence \( s \) of strings entails string \( t \), \( s \models t \), iff for every valuation \( V \):

\[
V \models s \text{ implies } V \models t.
\]

\[
V \models_{cpl} p \text{ iff } V \models_{cpl} p^{\bigodot} \text{ iff } V \models_{cpl} p^{\bigcirc}.
\]
### Definition (EPL Deduction)

A conclusion $t$ can be deduced from a sequence of assumptions $s$, $s ⊢ t$, if $s = s_1, \ldots, s_i$ and there is a derivation:

1. $s_1$ [ass.]
   - without pending assumptions
2. $s_i$ [ass.]
   - between lines $i$ and $n.$
3. $t$

A derivation is a list of strings each member of which is either an assumption, or licensed by one of the inference rules stated below.

### Inference Rules

#### Exclusion ($X$)

$\vdash$

1. $\bot$ [ass.]
2. $\bot$ [ass.]
3. $\bot$ [X, 1, 2]
4. $\bot$ [R]
5. $\bot$ [C]

#### Retraction ($R$)

$\vdash$

1. $r$ [ass.]
2. $r$ [ass.]
3. $\bot$ [ass.]
4. $\bot$ [R]
5. $\bot$ [X, 1, 4]
6. $\bot$ [R]

#### Contradiction ($C$)

$\vdash$

1. $\bot$
2. $\bot$
3. $\bot$
4. $\bot$
5. $\bot$
6. $\bot$

- And that is it.

### Results and Proofs

#### Sequitur Verum

$\vdash$

1. $\bot$ [ass.]
2. $\bot$ [ass.]
3. $\bot$ [ass.]
4. $\bot$ [R]
5. $\bot$ [X, 1, 2]
6. $\bot$ [C]

(L) The exclusion of the empty string leads to (is) a contradiction.
(R) Alternatively: the repetition of nothing.

#### Ex Falso Sequitur Quodlibet

$\vdash$

1. $\bot$ [ass.]
2. $\bot$ [ass.]
3. $\bot$ [ass.]
4. $\bot$ [R]
5. $\bot$ [X, 1, 4]
6. $\bot$ [R]

- The exclusion of anything excludes everything.
Derivable Rule: Subtraction (−)

\[ ... \\
  i. rst \\
  ... \\
  n. s \ [\cdot, i] \]

\(rst \vdash s\) subsumes that \(rs \vdash s\) ("R-conjunct Elimination")
\(st \vdash s\) ("L-conjunct Elimination")
\(s \vdash s\) ("Repetition")

Proof

\[ \begin{align*}
  1. & \ \ rst & [ass.] \\
  2. & \ \ r & [ass.] \\
  3. & \ \ s & [ass.] \\
  4. & \ \ t & [ass.] \\
  5. & \ (s) & [X, 3, 5] \\
  6. & \ () & [R] \\
  7. & \ (s) & [R] \\
  8. & \ (t(s)) & [X, 1, 10] \\
  9. & \ (st(s)) & [C] \\
  10. & \ (rst(s)) & [R] \\
  11. & \ (s) & [X, 3, 5] \\
  12. & \ s & [C] \\
\end{align*} \]

Derivable Rule: Addition (+)

\[ ... \\
  i. rt \\
  ... \\
  j. s \\
  ... \\
  n. rst \ [\cdot, i, j] \]

\(rt, s \vdash rst\) subsumes that \(r, s \vdash rs\) ("Conjunction Introduction")
\(t, s \vdash st\) ("Inverse Conjunction")
\(s \vdash s\) ("Repetition")

Proof

\[ \begin{align*}
  1. & \ \ rt & [ass.] \\
  2. & \ \ s & [ass.] \\
  3. & \ \ r & [\cdot, 1] \\
  4. & \ \ t & [\cdot, 1] \\
  5. & \ (rst) & [ass.] \\
  6. & \ (st) & [X, 3, 5] \\
  7. & \ (t) & [X, 2, 6] \\
  8. & \ () & [X, 4, 7] \\
  9. & \ (rst) & [X, 3, 5] \\
  10. & \ rst & [C, 9] \\
\end{align*} \]
**Generic Proof Format**

$s_1, \ldots, s_i \vdash t$ on the left can be shown by providing the derivation (TBP), i.e., by proving $\vdash (s_1 \ldots s_i(t))$ on the right, independently:

\[
\begin{array}{c|c}
\vdash s_1 \ldots s_i(t) & \vdash r_1 \ldots r_j(t) \\
\text{[ass.]} & \text{[ass.]} \\
\vdash \vdash & \vdash \\
\vdash r_j & \vdash \\
\text{[ass.]} & \text{[ass.]} \\
\vdash (t) & \vdash (t) \\
\text{[ass.]} & \text{[ass.]} \\
\vdash (r_1 \ldots r_j(t)) & \vdash (r_1 \ldots r_j(t)) \\
\text{[TBP]} & \text{[X]} \\
\vdash & \vdash \\
\end{array}
\]

where the concatenation of the series of premises $s_1 \ldots s_i$ equals the string of propositions $r_1 \ldots r_j$. Procedure? Next two frames.

---

**Generic Proof Procedure (1)**

If we already have that ( ) as, say, the $i$-th assumption, we solve:

\[
\begin{array}{c|c}
r_1 & r_1 \\
\text{[ass.]} & \text{[ass.]} \\
\vdash & \vdash \\
\vdash r_j & \vdash \\
\text{[ass.]} & \text{[ass.]} \\
\vdash (t) & \vdash (t) \\
\text{[ass.]} & \text{[ass.]} \\
\vdash (r_1 \ldots r_j(t)) & \vdash (r_1 \ldots r_j(t)) \\
\text{[TBP]} & \text{[X]} \\
\vdash & \vdash \\
\end{array}
\]

and we are done.

---

**Generic Proof Procedure (2)**

Or we have some $(r s)$ as, say, the $i$-the assumption, with $r$ a proposition. We then proceed by trying to solve (A) and (B):

\[
\begin{array}{c|c}
(A) & (B) \\
\vdash r_1 & \vdash r_1 \\
\text{[ass.]} & \text{[ass.]} \\
\vdash & \vdash \\
\vdash r_j & \vdash \\
\text{[ass.]} & \text{[ass.]} \\
\vdash (t) & \vdash (t) \\
\text{[ass.]} & \text{[ass.]} \\
\vdash (r_1 \ldots r_j(t)) & \vdash (r_1 \ldots r_j(t)) \\
\text{[TBP]} & \text{[X]} \\
\vdash & \vdash \\
\end{array}
\]

\[
\begin{array}{c|c}
\vdash (r) & \vdash (r) \\
\text{[ass.]} & \text{[ass.]} \\
\vdash & \vdash \\
\vdash r & \vdash \\
\text{[ass.]} & \text{[ass.]} \\
\vdash (r) & \vdash (r) \\
\text{[ass.]} & \text{[ass.]} \\
\vdash (r_1 \ldots r_j(t)) & \vdash (r_1 \ldots r_j(t)) \\
\text{[TBP]} & \text{[X]} \\
\vdash & \vdash \\
\end{array}
\]

and we are done.

---

**Completeness**

$s \models t$ iff $s \vdash t$

Soundness (if) is obvious.

Completeness (only if) follows from:

- Semantic validity in EPL corresponds to semantics validity in CPL.
- Translations of the classical deduction rules for $\neg$, and $\land$ (and $\rightarrow$) are derivable from (X), (R) and (C).
- The classical system is complete.

$\gg$ (X), (R) and (C) characterize propositional logical validity.
Implicative versus Exclusive Logic

Classical and Exclusive logic are not distinct, logically speaking.

There is some difference in their conception though.

Classical logics support conclusions.

Exclusive logic grounds exclusions.

Implication, or . . . ?

- Classical logics have been built upon the implication, and people have worried about the “paradoxes” of material implication, e.g.:
  \[ p \vdash (q \rightarrow p) \quad \vdash ((p \rightarrow q) \lor (q \rightarrow p)) \]

  First: these are no paradoxes.
  Second: one can only be worried because one has some ‘essentialist’ understanding of →, that it indicates a certain connection.
  Third: EPL fails such implications, and constructions out of them.

Or . . . Exclusion!

- ‘Implication’ comes with uninvited implications.
  \[ p \vdash (q \rightarrow p) \]
  [If \( p \), then anything (e.g., \( q \)) implies \( p \).]

- ‘Exclusion’ comes without uninvited implications.
  Implicative \( p \vdash (q \rightarrow p) \) translates into exclusive \( s \vdash (r(s)) \).
  \[ s \vdash (r(s)) \]
  [If \( s \), then nothing (e.g., no \( r \)) excludes \( s \).]

Material Implication

- “Too many” implications.
  \[ \vdash ((p \rightarrow q) \lor (q \rightarrow p)) \]
  [At least one of any two propositions implies the other.]

- None “too many” exclusions.
  The disjunction of the implications translates into:
  \[ (\neg (r(s)) \lor \neg (s(r))) \]
  which easily reduces to:
  \[ (r(s)) \lor (s(r)) \]
  [The exclusion of an obvious contradiction; two actually.]
Excluded Middle?

\[ \vdash (p \lor \neg p) \]

The translation of this disjunction reads:

\[ \vdash (s ((s))) \]

This can be innocently simplified into:

\[ \vdash (s s) \]

Which says that the exclusion of \( s \) excludes \( s \).

Double Negation Elimination?

\[ \vdash (\neg\neg p \to p) \]

The translation of this implication reads:

\[ \vdash (\neg\neg s s) \]

The exclusion \((s)) of \( s \) excludes \( s \).

[An instance of \( \vdash (r) r \) (previous slide), with \( r := (s) \).]

A Proof of Both

Excluded Contradictions

\[ \vdash ((s)((s))) \]

\[ 1. (s) \text{ [ass.]} \]
\[ 2. ((s)) \text{ [ass.]} \]
\[ 3. () \text{ [X, 1, 2]} \]
\[ 4. (((s))) \text{ [R]} \]
\[ 5. ((s)((s))) \text{ [R]} \]

[The contradiction rule \( C \) is not required.]

Renouncing the Contradiction Rule

\[ ((s)) \vdash s \]

Such holds by Fiat, and some people are worried about it.

What do we give up if we renounce the contradiction rule \( C \)?

And what would it mean doing so?
Four Supporting Lemmas (1 & 2)

$C$ holds for exclusions, without $C$.

Repetition of an Exclusion, without $C$.

\[
\begin{align*}
1. & \quad (((r))) \vdash (r) \
2. & \quad ((s)) \vdash (s) \
3. & \quad (r) \vdash (s) \
4. & \quad ((r))) \vdash (r) \\
5. & \quad ((r))) \vdash (r) \\
6. & \quad (r) \vdash (s) \\
7. & \quad (s) \vdash (s) \\
8. & \quad (r) \vdash (s) \\
9. & \quad (s) \vdash (s) \\
10. & \quad (r) \vdash (r)
\end{align*}
\]

$EPL$ Without $C$

- What do we give up if we renounce the contradiction rule $C$?

Dropping $C$, all Conclusions remain Excluded from Exclusion.

If $r \vdash s$ with $C$, then $r \vdash ((s))$ without $C$.

Dropping $C$, all Concluded Exclusions remain Conclusions.

If $r \vdash (s)$ with $C$, then $r \vdash (s)$ without $C$.

Dropping $C$, all Conclusions are Exclusions.

- And what would it mean doing so?

It is radically intuitionist, while zero constructivist.

Four Supporting Lemmas (3 & 4)

$r \vdash (s) [A]$ implies $((r)) \vdash (s)$

\[
\begin{align*}
1. & \quad (((r))) \vdash (s) \
2. & \quad ((s)) \vdash (s) \\
3. & \quad (r) \vdash (s) \\
4. & \quad ((r))) \vdash (r) \\
5. & \quad ((s)) \vdash (s) \\
6. & \quad (r) \vdash (s) \\
7. & \quad (s) \vdash (s) \\
8. & \quad (r) \vdash (s) \\
9. & \quad (s) \vdash (s) \\
10. & \quad (r) \vdash (r)
\end{align*}
\]

These two lemmas imply that it is immaterial whether it is $r$ or $((r))$ which is available in the exclusion and retraction rules (X) and (R).

Summarizing

What Have We Been Doing Today?

- Our aim: Theory of Interpretation.
- We had: Mutual Understanding.
- We wanted: Minimal Logical Formalism.
- We need First Order Analysis of Propositions.
- Maintaining Total Understanding and Total Control.
- Providing a Logical Motivation for doing $DRT$.
Theory of Interpretation (Day Two)
Indexical Inference

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There is this proof, so something exists.

Day Two: Indexical Inference

Where Are We Today?

• Our aim: Theory of Interpretation.
• We require: A Logical Formalism
• We have: A Minimal Propositional Logic.
• We want: A First Order Analysis of Propositions.
  • Indexical Predicate Logic.
• Today is the Tough Day, but Rewarding.

Day Two: Indexical Inference

First Order Coordination

Predicate Logic

What are (elementary) propositions?

A logical coordination of terms.

Predicates (Aristotle) and Relations (Frege).

[For the time being, no names (Russell, Quine).]

Predicative expressions (including relational ones) have various logical instantiation possibilities.

Mere (non-)juxtaposition of terms is not fine-grained enough.

There is a variety (the fabulous four) of logical combinators (Aristotle) and (n-ary) relations vary along a multitude (of n) dimensions (Frege).

Arguments and Instances

Widespread assumption in the area of Logic and Language.

Predicative terms have roles, or arguments slots, which can be instantiated and coordinated.

We need a device to coordinate argument slots. And we need an instantiation device.

We employ:

⇒ Heim/Vermeulen instance DECLARATIONS,
⇒ and de Bruijn INDEXICAL COORDINATION.
Day Two: Indexical Inference  First Order Coordination

Declarations

An instance serves as a mere point of coordination. Its declaration establishes the possibility of coreference, and thus serves as a logical, not actual, “point of reference”.

We will employ the numeral “1” as an atomic device, an atomic proposition, that just does this: declare an instance.

[Formally this device corresponds to Irene Heim’s “Introduction of a File Card,” and to Kees Vermeulen’s “Declaration of a Discourse Referent.”]

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Day Two: Indexical Inference  First Order Coordination

Coordinations

We need to be able to coordinate arguments unambiguously with all instances available.

Indices 1, 2, . . . , i, j, . . . (small script) do precisely this. An index in an argument establishes its being instanced through the i-th active declaration before the occurrence of the index.

!! Every declaration in a proof or discourse can, thus, be, unambiguously, identified !!

[They are really Nicolaas Govert De Bruijn’s Indices, used to escape the need of any variable conventions, and figure in a Pauline Jacobson style Variable Free Semantics, avoiding the “antinomy of the variable,” so-called.]

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Day Two: Indexical Inference  First Order Coordination

Illustrations

• A string ‘1 D1’ reads that there is something and it is D.

• A string ‘1 D1 O21’ reads that there is a D and the thing declared before it Os it.

• A string ‘1 F1 1 D1 O21’ reads that there is an F and a D and the F Os the D.

• A string (proposition) ‘(1 F1 (1 D1 O21))’ reads that it is excluded that something is an F and such that it is excluded that there is a D such that the F Os the D, i.e., For no F there is not a D that he Os, e.g.,

  * Every Farmer Owns A Donkey.

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Let $R$ be a set of sets $R^j$ of relation constants of arity $j$.

**Definition (IPL Syntax)**

$L_{IPL}$ consists of propositions $p$ and strings $s$ such that, for $R^j \in R^j$ and sequences of $j$ indices $i_1 \ldots i_j$:

$$p ::= 1 \mid R^j i_1 \ldots i_j \mid (s)$$

$$s ::= p^*$$

**Translation**

For a regimented, but complete, part of FOPL we can present a translation $\mathcal{S}$ to a static fragment of IPL, and back $\mathcal{S}$.

[The translation is for closed FOPL-formulas stated using $\bot$, $\rightarrow$, $\forall$ only.] [The reverse is for resolved IPL-exclusions, with no active declarations.]

$$\mathcal{S} A t = A t$$
$$\mathcal{S} \bot = ( )$$
$$\mathcal{S} [\phi \rightarrow \psi] = (\mathcal{S} \phi, \mathcal{S} \psi)$$
$$\mathcal{S} \forall x \phi = (1 [(i/x)^+ \mathcal{S} \phi])$$

[The operations $+1$ and $-1$ can only be explained later.] [There are obvious conventions on the proper substitution of variables.]
Accommodation

- The semantics of IPL is not really that interesting.
- The proof theory is. We don’t really need any new rules.
- We adapt our Two Rules (sic!), to accommodate new structure.
- No new connectives, so no new logic.

[In the mean time we keep on using rule (C), for convenience.]

Accommodating Updates

Wenn jemand heute dasselbe sagen will, was er gestern das Wort “heute” gebraucht ausgedrückt hat, so wird er dieses Wort durch “gestern” ersetzen.

[When one wants to say today, what one has expressed yesterday using the word ‘today’, one should replace this word by ‘yesterday’.

[Gottlob Frege, 1918, “Der Gedanke”, p. 64]

... “FINE[Yesterday]” ... “FINE[Yester_Yesterday]”

- The proposition that $F_i$ has to be updated to the proposition that $F_{i+m}$ if $m$ instances have been introduced in the meantime.

...$F_i$...$F_{i+1}$

Utility (Third of Four)

- For any string $s$ at some line $j$:

  $i+m s$ the string that says what the string $s$ said, or would have said, now that $m$ more instances have been declared.

  $i\leq s$ is the string that says, at line $j$, what $s$ said or would have said if used at line $i < j$.

- Obvious, but tedious, definitions omitted. (Comments supplied.)

  Indices resolved within $s$ remain unaffected by $i+m s$.

  Update $i\leq s = i+1\leq s$ says what $s$ would have said directly after line $i$. 
Retraction Adapted

Retraction (R)

\[ \cdots \]

i. \( r \) \[ass.\]

\[ \cdots \]

\[ n-1. \] \( ^<_{(s)} \) \[ass.\]

\[ n. \] \( (rs) \) [R]

[The conditional conclusion \( ^<_{(s)} \) at line \( n-1 \) says what \( (s) \) would have said directly following \( r \), disregarding items that have been introduced in the meantime.]

Illustration (Screwed)

No farmer who owns a donkey beats it.

Every farmer owns a donkey.

\[ ^\uparrow \text{If Ian is a farmer he doesn’t beat it.} \]

Screwed

1. \( (1 \; F_1 \; 1 \; D_1 \; O_21 \; B_21) \) \[ass.\]

2. \( (1 \; F_1 \; (1 \; D_1 \; O_21)) \) \[ass.\]

3. \( i \; F_1 \) \[ass.\]

4. \( (1 \; D_1 \; O_21 \; B_21) \) \[X, 3, 1\]

5. \( ((1 \; D_1 \; O_21)) \) \[X, 3, 2\]

6. \( 1 \; D_1 \; O_21 \) \[C\]

7. \( (B_21) \) \[X, 6, 4\]

8. \( (i \; F_1 \; B_21) \) \[f\]

An hypothetically inferred donkey evaporates.

Illustration (Unscrewed)

No farmer who owns a donkey beats it.

Every farmer owns a donkey.

\[ ^\uparrow \text{If Ian is a farmer he doesn’t beat a donkey that he owns.} \]

Unscrewed

1. \( (1 \; F_1 \; 1 \; D_1 \; O_21 \; B_21) \) \[ass.\]

2. \( (1 \; F_1 \; (1 \; D_1 \; O_21)) \) \[ass.\]

3. \( i \; F_1 \) \[ass.\]

4. \( (1 \; D_1 \; O_21 \; B_21) \) \[X, 3, 1\]

5. \( ((1 \; D_1 \; O_21)) \) \[X, 3, 2\]

6. \( 1 \; D_1 \; O_21 \) \[C\]

7. \( (B_21) \) \[X, 6, 4\]

8. \( (1 \; D_1 \; O_21 \; (B_21)) \) \[ass.\]

9. \( ((B_21)) \) \[X, 6, 8\]

10. \( (\; \;) \) \[X, 7, 9\]

11. \( ((1 \; D_1 \; O_21 \; (B_21))) \) \[R\]

12. \( (i \; F_1 \; (1 \; D_1 \; O_21 \; (B_21))) \) \[R\]

Hypothetically inferred donkey can be re-inferred.

Accommodating Reinstantiations

1. \( 1 \; P_1 \) \[There is a Prince.\]

\[ \cdots \]

i. \( 1 \; D_1 \) \[There is a Dragon.\]

\[ \cdots \]

j. \( (1 \; P_1 \; 1 \; D_1 \; S_21) \) \[No Prince Shuns a Dragon.\]

\[ \cdots \]

n-1. \( (1 \; D_1 \; S^?k1) \) \[So He Shuns No Dragon.\]

n. \( (S^?m) \) \[So He doesn’t Shun It.\]

\[ [ \; k = ^<_{(1)} \; = \; l; \; m = ^<_{(1)} ] \]
Day Two: Indexical Inference

Utility (Last of Four)

- For a string \( r \) at line \( i \) and \( s \) excluded by an update \( i \leq r \) of \( r \) at line \( j \):

  \( r_{i \rightarrow j} s \) is the adaptation of the string \( s \) to the fact that the occurrence of \( r \) at line \( i \) provides an instantiation of the occurrence of \( r \) at line \( j \); it accommodates the indices unresolved in \( s \) to the fact that the occurrence of \( r \) at line \( i \) has taken the place of the one at line \( j \).

- Obvious, but very tedious, definition omitted. (Comments supplied.)

Indices resolved within \( s \) remain unaffected by \( r_{i \rightarrow j} s \).
Indices resolved by \( r \) within \( s \) are re-directed by \( r_{i \rightarrow j} s \).
Indices resolved otherwise are updated by \( r_{i \rightarrow j} s \).

Exclusion Adapted

Exclusion (X)

\[
\begin{align*}
\text{Line } i & : r \\
\text{Line } j & : (i \leq r) \\
\text{Line } n & : j \leq (r_{i \rightarrow j} s) [X, i, j]
\end{align*}
\]

Illustrations (Elementary)

There is a farmer.
Every farmer adores him.
So, he adores himself.

There is a prince.
There is a dragon.
No prince shuns it.
So, he does not shun it.

1. \( F_1, (F_1 (A_{12})) \vdash A_{11} \)  
2. \( F_1 \)  
3. \( (A_{11}) \)  
4. \( A_{11} \)

1. \( P_1, (P_1 (S_{12})) \vdash S_{21} \)  
2. \( P_1 \)  
3. \( (P_1 S_{12}) \)  
4. \( (S_{21}) \)

Paul Dekker (ILLC, UvA)  Theory of Interpretation (ESSLLI)  Galway 22, Aug 9 61 / 80
**Derivable Rule Schemes (Adapted)**

**Addition (+)**

**Subtraction (−)**

- i. \(rt\)
- i. \(rst\)
- n. \(t \leq s [-, i]\)
- n. \(t \leq p^{n-1}s \leq t [+, i, j]\)

[Proof: by updating the proofs above.]

**Derivable Rule Scheme (Dynamic Conjunction)**

**Dynamic Conjunction (DC)**

There is a farmer.  
There is a dog.  
He owns it. So?  
There is a farmer who owns a dog.

1. \(F_1\)  
2. \(D_1\)  
3. \(O_{21}\)  
4. \(F_1 \cdot D_1 \cdot O_{21}\)  

[DC, i, j]

[A Form of Existential Generalization.]

- n-1. \(F_j A_j\)  
- n. \(F_1 A_{13}\)  

[It would be "classical" to drop the proviso above.]

**Introduction (I)**

- i. \([j/0]s\)  
- n. \(1 \cdot [1/0] \cdot s [I, i(j)]\)

[Provided that \(j\) is resolved at line i.]

**Universal Instantiation**

- i. \((1 \cdot [1/0] (t + 1s))\)  
- n. \([j/0] \leq s [UI, i(j)]\)

[This rule (non-classically) requires \(j\) to be resolved at line n.]
**Derivable Rule Schemes (Universal Generalization)**

*Universal Generalization*

\[
\begin{align*}
&\vdots \\
i. & 1 \quad \text{[ass.]} \\
n-1. & i \leq s \\
n. & (1 (s)) \quad \text{[UG]} \\
\end{align*}
\]

[The truly appealing idea of this rule scheme is due to Kees Vermeulen.]

[It turns out to be a derivable rule here.]

---

**Key Results**

- With the unrestricted (non-derivable) Introduction Rule (I):

\[
s \vdash t \text{ if and only if } s^* \vdash t \text{ if and only if } s^- \vdash t.
\]

- Independently:

\[
s \models t \text{ if and only if } s^* \models t \text{ if and only if } s^- \models t.
\]

- Moreover:

The logic of Nude Normal Forms is classical.

---

**Normalization**

- The *Kamp Normal Form* of a string \( s \) is recursively defined as an, equivalent, string \( s^* \) starting with \( n_s \) declarations and a subsequent string of atoms and exclusions of normal forms.

\[
\begin{array}{c}
1 \ 1 \ 1 \ \ldots \\
\vdash At, At, \ldots \\
\vdash At, \ldots \\
\vdash \neg At, \ldots \\
\vdash \ldots \\
\end{array}
\]

- The *Nude Normal Form* \( s^- \) of \( s \) is the normal form \( s^* \) with its initial \( n_s \) declarations stripped of.

---

**Soundness and Completeness**

\[
s \models t \text{ if and only if } s \vdash t.
\]

Proof (Outline):

\[
s \models t \text{ if (contraposition)}
\]

\[
s(t) \models () \text{ if (normalization)}
\]

\[
s(t)^- \models () \text{ if (classical)}
\]

\[
s(t)^- \vdash () \text{ if (normalization)}
\]

\[
s \vdash t \text{ (contraposition)}
\]

\[(X), (R), (C) \text{ and (I, unrestricted) characterize validity.}\]
On the Antinomy of the Variable

If distinct variables $x$ and $y$ cannot be taken to be synonymous, then how can it be that $\forall x \lbrack x/z \rbrack \phi$ and $\forall y \lbrack y/z \rbrack \phi$ "as Fine says, (…) are mere 'notational variants'?" [Bryan Pickel and Brian Rabern, 2016, The Antinomy of the Variable: A Tarskian Resolution, The Journal of Philosophy 113.]

- The problem is trivially solved here. We cannot even distinguish the latter two formulas, having no variables available. And we can, if need be, give an independent specification of the meaning of any variable-token; it reads:
  "the most close-by antecedent coindexed with its occurrence."

On Dynamic Contraposition and Dito Entailment

IPL supports a general, but order-sensitive, form of contraposition.

\[
rs \vdash t \iff rs(t) \vdash () \iff r \vdash (s(t))
\]

\[
r \vdash (1 \ M_1 \ (S_{11})) \explicits \iff \ r, 1 \ M_1 \vdash S_{11} \ \explicits \ (C \vdash \text{Every man shaves himself.})
\]

[Jeroen Groenendijk and Martin Stokhof, 1991, have dubbed such “Dynamic Entailment”, a form originally identified (and discarded) by Peter Geach, 1962.]

(28) A man has just drunk a pint of sulphuric acid. Nobody who drinks sulphuric acid lives through the day. Very well then, he won’t live through the day.

\[
1 \ M_1 \ DA_1, (1 \ DA_1 \ L_1) \vdash (L_1)
\]

On Free Logic

- We can free ourselves from any Existential Import—a remnant of Aristotle’s legacy—, by accepting the mentioned proviso on the Introduction Rule.

  There does not even seem any logical necessity why there should be even one individual—a why, in fact, there should be any world at all.


- Neither Names, nor Variables require us to resort to an ontological commitment to a non-empty domain. (Cf., also, tomorrow.)

On Inclusive Logic

We actually do not need any Rules of so-called Quantification.

The ("∃")Elimination Rule is a species of Subtraction.
The ("∃")Introduction Rule is a species of Dynamic Conjunction.

Basically, (X) and (R) characterize first order validity on ALL models.
On The Autonomous Self

• A zero index \( 0 \) is naturally taken to be a self-declaration.

• It relates to its own occurrence and, derivatively, perhaps, to
  • the sentence/proof in which it occurs,
  • the (discourse) time at which it is used,
  • the speech act it is used to perform,
  • its author, and the world she lives in, . . . .

• Self-reference implies self existence.

• Thus improving on René Descartes* and DRT† (tomorrow).

Quid Erat Demonstrandum

This* proves that something exists.
1. \([0/0] \text{[Seq.Ver.]}
2. 1 [1/0] \text{[I, 1(0)]}

[https://techgnosis.com/synthetic-meditations-in-the-matrix/]

Without (C), we can only prove that it is excluded that nothing exists.

\( \vdash ((1)) \)

The proof is logico-philosophically undubitable.

Current Logical and Customary Practice

• Standard systems of natural deduction are characteristically
  indexical. Reference is made to occurrences of propositions and
  the annotation is essentially indexical.

“\([X, 7, 9]\)” (e.g., on slide 59) should be understood to refer to
the most recently defined rule labeled “\(X\)” in the current
presentation, and to the lines 7 and 9 in the very proof where
this token of “\([X, 7, 9]\)” occurs.

[All of this is being explained in the language we are speaking.]
[We are who are reading this, slide 79 of this presentation.]

What Have We Been Doing Today?

• Our aim: Theory of Interpretation.
• We had: A Minimal Propositional Logic.
• We wanted: A First Order Analysis of Propositions.
  Indexical Predicate Logic.

• It wasn’t that Tough after all?
• We can Turn it into more Familiar and Richer Formalisms.
• Maintaining Complete Understanding and Total Control.
A meaning is a type of interpretation of an occurrence of a type of expression.

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August 8-12, 2022

Where Are We Today?

• Our aim: Theory of Interpretation.
• We have: A Logic for a Language of Translation.
• We want: An Interpretation for the LoT.
  • Discourse Re-Presentation Theory.

Consider the following criteria for a good notation:
(i) easy to write and easy to read for the human reader;
(ii) easy to handle in metalingual discussion;
(iii) easy for the computer and for the computer programmer.

The system we shall develop here is claimed to be good for (ii) and
good for (iii). It is not claimed to be very good for (i); this means
that for computer work we shall want automatic translation from
one of the usual systems to our present system at the input stage,
and backwards at the output stage.

[Nicolaas Govert de Bruijn, 1972, “Lambda Calculus Notation with
Nameless Dummies”, Indagationes Mathematicae 34, pp. 381–392.]

Meaning and Use of Variables

We have so far chosen not to adopt variables,
• because they haven’t any meaning in themselves.
But for this very reason they are also harmless,
• and therefore can be employed, if properly.
Labelling IPL . . . ,

Here is a proposal.

[A.] Let us label, superscript, declarations, occurrences of “1”, with (distinct) VARIABLES, x, y, . . . , from a set of variables V, .

[Instead of “1” we may even choose to simply write “∃x”.

[Logically and semantically “∃x” is really equivalent to “1”.

[B.] Let us index, subscript, any numeral i with a variable x iff the declaration that it relates back to has been labeled x.

[Instead of “ix” we may even choose to simply write “x”.

[Logically and semantically this use of “x” represents this use of “ix”.

We have only changed the presentation.

As a result we recover the system of DPL. [Jeroen Groenendijk and Martin Stokhof, 1991, “Dynamic Predicate Logic”, Linguistics and Philosophy 14, 39–100.]

But with a sound and complete proof system underneath. [Frank Veltman, 2011, “An Appendix to DPL”, in: Jaap van der Does and Catarina Dutilh Novaes, This is not a Festschrift, http://festschriften.illc.uva.nl/M60; cf., https://staff.fnwi.uva.nl/f.j.m.m.veltman/DPL2000.pdf]

Some man. He shaves himself. So, someone shaves him.

[Adopting the Generic Proof Format from Day One.]

\[
\begin{align*}
1 M_1 &\; 1 D_1 \; O_21 \iff A \\
\exists x \; M_1 &\; \exists y \; D_1 \; O_21 \iff B \\
\exists x \; M_2 &\; \exists y \; D_2 \; O_2y.
\end{align*}
\]

Presentation and readability are further enhanced once we decorate IPL strings in Kamp Normal Form.

\[
\begin{align*}
\text{\textit{x}_1, \ldots, \textit{x}_j} &\; \quad \text{CON}_1 \\
\quad \ldots &\\
\text{\textit{x}_n} &\; \quad \text{CON}_n
\end{align*}
\]

Even if it is not directly visible, we have chosen to present the declarations in reverse (mirror) order, so that a merge \(\oplus\) can be defined thus:

\[
\begin{align*}
\text{\textit{x}_1, \ldots, \textit{x}_j} &\; \quad \text{CON}_1 \\
\quad \ldots &\\
\text{\textit{x}_n} &\; \quad \text{CON}_n
\end{align*}
\]

\[
\begin{align*}
\oplus &\; \quad \text{CON}'_1 \\
\quad \ldots &\\
\oplus &\; \quad \text{CON}'_m
\end{align*}
\]

\[
\begin{align*}
\text{\textit{x}_1, \ldots, \textit{x}_j} &\; \quad \text{CON}_1 \\
\quad \ldots &\\
\text{\textit{x}_n} &\; \quad \text{CON}_n
\end{align*}
\]
So What’s The Big Deal?

Upon my, private, understanding of DPL and DRT they have a sound and complete proof-theory.

If you like, you can prefer to use the type of formulas of DPL, now that a proof theory is available for the logic underlying it.

You don’t have to motivate all this saying that meanings are dynamic, because we don’t even need to talk about meanings here.

If you want, you may as well prefer to use the formal architecture of DRT.

You don’t have to motivate this saying that meaning requires representation, because we are only dealing with presentations here.

But if we, therefore, can just safely employ DPL and DRT, for that matter, why bother about this unorthodox unfamiliar “underlying” notation?

It allows straightforward extensions with nominal declarations, knowledge of individuals, and talk about intentional objects.

What To Do with Names?

Philosophers and linguists find it convenient to agree that proper names do nothing but refer—are used for no other main purpose than that of referring—while no logic, can secure them a reference.\(^a\)

The existence and identity of the referents of names can only be established empirically, perhaps conventionally, but not logically.\(^b\)

So how can names in logic do anything like they are supposed to do, if it is only to do something that logic cannot enable them to do?

We just have to assume that names, in use, do what they are supposed to do, refer to something, and leave it open what that something is, besides assuming that the thing is named that way.

\(^a\)Despite Gottlob Frege’s brave logical attempts in his *Grundlagen 1884*.

\(^b\)Even though there is surely something analytic about the proposition that a name, e.g., “Donald Trump” refers to some thing so-named, e.g., Donald Trump. More perhaps on this below.

Referential and Predicative Uses of Names

- Scholars have discussed referential and predicative uses of names, and quarrel about which one is basic. [E.g., Delia Graff Fara, 2015, “‘Literal’ Use of Proper Names”, in: Andrea Bianchi, *Reference*, OUP.]

- Obviously, names are typically meant to be referring, i.e., typically used with the intention to refer, to something, so-called. While also obviously, being so-called is a kind of a property.

  That’s all. Logically speaking it is what we can know and need to know.

A name can be used to present a so-named instance in a discourse, that can be related back to—like other declarations can. The difference is, merely, that names are typically not used to declare arbitrary anonymous instances, but only instances so-called, and often familiar.

The Declarative Use of Names

We take as a ‘proto-typical’ use of a name a declaration:

“There is Ian.”

- The use of a name is a nominal, as distinct from anonymous, declaration.

\[\text{[syntactic amendments required on slide 49]}
\]

\[\ldots \text{and let } N \text{ be a set of names} \]

\[\ldots \text{and for } a \in N \ldots \]

\[p ::= 1|a|Rt_1 \ldots t_j |(s) \]

\[\ldots \]

\[\text{[proof-theoretic amendment on slide 62]}
\]

any specification \(\gamma r\) of \(r\) [rule X, line i] initiates the exclusion

[A specification \(\gamma r\) is any string obtained from \(r\) by specifying some (possibly none) anonymous, non-excluded, declarations in \(r\) by means of nominal ones.]

- That’s all, folks!
We Have Already Seen this Before

* Ian is a man. No man is immortal. So, he is not immortal.

1. $i M_1$ [ass.]
2. $(1 M_1 I_1)$ [ass.] $[i M_1$ is a specification of $1 M_1$]
3. $(I_1)$ [X, 1, 2]

The de- and re- construction of strings with nominal declarations proceeds like that of those with anonymous ones.

- More interesting are issues of **existence** and **identity**.

  Existence is actually for Friday.

  Now: a minimal logic of identities.

Identity Rules

* **Indiscernibility**

  $i = j$

  $=$

  $[k \leq j] s$

Symmetry and Transitivity

* **Leibniz (L) secures symmetry (symm.) and transitivity (trans.).**

  $x = y, y = z \vdash x = z$

  $x = y \vdash y = x$

  $k. x = y$

  $y = z$

  $n. y = x$ [symm., $k$]

  $n+1. [\gamma / a] a = x$ [refl.]

  $n+1. [\gamma / a] a = x$ [L, k, n]

  $n+2. [\gamma / a] a = z$ [L, n, n+1]

[Besides the restriction to declared instances, this is all classical.]
Naming and (Logical) Necessity

- Victor, if existent, is necessarily self-identical. $v^x \vdash x = x$
- But it is not necessary that Victor is self-identical. $\nvdash v^x \ x = x$
- Because it is not necessary that Victor exists. $\nvdash v$
- And while Victor, if existent, is necessarily Victor. $v^x \vdash v^x \ x = z$
- Yet Victor and Victor are not necessarily identical. $v^x \neq v^x \ x = z$

[One may redo this exercise with “Venus” as and with “Vulcan” as]

- Planet or Goddess.
- Hypothetical Planet or Mythical Figure or Gay Magazine.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Paul Dekker (ILLC, UvA)</td>
<td>Theory of Interpretation (ESSLLI)</td>
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</tbody>
</table>

Discourse Presentation Theory

- Let us summarize a possible re-presentation of our formalism.

**Definition (DPT Syntax)**

$\mathcal{L}_{DPT}$ consists of presentations $K$ such that, for $x \in V$, $a \in N$, $R^j \in R^j$,

- $r := 1^x | a^x$
- $C := x = x | R^j x_1 ... x_j | \neg K$
- $K := (r^x, C^x) | K \oplus K$

and such that all declarations in $K$ are labeled by distinct variables.

[We also use $x$ for $1^x$, $a$ for $a^x$, if unambiguous.]

- A presentation is often displayed in box-format: $r^x \in C^x$.

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DPT Logic

DPT has, below the surface, a sound and complete logic with exclusion (X), retraction (R), Identity (=), Leibniz (L) and (not necessarily) Contradiction (C).

- Suppose Ron is a clown, that if a clown sees a duck, she teases it, that Sandy is a duck, and that Ron sees Sandy.
- It follows that that Ron teases that Sandy, of course. And indeed:

  - $r^x \vdash u \in \mathcal{L}_{DPT}$
  - $CLu \Rightarrow DUy \vdash TEAuw$
  - $DUy \Rightarrow SEEwy \vdash TEAz$
  - $r^x \in C^x$

[Where $K_1 \Rightarrow K_2$, familiar from DRT, is used to represent $\neg(K_1 \oplus K_2)$.

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Models $M$ for $\mathcal{L}_{DPT}$ extend those for $\mathcal{L}_{IPL}$ with an interpretation $I(a) \subseteq D$, for any name $a$. (A set, possibly empty, of those named $a$.)

- Satisfaction is defined relative to models $M$, variable assignments $g$, and sequences $ce$ of witnesses and so that:

**Definition (DPT Semantics)**

- $M, d \models 1^x$ iff $d \in D$;
- $M, d \models a^x$ iff $d \models I(a)$.
- $M, g \models x = y$ iff $g(x) = g(y)$;
- $M, g \models R x_1 ... x_j$ iff $(g(x_1), ..., g(x_j)) \in I(R)$;
- $M, g \models \neg K$ iff for no $c \in D^*$: $M, g, c \models K$.
- $M, g, e_1, e_{i+1} ... e_j \models \langle r_1 \ldots r_j, C^* \rangle$ iff $M, e_i \models r_i$ (for all $i \leq j$) and $M, g[\langle r_i, c \rangle), e \models C$ (for all $C \in C^*$);
- $M, g, ce \models K_1 \oplus K_2$ iff $M, g, e \models K_1$ and $M, g[\langle r_i, c \rangle), e \models K_2$.

[Here $g[\langle r_i, c \rangle)$ is used for the assignment $h$ different from $g$ at most in that for all $r_i, h(x) = e$, if $x$ is the label of $r_i$.]

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Truth, Meaning and Understanding

(...)

There is no need to suppress, of course, the obvious connection between a definition of truth of the kind Tarski has shown how to construct, and the concept of meaning. It is this: The definition works by giving necessary and sufficient conditions for the truth of every sentence, and to give truth conditions is a way of giving the meaning of a sentence.


- After some informal explanation of the IPL / DRT / DPT formalism, and perhaps even more clearly after the statement of their semantics, we may assume that we understand what these “Sequences” or “Discourse (Re-)Presentation Structures” mean, or better: present.

- They are ways of presenting the truth, if true, as Frege would have it. And they are taken to present the truth by presenting individuals that are presented to stand in presented relations. Any such presentation can be considered a Sinn; its realization, if any, its Bedeutung.

Representation and Re-Presentation

In the original formulation of DRT (...a new piece of discourse ...) updates the representation of the already processed discourse (...).

[Hans Kamp et al., 2011, “Discourse Representation Theory”, in: D. Gabbay and F. Guenthner (eds.), Handbook of Philosophical Logic, 15, Reidel, §2.3]

- This is standard jargon, but hard to understand if taken literally.

  But DRSs do more: they not only represent propositional content, but also provide the context against which new sentences in a discourse are interpreted.

  [Kamp et al., §3.2]

- Also standard, and hard, unless “represent” really means: present.

  One way to think of this DRS is as a structure which the interpreter forms in his mind and which for him identifies the content of the interpreted statement.

  [Kamp et al., §5.1]

- It better be a public structure which defines, or presents, such content.

Anaphoric Rigidity

While different uses of a name may systematically relate to different instances, in ordinary discourse such typically does not happen, if it is not somehow signaled otherwise.

- We can of course explicitly enforce identity of different uses of a name, and we may prefer to even encode that such an identity with a familiar use of the name is presupposed.

- We can write declarations like $a_i$, short for $a_i \equiv_{df} a = 1 + i$. Easier:

One More Utility

We use $1_x$ and $a_x$ to render the locutions “$x$ exists” and “$x$ is Alfred”:

$1_x \equiv_{df} \exists x \quad a_x \equiv_{df} a^x \quad x = z$

[Most sensibly when those $x$’s are declared in another modality.]

Contingencies of Nominal Reference

The referent of a certain use $v$ of the name $n$ is [assumed to be] the one and only object that has originally been dubbed $n$ in the causal-intentional tradition which this use of the name is [taken to be] participating in. [roughly along the lines of Manuel García-Carpintero, 2018, “The Mill-Frege Theory of Proper Names”, Mind]

Existence of the object of a referential intention is pragmatically presupposed; it cannot be secured by logical means.

The intended object can be assumed to be familiar from a “resource situation” presented by a conceptual window through which we see it.

The formulation of this requires us to adopt a modal perspective, in which these situations can be defined.

Those windows come tomorrow. We prepare for this today, inspecting naming and necessity.
Modest Modal Logic

Let us introduce the locution ‘necessarily $p$’ [$\Box p$], and its dual ‘possibly $p$’ [$\Diamond p$], for any proposition $p$ and modality $i$.

$\Box \equiv q \ \Diamond 0$ is assumed to be as universal as possible.

Distributed Necessitation

if $p_1,\ldots, p_j \vdash q$, then $\Box p_1,\ldots,\Box p_j \vdash \Box q$

[I here profess professional ignorance of the actual extension of the locutions “... is a possibility” and “... are necessities.”]

Logic does rule modal space, but does not define it.

Modal Space

$S5 \ & \Box \subseteq \Box i$

[for any other contingent modality or modal base $i$]

Anaphoric Rigidity

[It is because we can refer (rigidly) to Nixon, and stipulate that we are speaking of what might have happened to him (under certain circumstances), that ‘transworld identifications’ are unproblematic (...).]

Saul Kripke, 1972, Naming and Necessity, p. 49

What Logic Does and Does Not Tell Us

Consider a possibility with somebody present who is actually Nixon.

What do we know about such individual in such a possibility?

Only what we know that she necessarily must be.

An individual is not necessarily something that it actually is, and it is not necessarily distinct from something that it actually is not.

[We are not intending the same object in different, real, circumstances — mind you! — but an object in other, non actual, possibilities.]

Non-Logical Metaphysical Conclusions need Like Assumptions.

Regarding Nixon, we can stipulate that he exists in a possibility, and also that he is necessarily Nixon, if he exists, and that, necessarily, if anything is Nixon it is him.

$n x \Diamond P$

i. $\Diamond A$: (be is necessarily animate, ass.)

j. $\Diamond S R$: (be is necessarily rational, ass.)

k. $\Diamond S H$: (be is necessarily human)

[The occurrence of “$\Box” presents someone who is actually that Nixon in a possibility.]
### Teams

Who’s playing?
The Dutch play the Belgians.

Who is who?
The Orange Ones are the Dutch.

---

### Live Predications

- The iterated use of one and the same predicate is normally supposed to be meaning preserving, as iterated uses of names, by default, relate to one the same instance.
- But it is evident that one and the same predicate allows of many distinct uses, certainly at different occasions.
- We can deal with varying use of one and the same predicate, in the way we have dealt with names. A predicate term $P$ can be used to declare any non-empty subset of what very generically may counts as $P$’s in the domain.
- This declaration may allow subsequent reference back to the set of $P$’s thus introduced, and it may itself be identified with a sort of $P$’s previously introduced.

---

### Kinds of Students

“Ron is a student.”

- She studies a lot.
- She has officially registered.
- She belongs to the student movement.
- She is really investigating this thing.
- ... 

\[ \text{ron}^*_x, \text{STUDENT}^*_y \]

\[ Y[x] \]

- Which type of student is clear or doesn’t matter, normally.
- Sometimes it does, though.

---

### Students and Art

Most students are real students.

Not all students are actually students.

No students are students.

There is art and art.
Red Apples

We’re at a county fair picking through a barrel of assorted apples. My son says ‘Here’s a red one,’ and what he says is true if the apple is indeed red. But what counts as being red in this context? (. . . )

Suppose now that we’re sorting through a barrel of apples to find those that have been afflicted with a horrible fungal disease. This fungus grows out from the core and stains the flesh of the apple red. My son slices each apple open and puts the good ones in a cooking pot. (. . . ) Cutting open an apple he remarks: ‘Here’s a red one.’

What he says is true if the apple has red flesh, even if it also happens to be a Granny Smith apple. [Anne Bezuidenhout, 2002, “Truth-conditional pragmatics”, Philosophical Perspectives.]

\[
P^x, \text{RED}_1^y \quad Y^x \quad \text{¬} Y^z, \text{RED}_2^z
\]

Romanovs

Joe Romanow (my barber) is not a Romanov.


- If being a Romanov equals being called a Romanov this is inconsistent.
- But if the interpretation of the predicate “being a Romanov” is contextual like “being a student” is, then being called a Romanov is consistent with not being a Romanov.

We can very well restate the above:

Joe Romanov is a Romanov, of course, but he is not a Romanov.

\[
P^y, R^1_x \quad R^2_x \quad Y^x \quad \text{¬} Z^x
\]

Hamburgers

“The Hamburger. That’s me.”

“Ich bin ein Hamburger”

“Hamburger”

Logicians

Ham: The hamburger [red-neck] wants to pay.
Log: You mean the person who ordered a hamburger?
Ham: That’s what I say.
Log: But a person who ordered a hamburger is not the same as a hamburger.
Ham: Yeah, it’s the same.
Log: But you don’t cook persons who order hamburgers, do you?
Ham: What, are you crazy? I cook meat, not people.
Log: So, then, the hamburger who wants to pay, he ordered a hamburger, that means, he ordered a person who ordered a hamburger?
Ham: No, silly. Then he would have ordered a person who ordered a person who ordered a hamburger. You have a lot to learn.

Le Jeu de l’Oie

I am parked on square 43. I am the red pawn. And I am bored.
The red pawn is bored.

Lisa (Green) escaped from prison, but she isn’t happy.
Lisa (Green) escaped from prison, so she can move again.
A deferred ‘escape’ or a deferred ‘Lisa’, or both?

Not in any way a logical problem; but at most a practical one.

Indexical Compositionality

- What goes into the machinery of compositional processing, and translation, is what a term is taken to mean on its occasion of use.
- How it fits in there is construed INDEXICALLY, relative to the context of its occurrence and the agents that employ it.
- HOW we determine how to fit things in what is up for deliberation and negotiation about what norms and practices to resort to.
- What Philosophers and Linguists have on offer is, at best, and only, a formal framework for the presentation of the results, Interpretations, throughout contextual, and indexical at the heart.

What Have We Been Doing Today?

- Our aim: Theory of Interpretation.
- We had: A Logic for a Language of Translation.
- We wanted: An Interpretation for the LoT.

Discourse Re-Presentation Theory.

- We defined and explained the use of Presentations.
- Understanding (Sinn) of Satisfaction (Bedeutung) Conditions.
- Emphasized the Use and Logic of Occurrences of Them.
Theory of Interpretation (Day Four)
Propositional Attitudes

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National University of Ireland
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Realist Idealism doesn’t make sense
Idealist Realism does.
(Or was it the other way around?)

Where Are We Today?

• Our aim: Theory of Interpretation.
• We have: an Understanding of the Sinn, which is the Determination of a Bedeutung of a LoTr (not LoTh).
• We want: An Understanding of Intentional Discourse.

Intentional Presentation Theory.

• Why do we want that?

Where We Are Today

We are at ESSLLI in Galway.

A lot is happening on the logic of statements of the form “Agent a knows that p,” or “Agent x has information that so and so,” where in particular “p” and “so and so” are used to characterize beliefs and information they or other agents have.

But what are these locutions supposed to mean?

Major Issue Today

What is it that we actually say when we say something of the form

“x attitudes that p”?

What would solicit us to say that?
Some Generic Beliefs

The Babylonians believed that Marduk, the chief god, created the world, the first calendar, and humans.

The Ancient Greeks believed that when you died a ferryman called Charon rowed your spirit across a river called the Styx to the entrance of the underworld.

Christians believe God sent his son Jesus, the Messiah, to save the world.

[The Dutch believe that one receives status and respect by hard work and studying, not by familial ties or age.] Quomodo insipiens dixit in corde, quod cogitari non potest.

[Anselm, 11-th cent., Proslogion, ch. 4]
Disposition to assent \( \neq \) true belief.
[Adding “honest and sincere” does not help.]

Some Particular Beliefs

Cleopatra believed that she was the reincarnation of the Goddess Isis.

Joan of Arc believed that God had chosen her to lead France to victory in its long-running war with England.

Thomason believes that semantics is a branch of mathematics.

Loar believes that semantics is a branch of psychology.

Noam Chomsky believes that children are born with an inherited ability to learn any human language.

The worlds in which different societies live are distinct worlds (\ldots). [Edward Sapir, 1929, “The Status of Linguistics as Science”, Language]

“The Very Idea of a Conceptual Scheme” [Donald Davison, 1974, Inquiries into Truth and Interpretation]

If people live in different worlds, how can we “see” the world they “live” in?

What does David Believe?

The ex-Arsenal director David Dein, who worked so well in tandem with Arsene Wenger before he was relieved of his duties by the Board, has revealed that he still believes Arsene is the right man to lead the Gunners to success, or at least to enable them to challenge the Big Three for a few trophies. He also believes that Wenger is aware of the distance between Arsenal and Man United and will do something to redress the situation this summer. Dein said:

“It’s a long hard season and I am a great supporter of Arsene Wenger as you can imagine. I think he has done a phenomenal job and continues to do so year-on-year. I think he has the fire in his belly. I don’t see any reason why he shouldn’t be there. (\ldots) Arsene’s on course now to get the team to qualify for the Champions League for next season. He does that remarkably well and the club is financially very, very strong. But clearly they have fallen 20-odd points behind Manchester United which is not good enough. (\ldots)”
Day Four: Propositional Attitudes

Understanding Attitude Reports

We Are No Mind Readers

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Theory of Interpretation (ESSLLI)
Galway 22, Aug 10
129 / 160

Day Four: Propositional Attitudes

Understanding Attitude Reports

We Are Mind Writers

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130 / 160

Day Four: Propositional Attitudes

Understanding Attitude Reports

Classical Relational Construal

Agent a believes (know, desires, ...) that p.

A relation between agents and a kind of object.

Often construed as the factual obtaining of a relation between an epistemic agent and a proposition.

Ann believes that p.

Ann hired Ron.

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131 / 160

Day Four: Propositional Attitudes

Understanding Attitude Reports

Objects of Belief

What are these propositions? (And what is this relation?)

- Fregean thoughts,
- Russelian structured meanings,
- Sentences, external or internal to the mind,

› ‘Snapshots’ of an imaginary world,

... ?

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132 / 160
Preliminary Assumptions

- We share a world and a language to talk about it. [Something I assumed we assume on Monday.]
- We share an understanding of the tokens produced. [Something I assumed we assume on Tuesday and Wednesday.]
- We share an understanding of our fellow agents. [Something I assume we assume Today and Tomorrow.]

Understanding of our world-mates involves the notion of their being in a world, and a notion of a world they could be being in, and in which the agents’ actual behavior would make sense to us.

Meaning of Attitude Reports


The world as, I think, it subjectively appears to her, if I want to make sense of her overt proceedings, and the world as, I think, it objectively appears to her, if I imagine her inner proceedings. [Whatever ;—]

When we ascribe a belief in the idiom ‘x believes that p’ (…) we reflect on the believer’s behavior, verbal and otherwise, and what we know of his past, and conjecture that we in his place would feel prepared to assent, overtly or covertly, to the content clause.


Intensional Models

\[ M = \langle W, D, \{R_i\}, I \rangle \]

- A model \( M \) is a quadruple consisting of a set of possibilities \( W \), a domain function \( D(\_\_\_) \), a family of accessibility relations \( \{R_i\} \), for any modality \( i \), satisfying suitable constraints, and an interpretation function \( I(\_\_\_) \), so that \( (D, I) \) is an extensional FO model.

\[ M, w, g, e \models s \]

Satisfaction in a model is defined relative to a world \( w \) and a variable assignment function \( g \) and a sequence of witnesses \( e \), where the values of \( g \) and the witnesses in \( e \) all are individual concepts: function from worlds to individuals in those worlds. Relativization to worlds, e.g., in:

\[ M, w, g \models Rx_1 \ldots x_j \text{ iff } \langle g(x_1), \ldots, g(x_j) \rangle w \in I_w(R) \]
Generally Shared Worry

Does everybody necessarily believe all logical consequences of everything she believes? Like every logical truth? That cannot be right!

On the possible worlds evaluation of Attitude Reports, indeed:

\[
\vdash B_a \top \\
\exists_a (\phi \rightarrow \psi), B_a \phi \vdash B_a \psi \\
B_a \phi \vdash B_a \psi \text{ when } \phi \vdash \psi
\]

[It is impossible not to believe any logical truth.]
[It is impossible to believe some, but not all, logical falsehoods.]

Some First Responses

- Upon the possible worlds approach it is not primarily sentences or presentations that people are said to believe.
- We report what, we think, characterizes possibilities that suit an agent.
- A logical truth characterizes no possibilities that suit an agent, because a logical truth characterizes no possibilities.
- It is not felicitous to say that an agent believes \( \phi \), because it doesn’t say anything. It is just as infelicitous as to say, and mean, \( \phi \).

Tautologie und Kontradiktion sind sinnlos. [Ludwig Wittgenstein, 1921, Logisch Philosophische Abhandlung, #4.461.]

Some Second Responses

- We explain and understand our world-mates against the background of beliefs assumed to be shared, and we particularly point out the deviances, in assumptions and beliefs not shared.
- There is no point in stating the assumed obvious, or in stating that someone believes it, if not questioned for some reason.
- If somebody is relevantly said to believe something, the report of any entailed belief, even if entailed, runs the risk of not characterizing the behavior —quod est explanandum.

What is private about belief is not that it is accessible to only one person, but that it may be idiosyncratic. [Donald Davidson, 1974 (1984), “Belief and the Basis of Meaning”, Synthese.]

Some Third Responses

- It is sentences, or presentations, that are or are not equivalent; they are not primarily the kinds of things we are said to believe.
- A belief in a logical or mathematical proposition is a belief to live in a world as characterized by the proposition, reflecting on its statement.
- It is not the belief to live in a world as presented (which would be vacuous), it is a belief about the presentation of it.
- Distinct, logically equivalent, presentations are distinct presentations. One can obviously judge one of them true and the other one false.
Some Fourth Responses

Apparent tautologies and contradictions naturally invite reinterpretations.

[Ann believes that] Some students are students.

[Ann believes that] He is an actor or he is not an actor.

I (…) question the (…) assumption, that sentences that appear to be necessarily equivalent really are, in the relevant context, equivalent. [Robert Stalnaker, 1987, “Semantics for Belief”, Philosophical Topics, p. 178.]

• Stalnaker’s solution by diagonalization is ad hoc, compared to our solution, the possibility of which has been built in right from the start.

[Ann believes that] Not all students are students

Some Final Response

• People are actually said to believe apparent impossibilities, or disbelieve logical and mathematical truths.

• My energy will be spent in efforts to make sense of such a report so that, indeed, it is construed as a belief in something possible after all, so that no belief in a genuine impossibility would remain.

For that would be tantamount to believing that something was not the same as itself, and surely I could never believe that. (…) [My] assent to a sentence false in all possible circumstances does not carry into a belief. [Ruth Barcan Marcus, 1983, “Rationality and Believing the Impossible”, Journal of Philosophy, p. 330/8.]

Was wir nicht denken können, das können wir nicht denken; wir können also auch nicht sagen, was wir nicht denken können. [Ludwig Wittgenstein, 1921, Logisch Philosophische Abhandlung, #5.61]
David Kaplan has attempted to re-subsume the real De Re under the ideal De Dicto.

“Ralph believes that Ortcutt is a spy.” is rendered as:

\[ \exists \alpha [ R(\alpha, o, r) \land B(\alpha) \text{ is a spy} ]. \]

Hereby \( R(\alpha, x, r) \) says that \( \alpha \) represents \( x \) to Ralph, i.e., that:

(i) \( \alpha \) denotes \( x \),
(ii) \( \alpha \) is a name of \( x \) for Ralph, and
(iii) \( \alpha \) is (sufficiently) vivid.

[David Kaplan, 1969, “Quantifying In”, in: Donald Davidson and Jaakko Hintikka (eds.), *Words and Objections*, Reidel, §IX]

The notion of a vivid name is intended to go to the purely internal aspects of individuation. (. . . ) Look only at the conglomeration of images, names, and partial descriptions which Ralph employs to bring \( x \) before his mind. Such a conglomeration, when suitably arranged and regimented, (. . . ) depends only on Ralph’s current mental state (. . . ) The vivid names “represent” those persons who fill major roles in that inner story which consists of all those sentences which Ralph believes. I have placed ‘represent’ here in scare quotes to warn that there may not actually exist anything which is so “represented”. Ralph may enjoy an inner story totally out of contact with reality, but this is not to deny it a cast of robust and clearly delineated characters. According to my analysis, Ralph must have quite a solid conception of \( x \) before we can say that Ralph believes \( x \) to be a spy.

[Kaplan, 1969, “Quantifying In”, p. 201/4]

Kaplan’s seems to set out to solve an exportation problem.

*Under what conditions can we conclude ‘\( \exists \phi(z) \) of \( o \)’ from ‘\( \exists \phi(o) \)’?*

[The term “exportation” is from Quine, 1956, p. 182.]

But no representans forces a representandum. Representations are made and understood to represent, they don’t do so by themselves.


*How does someone’s belief come to be a belief about Ortcutt?*

That’s the wrong question. The question should be:  

*How does Ortcutt come to be something someone has a belief about?*

Pierre’s London from Kripke

Pierre can be said to believe that London is pretty, because of what he has learned about London from travel guides and stuff when he was young, and also that London is not pretty, because of the way he has once experienced the city. Such is, roughly the state Saul Kripke has put him in.

*But none of this answers the original question. Does Pierre, or does he not, believe that London is pretty? I know of no answer to this question that seems satisfactory. It is no answer to protest that, in some other terminology, one can state ‘all the relevant facts.’*


Kripke seems to presuppose that there is an answer to his question.

Relying on the familiar idea that it is *sentences* that are true or false.
Two Views, Two Judgments

We can agree that we, as Kripke says, understand the situations.

That it is entirely appropriate to say, when Pierre is busy sorting out how to get to this city, London, he once read about in his French years, that he believes London is pretty, and that he does not believe it is not so.

That it is entirely appropriate to say, when Pierre is eagerly trying to leave the town he is in, London, that he believes London is not pretty, and that he does not believe it is.

Why should we insist that the sentence, not in use, has a determinate truth value?

Sharing Resource Situations

When Quine & Kripke explain the vicissitudes of Ralph & Pierre, they do not do so by sketching their individual cognitive configurations, they do not do so by copying their private mental conception of the world, they do so by sketching the situation that they all find themselves in, and that we understand them to be cognitively responding to.

We elaborate on an assumed shared common ground which hosts individuals that our world-mates are cognitively engaged with.

We do not do so by sketching their individual cognitive configurations, we do not do so by copying their private mental conception of the world, we do so by sketching the situation that they all find themselves in, and that we understand them to be cognitively responding to.

Sharing Resource Situations

What we ‘see’ is an assumed real situation, not a mental entity or idea.

A resource situation is what we see through a conceptual window.

Pragmatics of Belief Reports

According to a Hintikka/Aloni approach, the proper interpretation of De Re Belief Reports and of Identification Questions is guided and informed by a perspective on the domain.

The choice of a perspective or conceptual cover renders the report or the question informative rather than trivial or even inconsistent.

Trivial readings, of course, are unacceptable, but they are readings.


Kripke’s Puzzle

Here is Pierre’s situation according to Kripke.

\[
\begin{array}{c}
\sigma^w r^w \\
\end{array}
\]

\[
\begin{array}{c}
x = y \\
B_w \neg P_z \neg P_z \\
\end{array}
\]

Why does Kripke want to say that Pierre’s London is the real London?

[There are several approaches that seek to answer this question in terms of properties of entity representations, belief objects, concepts and covers, . . . Husserl famously argued against such a line of explanation.]

Like Pierre, and unlike Kripke, we can see London through two different Conceptual Windows.

[Conceptual Windows are like Conceptual Covers, but they behave better ontologically and cognitively speaking.]
Two Windows on London

Pierre’s behavior (verbal, or otherwise) indicates that the city advertised in the magazines and travel guides (defining a resource situation) is pretty to him. This city is London, so we report that he believes that London is pretty.

We assume we agree on there being a real situation in which we agree with Pierre there is something that we happen to know to be London, and he likes it.

<table>
<thead>
<tr>
<th>l^w p^w</th>
<th>l^w p^w</th>
</tr>
</thead>
<tbody>
<tr>
<td>r :  Fu Rwu</td>
<td>s :  Gv Swv</td>
</tr>
<tr>
<td>x = y B_w y</td>
<td>x = z B_w z</td>
</tr>
</tbody>
</table>

Or, for instance, he signals that the city he is currently inhabiting (another situation we share information about) is appalling. The city is London. So we report that he believes that London is not pretty.

We assume we agree on there being a real situation in which we agree with Pierre there is something that we happen to know to be London, and he doesn’t like that.

Conceptual Windows

We quantify objects relative to indexed perspectives (Aloni 2005, et al.) ‘r’1 declares an individual seen through window r, a set of conceptions.

<table>
<thead>
<tr>
<th>Indexically Restricted Declarations</th>
</tr>
</thead>
<tbody>
<tr>
<td>syntax: p := ‘1’</td>
</tr>
<tr>
<td>logic: Minds the index. ( \vdash (‘1’ (1_y)) )</td>
</tr>
</tbody>
</table>

Windows are Transparent. Whatever we see through a window exists.

Rigidity Postulates

<table>
<thead>
<tr>
<th>Rigidity Postulates</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Box (‘1’ \forall x = y (\Box_x x = y)) )</td>
</tr>
<tr>
<td>[NB. ( \Box_s s \equiv q \Box (1_s(s)) )]</td>
</tr>
</tbody>
</table>

Windows provide a Clear and Distinct view. A window settles all question of identity of all individuals in the window—in, and not beyond.

No Puzzle About Belief

Formally all we need is this.

London has been declared and judged through two conceptual windows.

<table>
<thead>
<tr>
<th>No Puzzle About Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I^w p^w )</td>
</tr>
<tr>
<td>‘1’ x = y B_w</td>
</tr>
<tr>
<td>( \forall^x x = z B_w</td>
</tr>
</tbody>
</table>

Conceived resource situations have been specified further (e.g., by Kripke).

<table>
<thead>
<tr>
<th>Conceived Resource Situations</th>
</tr>
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<tbody>
<tr>
<td>( (‘1u (\Box_u</td>
</tr>
<tr>
<td>( (‘1v (\Box_v</td>
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</tbody>
</table>

Such entails that Pierre believes the shiny city that he read about to be pretty.

<table>
<thead>
<tr>
<th>Propositional Attitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vdash (e.g.) B_w</td>
</tr>
</tbody>
</table>

Meta-linguistically speaking, there are “three Londons”.

Actually, ‘there’, ‘there’ and ‘there’ is only one.
Day Four: Propositional Attitudes

Conceptually Restricted Quantification

For $w = r$ or $s$ or $t$:

\[(1^r x Bx (1^r x z))\]

\[(1^s (\Delta z Bz))\]

There is someone who is actually Messi, and Alf believes he scored,
but there is no Messi such that Alf believes he scored.

\[(1^t m x Bm [1^t x s])\]

(1^t m z Bm [1^t z S])

Alf does know who is who, but not who is who.

(1^r x' 1^r y = z (B_a[y = z]))

(1^r x' 1^t x = z B_a[x = z])

---

Conceptual Windows in stead of Conceptual Covers

- We can, but we should not, stipulate our windows to collapse into Aloni’s conceptual covers.

- We can harvest all the benefits of the latter, without its dubious commitments.

  No need for every individual to be identified in every possibility;
  no need to make background assumptions into necessary laws.

- Conceptually restricted quantification is a proper improvement over quantification through conceptual covers.

---

No Realist or Representationalist Commitments

- We don’t quantify over conceptual representations, like David Kaplan does, and recently Percus and Uli Sauerland, and Simon Charlow and Yael Sharvit.1

- We don’t objectify possible objects, like David Lewis does, and recently Dilip Ninan.2

- More on talk about things non-existent tomorrow!


---

What Have We Been Doing Today?

- Our aim: Theory of Interpretation.

- We had: An Understanding Language of Translation.

- We wanted: An Understanding of Intentional Presentations.

  Intentional Presentation Theory.

- We explained the *Bedeutung* and *Sinn* of Intentional Presentations through our Understanding of Extensional Presentations.

  It is Essentially Extensional and, hence, Realist.

  It is Essentially Intentional and, hence, Idealist.
Non-existent objects don’t make any sense; Talk about non-existent objects does. (At least sometimes.)

Where Are We Today?

• Our aim: Theory of Interpretation.
• We have: an Understanding of Intentional Presentations.
• We want: An Understanding of Intentional Discourse.

Making Sense of Talk About Things Non-Existential.

Aristotle Talking of What Is

to say that what is not, (...) is false; but to say that what is is, (...) is true; to μὴν γερά λέγειν τὸ δὲ μὴ εἶναι (...) φευδός, τὸ δὲ τὸ ἐν εἶναι (...) ἀληθικός. [Aristotle, The Metaphysics IV, VII, 1011b26–28]

• That Abraham Lincoln had no pets, is false.
• That Donald Trump has hair, is true.
• That Fido is called “Fido”, is true.
• That Michael D. Higgins does not exist, is false.

(Ετς.)

That’s easy.
Aristotle Talking of What Is Not

To say that (...) what is not is, is false; but to say that (...) what is not is, is true.

Aristotle, The Metaphysics IV, VII, 1011b26–28

- That Donald Trump has pets, is false.
- That Michel Foucault has no hair, is true.
- That Mido isn’t called “Fido”, is true.
- That the flying horse Pegasus exists, is false.

But what is it that is not, and that is -falsely- said to be, or -truly- said not to be?

Plato and Aristotle on What Is Not

Not being (...) is inconceivable, inexpressible, unspeakable, alogic. (.)

[Plato, Sophist, 238C]

Therefore we even say that not-being is not-being.

[Aristotle, Metaphysics, B. IV, Ch. 2, 1003b10]

Vulcan (in WIKIPEDIA)

“Vulcan” may refer to:

- the god of fire, . . . , an album by Chris Wood, a fictional race in Star Trek, a gay pornography magazine, . . . , a materials manufacturing company, . . .
- . . . , a hypothetical planet between Mercury and the Sun (.), a programming language now known as dBase, . . .
- a South Devon Railway Buffalo class steam locomotive, . . .

Vulcan (Hypothetical Planet)

Vulcan is a small hypothetical planet that was proposed to exist in an orbit between Mercury and the Sun. Attempting to explain peculiarities of Mercury’s orbit, the 19th-century French mathematician Urbain Le Verrier hypothesized that they were the result of another planet, which he named “Vulcan.”

Claiming that *Holmes distracted him* "from better things," Conan Doyle famously in 1893 ("The Final Problem") attempted to kill him off; during a violent struggle on Switzerland’s Reichenbach Falls, both Holmes and his nemesis, Professor Moriarty, are plunged over the edge of the precipice. Popular outcry against the demise of Holmes was great; men wore black mourning bands, the British royal family was distraught, and more than 20,000 readers cancelled their subscriptions to the popular Strand Magazine, in which Holmes regularly appeared. By popular demand, Conan Doyle resurrected his detective in the story “The Adventure of the Empty House” (1903).

[https://www.britannica.com/topic/Sherlock-Holmes]

---

If we combine a one-place predicate like `snore` and a proper name like `Jane` as in (1), we predicate the property of snoring of `Jane` (\ldots)

\begin{itemize}
  \item \(\text{Jane snores.}\)
  \item \(\text{[Snore}(j)] = 1 \text{ if } [j] \in [\text{Snore}]\)
  \item \(\text{Jane(Snore)}\)
  \item \(\{X \subseteq U \mid j \in X\}\)
\end{itemize}

\((2a)\) claims that the property of snoring is one of the properties `Jane` has.


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Bellows Falls Bridge (1905—1982)

*This is the plan of the bridge they were building in 1904.*

The bridge did not exist then, it does not exist now, but they were building it.
Philosophical Habilitations

- Intentional Objects have achieved the status of honourable objects, not only in literary circles, but also in philosophical and linguistic discourse.


- Declared or Undeclared Neo- or Non-Meinongians, they all postulate things and stuff that does not ‘regularly’ exist.

If Only Intentional, then Actually Not

*Objects of thought are not, as such, entities. An object of thought is just anything which is thought about (…). Some objects of thought exist, and some do not. (…) When an object of thought exists — for example, when I think about the planet Neptune — then the object of thought simply is the thing itself (Neptune itself). When the object of thought does not exist, it is nothing at all (cf. Husserl 1900-01: 99).* [Tim Crane, 2012, “Existence and Quantification Reconsidered”, Cambridge]

[Intentional objects can be made available only by the presence (…) of intentional acts on which the intentional objects depend. Positing intentional objects thus does not mean taking unsuccessful acts of reference to in fact be successful, referring to intentional objects. (…) Intentional objects are not part of the ontology; they are mere projections of intentional acts, which is why they have the status of nonexistent. [Friederike Moltmann, 2015, “Quantification with Intentional and with Intensional Verbs”, p. 147, 150, 152]*

A Compulsive Disorder

Even those who do not want to commit to their existence, cannot but do so.

(…) [If we aim to give a systematic account of our actual thought and language, then we have to make room for quantification over the non-existent.]

[Tim Crane, 2012, “Existence and Quantification Reconsidered”]

(16b) John mentioned, [a woman,]

(16c) $\exists e, x (\text{mention}(e, John, x) \land \text{woman}(x, e))$

“buildings that John could have built”:

$\lambda x [\forall (\text{building}(x) \land \text{build}(John, x))]$

[Friederike Moltmann, 2015, “Quantification with Intentional and with Intensional Verbs”, p. 147, 150, 152]

The attraction of intentional objects appears irresistible, even inescapable, even to the unwilling.

Possibly, but Not Really

- Do future bridges exist? Not yet. (Only in the future.)
- Do past bridges exist? No longer. (Only in the past.)
- Do possible individuals exist? Not actually. (Only in a possibility.)
- Are there imaginary objects? Not really. (Only in the imagination.)
- Are there intentional objects? Only intentionally.
Intentional Presentation (Natural Language)

**Intentional objects exist in an intentional presentation.**

That there is a certain intentional object says nothing more, nor less, than that it has been intended that there be that kind of object.

- A possible intruder: it is possible that there is an intruder.
- A planned bridge: it is planned that there be a bridge.
- A hypothetical planet: it is hypothesized that there is a planet.
- A needed postdoc: it is required that there be a postdoc.
- ...

\[ P : \begin{array}{c}
  \text{BRDG} \[ x \\
  \ldots
\end{array} \]

We understand this, or so we assume.

P: [BRDG \[ x \] \ldots]

We understand this, or so we assume.

Paul Dekker (ILLC, UvA)  
Theory of Interpretation (ESSLLI)  
Galway 22, Aug 12  
177 / 200

Intentional Presentation (Textbook)

\[ P : \begin{array}{c}
  \text{PLNT} \[ x \\
  \ldots
\end{array} \]

(pictured) (postulated) (hypothesized)

The cat sits. Jane snores. Vulcan could be a dwarf planet.

We understand all this. We see the cat. We reason about Jane. We speculate about Vulcan. But do any of them exist? No!

Paul Dekker (ILLC, UvA)  
Theory of Interpretation (ESSLLI)  
Galway 22, Aug 12  
178 / 200

Dynamic Intentional Presentation (DIPT)

We Actually Conceive Possibilities

*We think things, and the things we think can be the way things are.*  
[These are different “things”.

*We picture, imagine and conceive “possibilities”.*  
[Which is not to say that there are possibilities that we conceive.

Some of them are real, or actual, some are not.  
[How we manage to do this, is for another discipline to explore.

To speak truly of something that is not, is not to assert the being of something not being, but to assert the non-being of something not being.  
[To deny the being of what can be thought to be real but really is not.]

Paul Dekker (ILLC, UvA)  
Theory of Interpretation (ESSLLI)  
Galway 22, Aug 12  
179 / 200

Non-Existence, Possibly

Here is a presentation that presents Pegasus, as existing.

\[ P \]

*There is Pegasus*

[Pegasus’ existence is a possibility which could be not realized.]

Paul Dekker (ILLC, UvA)  
Theory of Interpretation (ESSLLI)  
Galway 22, Aug 12  
180 / 200
Non-Existence, Factually

Here is a presentation that presents Pegasus as not existing.

\[ \neg p \]

There is no Pegasus

[There is no realization of a Pegasus presentation.]

[This interpretation properly serves our logical purposes. (Wednesday)]

Non-Existence, Discursively

Here is a presentation that presents a familiar Pegasus as not existing.

\[ \ldots \neg p \ldots \]

This Pegasus does not exist.

[A presupposed presentation of Pegasus is not realized.]


Question: What does \textbf{THIS} mean, and how does \textbf{it} relate to this?

Dynamic Intentional Presentation (Syntax and Logic)

- Syntax.

\[
K ::= \ldots | I_x K
\]

- Logic.

If \( K_1, \ldots, K_j \vdash C \), then \( I_x K_1, \ldots, I_x K_j \vdash I_x C \)

[Intentional objects are possibly non-existent but remain intentionally present. They allow for intentional reification. Even so, there is no support for dynamic existential entailments.]

Dynamic Intentional Presentation (Semantics)

- Models.

Quintuples \((W, U, \{C_i\}, \{R_k\}, I)\) with a set \(U\) of points coordinating our talk about possible individuals and a family \(\{C_i\}\) of methods of individuation, each method, or \textsc{Window}, consisting of a set of individual conceptions — functions, possibly partial, from \(W\) to \(U\).

\(C_0\) defines the domain \(D_v\) of a world \(v\). A window provides a view on existing individuals only: for any \(c \in C_i : c \in D_v\), if defined.

- Semantics.

\[
M, w, c \models '1' \quad \text{iff} \; c \in C_i \quad \text{and} \quad c \in D_v
\]

\[
M, w, g, e \models I_x K \quad \text{iff} \; v \in I_w(1)_{g(x)} \quad \text{only if} \; M, v, g, e \models K \quad \text{for any} \; v
\]

[Notice that \(e\) provides witnesses for \(K\), but not necessarily in \(w\).]
Christo’s Inclinations (Schematic)

An architect plans to build a bridge, and Christo wants to wrap it up.

\[
\begin{array}{c|c|c}
\text{P}_a : & t_x & y_x \\
\text{BRDG}[x] & \ldots & \text{WRAP}^c[c; y] \\
\hline
\text{I}_c : & \ldots \\
\end{array}
\]

The two presentations are jointly satisfied if architect \(a\) has a plan, and Christo \(c\) has an intention, so that any realization suits both plan and intention only if it has a bridge made by \(a\) and wrapped by \(c\).

[No bridge needs to be ever actually realized, for this to make sense. Neither need there be two “bridges”, which are here identified.]

How the two are assumed to be jointly ‘related’ to ‘the bridge’ can be framed employing the scheme \(l(\forall z (\exists z R[acz]))\).

Hob and Nob’s Beliefs (Schematically)

We have intentional identity when a number of people, or one person on different occasions, have attitudes with a common focus, whether or not there actually is something at that focus.


Hob thinks that a witch has blighted Bob’s mare, and Nob thinks she also killed Cob’s sow.

\[
\begin{array}{c|c|c}
\text{B}_h : & t_x & y_x \\
\text{WITCH}[x] & \ldots & \text{KCS}[y] \\
\text{BBM}[x] & \\
\hline
\text{B}_n : & \ldots \\
\end{array}
\]

These presentations are satisfied if Hob and Nob have beliefs which are jointly satisfied by a possibility only if it hosts a witch guilty of both crimes.

\(\Box(\forall z (\exists z S[hnz]))\) can be employed to specify how the two are supposed to ‘be familiar’ with ‘the witch’.

The Vulcan Hypothesis (Schematically)

- A planet has been hypothesized. It is called Vulcan. It does not exist.

\[
\begin{array}{c|c|c|c}
\text{H} : & t_x & y_x & z_x \\
\text{PLANET}[x] & \ldots & \text{OBS}[y] \\
\hline
\text{A} : & \ldots \\
\end{array}
\]

- We do not say that every use of “Vulcan” is without Bedeutung.
- It is a specific use of the term “Vulcan” which is denied Bedeutung.
- It is nevertheless ‘meaningful’, e.g., in explanations of human research.

A few, however, remained convinced that not all the alleged observations of Vulcan were bogus. [https://www.chemeurope.com/en/encyclopedia/Vulcan_(hypothetical_planet).html]

Intentional Reification

Such Intentional Reification is a well-established practice.

A cake that is to be baked—it is possibly going to be stuffy. A bridge that has been planned—it has a scheduled height.

A rose that is painted—it is painted red. A PhD student that is needed—she must complete her thesis in time. A unicorn that is imagined—it may and may not have any wings.

hypothetical planets, daggers perceived, missing individuals—

they are all intentionally reifiable.
Intentional Yet Referentiable

Does not what I have said about [Dragons do not exist.] require that dragons be even though they don’t exist? Must not dragons have some mode of being, exist in some universe of discourse? To these rhetorical questions it is sufficient to reply with another:

What, beyond the fact that it can be referred to, is said of something when it is said to have some mode of being or to exist in a universe of discourse?


Non-Existent Yet Identifiable

Questions about the identity of an intentional object, when this cannot be reduced to the identity of a material object, are obviously of some interest. How do we decide that two people or peoples worship or do not worship the same god? (…) The fact that we can use the concept of identity in connexion with intentional objects should not lead us to think there is any sense in questions as to the kind of existence—the ontological status—of intentional objects. All such questions are nonsensical.


The Final Puzzle ... 

Talking of non-existent objects, we relate to nothing, but we do relate to something of which we can say that it could exist, while it doesn’t.

Since it is taken not to exist, one may wonder:

What, if anything, would support our identification of two intentional objects (Q1), and what would their equation mean, if anything (Q2).

[The challenge being that it cannot be the establishment of a relation (of identity, …), because its terms are, supposed to be, non-existent.]

... and Its Solution

By the non-existence of merely intentional objects—there are no objects that are objects that are correlated—there is only the intentional correlation of intentional objects.

What would license the construal of such objects as one the same, (Q1)

What would their correlation mean? (Q2)

We construe the two intensional presentations as so correlated (A1), understanding the described agents as behaving so coordinated (A2).

Intentional Reification.

Viewing them as intentionally identical through intentional conceptual windows.
Coordinating Bridges

\[ \begin{align*}
P_a : & \quad BRDG[x] \\
\quad & \quad x = z \\
\quad & \quad \ldots
\end{align*}\]

\[ \begin{align*}
I_c : & \quad WRAP[c, v] \\
\quad & \quad v = w \\
\quad & \quad x = v \quad z \neq w
\end{align*}\]

- I can construe the intended objects as one and the same, which benefits my understanding of e.g., \( a \) and \( c \)'s coordinated behavior.
- I can construe them as distinct, which benefits my understanding of, e.g., the development of \( a \) and \( c \)'s individual plans and expectations, and respective changes in the bridges so conceived.
- I am able to do so, because —while there are bridges in my understanding of their individual plans— there are no bridges (no things) being characterized here.

Rediscovering Vulcan

In 1915, when Einstein successfully explained the apparent anomaly in Mercurys orbit, most astronomers abandoned the search for Vulcan. A few, however, remained convinced that not all the alleged observations of Vulcan were unfounded.

What if the few detect a clump of stuff, and insist it’s Vulcan?

We can decide that they are wrong, but not actually prove that the lump is not a certain non-existent object. No matter of fact settles that question.

It is subject to the construal of our understanding of the world.
Summarizing Today

- What is said to not exist, is some thing thought to possibly exist.

\[
I : \frac{\overline{y}x}{P[x]} \rightarrow \frac{z}{z = x}
\]

It was thought (...) that some thing exists. *That* object does not exist.

- We can conceive things without there being the things we conceive.
- Merely intentional objects only intentionally exist, and not really.
- Talk about things non-existent proceeds by intentional reification.
- This talk makes sense (*Sinn*) but has no meaning (*Bedeutung*).

Some Results, Perhaps . . .

- On the Negative Side, Critically.
  
  *We have done without implication, variables, quantification, representation, platonism, possibilism, and non-existent objects.*

- On the Positive Side, Constructively.
  
  *We have developed a minimalist logic, the concept of the declaration and coordination of referents, a Frege/Davidson/Kamp theory of Discourse Presentation, motivated by a nominalist logical understanding of intentional discourse.*

Thanks for your Attendance!

*I hope at least one of these days here you have gained some ideas, thoughts, questions, which are worthwhile to reflect upon for a substantial, non-empty, but not too long, period in your further hopefully enjoyable and successful intellectual life.*
There is no paradox of material implication if there is no material implication.