

# Exchange market pressure in interest rate rules

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## Abstract

Many central banks pursue some kind of exchange rate objective. We derive what variables the central bank should look at when setting the interest rate to implement a given objective. Exchange market pressure (EMP), the tendency of the exchange rate to change, emerges as the key variable. The resulting interest rate rule can implement many regimes, from floating to intermediate to fixed rates, and does so exactly, even after a structural change. It can be applied to many models, and we illustrate it in a New Keynesian model for a small open economy.

*Key words:* DSGE, exchange market pressure, exchange rate regime, fixed exchange rate, monetary policy, open economy Taylor rule.

*JEL classification:* E43; E52; F31; F33.

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# 1 Introduction

Most central banks engage in some form of exchange rate management, particularly in small open and emerging economies; IMF (2020). Our aim is to find the variable(s) the central bank should look at when setting its policy instrument to achieve a given exchange rate objective. We provide a novel derivation for that. It reveals an important role for exchange market pressure, EMP. This variable, due to Girton and Roper (1977), is the tendency of the exchange rate to change, where positive (negative) EMP means depreciation (appreciation) pressure.<sup>1</sup>

We derive this finding in a general setting. That is, by exploiting the information on exchange rate determination that is already present in the theoretical model at hand, we need only few additional assumptions. Moreover, the exchange rate objective can be a fixed, some intermediate exchange rate regime, or the (perfectly free) float, regarding the level or change of the exchange rate. Our finding also applies to several policy instruments, such as the interest rate and official forex intervention. Note that this paper focuses on the interest rate, because many countries use it and it provides the simplest framework for introducing our idea. We thus conclude that EMP results from a derivation that is applicable to many settings. Apparently, EMP is a key variable for central banking.

We take the exchange rate objective as given. So, we do not derive the degree of exchange rate management central banks should pursue, which depends on the imposed economic structure.<sup>2</sup> Instead, we start from what they *actually* pursue. In this sense the paper is not normative but positive. This also allows us to keep the analysis general, so that the insights are not model specific but hold in many settings.

Our derivation leads to a new interest rate rule that implements the exchange rate objective exactly. The rule has two main novelties. First, it extends a domestically-oriented rule, such as the Taylor rule, by adding EMP in deviation from the exchange rate change that is acceptable according to the objective. Excess pressure implies a

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<sup>1</sup>The formal definition of EMP follows in Section 2.3.1 and is taken from the EMP literature, in particular Weymark (1995). Interesting applications include the Financial Stress Index of IMF (2009), Frankel and Xie (2010), and Aizenman et al. (2017).

<sup>2</sup>Engel (2014) concludes that welfare-based optimal monetary policy analysis in open-economy models is still in the early stages, but that the analysis to date suggests a role for exchange rates in an optimal rule. Their importance depends on the economic structure. Schmitt-Grohé and Uribe (2016) show that in case of downward nominal wage rigidity optimal exchange rate policy calls for large devaluations during crises to ensure full employment. Davis et al. (2018) show that as central bank credibility falls and thereby the ability to commit to future policy, a highly open economy will quickly find it optimal to set the interest rate to peg the nominal exchange rate as the single mandate. Buffie et al. (2018) analyze less developed countries that pursue inflation targeting. In a float, currency substitution causes a high risk of indeterminacy (multiple equilibria), as well as escalation of inflation shocks. Both problems disappear by tight management of the nominal exchange rate.

high interest rate. This paper thus connects two strands of the literature, that on interest rate rules and EMP. It stresses that EMP, which is often used in applied work, also matters for theory and policy.

The second novelty of our rule is that the coefficient for EMP depends on the interest rate effectiveness to ward off depreciation: for given EMP, the policymaker should use the interest rate less intensively if it is more effective. If he does not adjust, the exchange rate objective is missed. This is a natural property but still a novel feature of our rule. It also helps us to reveal two structural parameters that underlie the coefficient for EMP, namely the effectiveness and the degree of exchange rate management. The former is determined by the model, while the latter is a policy choice.

What do central banks actually do? Some are explicit on this and consider what they call “pressure.” For example, Danmarks Nationalbank (2021) writes that “in situations with upward or downward pressure on the krone, Danmarks Nationalbank unilaterally changes its interest rates in order to stabilise the krone.” Likewise, the Hong Kong Monetary Authority (2009) describes its “automatic interest rate adjustment ... against downward pressure on the exchange rate.”<sup>3</sup> The idea is that high selling pressure requires a high interest rate. Calvo and Reinhart (2002) provide further examples. He et al. (2011) from the HKMA write that they monitor “foreign exchange market pressure” in their daily work. Mohanty (2013) reports that in a BIS survey among central banks almost 80% said that curbing speculative pressures on the exchange rate was the most important priority.

Despite the relevance of pressure in actual policy, there is no formalization yet. We show that EMP is close to what central bankers mean by pressure, and we derive that it is natural to have EMP in a policy rule. A model can then reveal building blocks of EMP, which can help policymakers to develop new indicators of pressure. So our derivation provides theoretical and practical support for actual policy. The reverse is also true: actual policy confirms the realism of our approach.

The traditional way to model the interest rate in case of an exchange rate objective is to add the exchange rate gap (actual minus target value) to a Taylor rule, as in Monacelli (2004), or to the foreign interest rate, as in Benigno et al. (2007). This has contributed to valuable insights, in other papers as well.<sup>4</sup> However, we cannot use such

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<sup>3</sup>Hong Kong has a currency board system based on an automatic interest rate adjustment mechanism. In case of downward pressure, the central bank purchases Hong Kong dollars from banks so as to increase market interest rates and thereby capital inflows and achieve exchange rate stability. Because of its focus on the interest rate, we use Hong Kong as an example in this paper. An alternative would be to model policy as unsterilized intervention, but then the main idea relevant for us would be similar: the central bank responds to pressure and exploits the interest rate.

<sup>4</sup>The former rule has also been used by Engel and West (2005), Corsetti and Müller (2015), and Galí

an approach of adding a preselected variable to answer our research question. After all, we want to know what variables to include, and how. That is why we *derive* our rule.

Although the scope of this paper is not to recommend a new interest rate rule, it is interesting to compare the rules. First, our rule can replicate both traditional rules as special cases. Second, it can cover the full spectrum of regimes, from floating to intermediate to fixed exchange rates. Third, in line with actual policy, we have EMP instead of the actual exchange rate gap. Fourth, our rule has separate parameters for the interest rate effectiveness and the exchange rate regime, which make that it automatically accommodates structural change, so that the objective keeps on being implemented. These structural parameters are hidden in the traditional approach.

These differences stem from our alternative procedure. We develop the rule within the economic model at hand, so that we end up with a consistent and coherent framework. That makes our rule attractive for use in theoretical analyses. It can act as a benchmark and provide insights into how the exchange rate is pinned down. The latter helps to derive a simple variant of the rule, one that only contains observables. It has the lagged instead of contemporaneous exchange rate, which improves implementability.

The structure of the rest of the paper is as follows. In Section 2 we derive the interest rate rule and show the relevance of EMP. Section 3 discusses its characteristics. In Section 4 we set out a New Keynesian DSGE model for a small open economy to illustrate our method and derive the interest rate rule and EMP for that model. Section 5 illustrates their characteristics using a simulation study. Section 6 concludes.

## 2 Interest rate rule

For a given exchange rate regime, the goal is to derive the variables the monetary authorities should look at when setting the interest rate to implement that regime. We do so by developing an interest rate rule in a two-country setting. The domestic monetary authorities, being the central bank throughout this paper, pursue some degree of exchange rate management as one of the policy goals (the float is a valid special case). Foreign authorities do not try to control the exchange rate.

The first section sets out the main ideas of our derivation in a simple setting, based on uncovered interest parity (UIP). After that, we present a more general framework, showing that our approach is broadly applicable.

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and Monacelli (2016), for example. The latter rule has been applied by Benigno (2004) and Born et al. (2013), among others. These papers focus on the nominal exchange rate. Instead, some authors add the real rate to an interest rate rule, as in Clarida et al. (1998) and Mimir and Sunel (2019). Leitemo and Söderström (2005) also study changes of nominal and real rates. All these papers add preselected variables, and we let Monacelli (2004) and Benigno et al. (2007) represent this approach.

## 2.1 Simple setting: UIP

### 2.1.1 First try

Let  $s_t$  be the (logarithm of the nominal) exchange rate at time  $t$ , which is the domestic currency price of one unit of foreign currency. If the central bank pursues a fixed exchange rate target  $s^t$ , a researcher can incorporate that by adding  $s_t = s^t$  for all  $t$  to the model. That can suffice for some fixed exchange rate analyses, but it will not tell us what variables the central bank should study to implement the target.

If UIP holds (in log-linearized form), the following method delivers the same regime endogenously. Let  $i_t$  and  $i_t^*$  denote the domestic and exogenous foreign interest rates, respectively, and  $\mathbb{E}_t \{s_{t+1}\}$  is the expected exchange rate conditional on information available in period  $t$ . The rule

$$i_t = i_t^* + \mathbb{E}_t \{s_{t+1}\} - s_t, \quad (1)$$

so with the target instead of  $s_t$  at the end, implies  $s_t = s^t$ , by construction.<sup>5</sup> It shows  $i_t^*$  and  $\mathbb{E}_t \{s_{t+1}\}$  are important variables for the central bank to implement the regime.

Our approach, presented below, improves on this. First, it provides additional insights into what variables matter for exchange rate policy. Next, it also applies to settings where UIP does not hold, so that we can obtain a more complete picture of the relevant variables. Moreover, from rule (1) it is not clear how to implement other exchange rate regimes, while our rule can deliver many regimes, including the float, a weighted combination of fixed and float, and leaning against the wind. Finally, many traditional rules have a domestically-oriented part to which an exchange rate part is appended. Our rule has that form as well, which facilitates comparison.

### 2.1.2 Derivation of the rule if UIP holds

Our method consists of three steps. First, we need the interest rate to be consistent with the exchange rate generated by the model at hand. In the current highly-simplified setting, this is automatically accomplished by UIP,

$$i_t = i_t^* + \mathbb{E}_t \{s_{t+1}\} - s_t. \quad (2)$$

The second step recognizes that  $i_t$  reflects not only forex policy but also domestically-oriented policy. Because we want to find what the central bank should focus on for

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<sup>5</sup>This rule is similar to the price level targeting rule for a closed economy in Adão et al. (2011), who use a Fisher equation with the current price level substituted by the target.

its forex policy, we split  $i_t$  into two parts. Let  $i_t^d$  denote the part that shows the rule for the interest rate the central bank would set without considering the exchange rate. We call  $i_t^d$  the domestically-oriented (part of the) interest rate rule. The remainder,  $i_t - i_t^d$ , reflects exchange rate policy. This does not introduce a different policy but just distinguishes two parts of the existing policy. Both parts typically depend on the actual economic situation, and thus on  $i_t$ , say. For example,  $i_t^d$  could be a standard Taylor rule based on actual inflation. In a float,  $i_t = i_t^d$ .

Rewrite (2) by adding and subtracting  $i_t^d$ , so that

$$i_t = i_t^d + s_t^d - s_t, \quad (3)$$

where

$$s_t^d = -i_t^d + i_t^* + \mathbb{E}_t \{s_{t+1}\}. \quad (4)$$

Hence, (3) shows that  $s_t^d - s_t$  reflects exchange rate policy. Definition (4) contains  $\mathbb{E}_t \{s_{t+1}\}$ , reflecting that exchange rate policy has to account for actual expectations of investors, not expectations based on  $i_t^d$  or so, because there is no policy change.<sup>6</sup> The presence of  $i_t^d$  instead of  $i_t$  in (4) together with actual expectations  $\mathbb{E}_t \{s_{t+1}\}$  shows that the contemporaneous impact of exchange rate policy is removed, that is, the impact not involving expectations.<sup>7</sup> Hence,  $s_t^d$  is the notional exchange rate that prevails when taking out the contemporaneous impact of exchange rate policy. As  $s_t^d$  combines part of  $i_t$  with the other determinants, we call  $s_t^d$  the intermedial exchange rate. It is just a convenient combination of existing variables, not a necessary variable in the model.

The final step of our derivation proposes the rule

$$i_t = i_t^d + s_t^d - s_t^o, \quad (5)$$

where  $s_t^o$  denotes the exchange rate objective. The intuition is that a high  $s_t^d$  reflects that, abstracting from the contemporaneous impact of policy, investors intend to sell the currency, and to the extent that it exceeds the objective  $s_t^o$ , the central bank has

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<sup>6</sup>Because — as always in the paper —  $i_t^d$  and expectations are both based on the actual interest rate  $i_t$ ,  $s_t^d$  is *not* the counterfactual exchange rate based on  $i_t^d$  and expectations and variables consistent with  $i_t^d$ . The latter would boil down to the exchange rate under a float, but that is not the regime in place, as we do not study a policy change. So  $s_t^d$  is not the exchange rate in another general equilibrium.

<sup>7</sup>Policy  $i_t - i_t^d$  can affect  $s_t$  through various contemporaneous channels. The impact through all of them is taken out to form  $s_t^d$ . Suppose, in contrast, we had kept in some fraction  $0 < \phi < 1$  to form the alternative  $\tilde{s}_t^d = -\phi (i_t - i_t^d) + s_t^d$ . Then  $s_t = \tilde{s}_t^d - (1 - \phi) (i_t - i_t^d)$ , so that (3) would have become  $i_t = i_t^d + \frac{1}{1-\phi} (\tilde{s}_t^d - s_t)$ . That would have yielded an equivalent rule with the same underlying economic mechanisms at work, it would not have affected the answer to the question of what variables the central bank should look at, and it would have made  $i_t$  on the left of the rule directly dependent on itself, blurring interpretations.

to set a high interest rate. For the fixed rate,  $s_t^o$  is the target  $s^t$ . In the float,  $i_t = i_t^d$ , so that  $s_t^o$  has to be set at  $s_t^d$ . Other regimes are handled by other choices for  $s_t^o$ , as described in Section 2.4.

It is clear that adding rule (5) to the model is equivalent to adding  $s_t = s_t^o$ . Hence, the rule implements the exchange rate objective exactly, by construction. The reverse is also true: to implement the objective, the central bank should use rule (5).

In the special case of the fixed exchange rate  $s_t^o = s^t$ , our rule is equivalent to rule (1). Still, our approach reveals another variable of interest for the central bank in its exchange rate policy, which is  $i_t^d$ , entering via  $s_t^d$  in (4). For example, a drop in  $i_t^d$  to stimulate the economy here increases excess supply of the currency and thus  $s_t^d$ , which the central bank has to offset by exchange rate policy. Actually, here the bank fully offsets the initial drop, in line with the incompatible trinity. The relevance of both  $i_t^d$  and  $i_t^*$  for exchange rate policy is also in line with the symmetry expected for a relative variable such as the exchange rate.

## 2.2 More general setting

Because of the potential relevance of imperfect capital mobility and risk premia, as of here we no longer assume that the exchange rate depends on UIP. Instead, in Section 2.2.1 we introduce a more general exchange rate function. The derivation of the interest rate rule itself is in Section 2.2.2.

### 2.2.1 Exchange rate function

We assume that it is possible to solve the exchange rate  $s_t$  from the particular model at hand as a function of the exchange rate determinants. The interest rate  $i_t$  is one determinant, and we cluster its effects on  $s_t$  in two groups. First,  $i_t$  operates via contemporaneous channels. For example, a high  $i_t$  attracts capital and thus lowers  $s_t$  (appreciation). Or a high  $i_t$  weakens current consumption, reducing the home price level, increasing foreign demand for home goods, and appreciating the home currency, all in period  $t$ .

The second impact of  $i_t$  on  $s_t$  goes via expectations. For example, a high  $i_t$  may increase the currently expected interest rate next period,  $i_{t+1}$ , which then weakens expected consumption at  $t+1$ , and similar to the causal chain above leads to expected home appreciation at  $t+1$ , appreciating the home currency at  $t$ .

We now write the exchange rate function  $s$  in the form

$$s_t = s(i_t, E_t), \tag{6}$$

where the  $i_t$  argument represents the contemporaneous channels, and the vector  $E_t$  consists of expectations  $\mathbb{E}_t \{.\}$  and all other exchange rate determinants. The  $(i_t, E_t)$ -separation will be convenient in the next section. It is not restrictive — it just splits the full impact of  $i_t$  on  $s_t$  into the two groups. For example, in the UIP Section 2.1 the function would be  $s_t = -i_t + i_t^* + \mathbb{E}_t \{s_{t+1}\}$ . Section 4.2 derives the  $(i_t, E_t)$ -form in our New Keynesian model, in particular formula (37), where (39) specifies  $E_t$ .

The variables in  $E_t$  depend on expectations, predetermined, and contemporaneous variables, but the separation implies that the latter no longer include the impact of  $i_t$ . So contemporaneous variables such as goods prices, interest rates concerning other maturities than the one underlying  $i_t$ , national income, and fiscal policy are first cleaned for  $i_t$  by moving the  $i_t$  dependencies to the  $i_t$  argument, and then the remainder enters  $E_t$ . For example, consider goods prices. The second example of contemporaneous channels above, that a high  $i_t$  lowers goods prices and appreciates the currency, is captured by the  $i_t$  argument. What remains in the  $E_t$  argument is, for example, that lower expected future income weakens current consumption, causing lower prices and appreciation, and that exogenous technological progress via lower prices causes appreciation.

Let's impose for simplicity the  $s$ -function is linear in its first argument, that is,

$$s_t = -wi_t + s(0, E_t), \quad (7)$$

where the scalar

$$w = -\frac{\partial s}{\partial i}(\cdot, E_t) \neq 0 \quad (8)$$

reflects the effectiveness of the interest rate to counteract depreciation via all contemporaneous channels.<sup>8</sup> We leave out the time subscript from  $w$  for notational simplicity. Linearity holds in the UIP section, where  $w = 1$ , and in our New Keynesian model, where  $w$  is constant, positive, and follows from existing parameters.

### 2.2.2 Derivation of the rule

The derivation resembles the structure of the UIP-based derivation in Section 2.1.2. It consists of three steps. First, realize that the model at hand determines how the interest rate  $i_t$  affects the exchange rate  $s_t$ , represented by  $s$ -function (7). To ensure that our rule implements the objective, we take that function and rewrite it as

$$i_t = \frac{1}{w} (s(0, E_t) - s_t). \quad (9)$$

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<sup>8</sup>One usually considers  $w$  to be positive, that is, an interest rate increase appreciates the currency. In intuitive explanations below we will do as if  $w$  is positive, but we do not impose it in the derivation.

The second step introduces the domestically-oriented interest rate  $i_t^d$ , as motivated in the UIP section, by adding and subtracting  $i_t^d$ . That yields

$$i_t = i_t^d + \frac{1}{w} (s_t^d - s_t), \quad (10)$$

where

$$s_t^d = s(i_t^d, E_t). \quad (11)$$

The interpretation of both equations is the same as given below (4).

Third, we propose the rule

$$i_t = i_t^d + \frac{1}{w} (s_t^d - s_t^o), \quad (12)$$

where  $s_t^o$  is again the exchange rate objective. If UIP holds, this becomes rule (5), and the intuition there also applies here. As another example, objective (16) implies rule (17), which in the New Keynesian illustration becomes (42).

Because the model at hand delivers exchange rate function (7), and the latter is rewritten into and thus equivalent to (10), adding rule (12) to the model is again equivalent to adding  $s_t = s_t^o$ . Hence, the rule implements the exchange rate objective exactly at every  $t$ . Conversely, to implement the objective, the central bank should use rule (12). This is also a consequence of the fact that our approach exploits information that is already in the model, fundamental equation (7) for the exchange rate. An additional insight compare to the UIP-based rule (5) is that the interest-rate effectiveness  $w$  matters, and thus the structure of the economy: the more effective the instrument, the smaller its required use, as expected.

## 2.3 EMP as the key determinant

Exchange market pressure (EMP) is a concept introduced by Girton and Roper (1977). Intuitively, it represents the reluctance of investors to hold the domestic currency at the forex market. This reluctance tends to affect the exchange rate, and that may trigger the central bank to act. This resembles the idea of our interest rate rule. Indeed, we will show that the rule implies a prominent role for EMP in policy.

### 2.3.1 EMP definition

The idea of the EMP concept is to split the actual (relative) depreciation of the home currency, resulting from the interplay of investors and authorities, into a part reflecting the reluctance of investors to hold the currency, called EMP, and the policy-based part,

which usually intends to counteract EMP. EMP applies to any exchange rate regime and can be positive as well as negative, where the latter means there is pressure on the currency to appreciate. One example is a fixed exchange rate that is under attack by speculators and where the attack is successfully offset by policy. Then EMP is positive, the policy-based counteracting depreciation is negative and offsets EMP exactly.

More formally,  $EMP_t$  is defined as the relative depreciation of the home currency at time  $t$  in the absence of exchange rate policy, while keeping expectations at the levels determined by actual policy. This is the standard definition in the EMP literature, due to Weymark (1995). The definition is unique and does not depend on a model. We here apply it to the setting where the interest rate is the policy instrument.<sup>9</sup>

One key element in the definition is the absence of exchange rate policy. The interest rate rule in this situation is  $i_t^d$ , the domestically-oriented rule introduced in Section 2.1.2. So EMP does not directly depend on the actual interest rate  $i_t$ .

Without the second key element in the EMP definition, the condition on expectations, the use of  $i_t^d$  would make EMP like the depreciation under a floating exchange rate regime. But that is not what EMP intends to capture; EMP is about the reluctance of investors to hold the currency in the *actual* regime. So EMP uses the same values for the expectations as  $E_t$  defined earlier.

We can thus express the standard EMP definition in our notation as

$$EMP_t = s_t^d - s_{t-1}. \quad (13)$$

### 2.3.2 EMP in the rule

Rewriting rule (12) using (13) yields

$$i_t = i_t^d + \frac{1}{w} (EMP_t - (s_t^o - s_{t-1})). \quad (14)$$

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<sup>9</sup>Klaassen and Jager (2011) set out how the interest rate appears in the EMP definition and provide empirical support for that. Our approach is also in line with that.

Apart from the unique definition of  $EMP_t$ , the EMP literature also offers two ways to quantify  $EMP_t$ . First, one can take a theoretical model and derive  $EMP_t$  as a function of fundamentals. This function varies across models, and Weymark (1995) and (13) with (41) in our paper provide examples. So, these are model-dependent quantifications of EMP, while the definition of the EMP concept itself is model independent.

Second, the EMP literature offers practical ways to measure  $EMP_t$ . In some papers, the term “EMP” not only refers to EMP itself, but also to the *measure* of EMP. EMP measures vary across papers and include the current value of the policy instrument, such as  $i_t$ . In contrast, in our paper “EMP” always means EMP itself, not the measure, and we use EMP to explain the current value of the policy instrument. Note that EMP measures often include the change in foreign reserves divided by money supply. Klaassen and Jager (2011) explain why that is in line with the EMP definition, for a central bank using forex intervention as instrument.

Given the derivation of the rule, we conclude that EMP emerges naturally as the key determinant of the interest rate if a central bank wants to implement a given exchange rate objective. In this sense, our derivation reveals the insight that central banks should look at EMP. So our rule guides policy.

The rule says that the central bank has to set  $i_t > i_t^d$  to ward off  $EMP_t$  insofar pressure exceeds the target depreciation  $s_t^o - s_{t-1}$ . The magnitude of  $i_t - i_t^d$  is the amount of excess pressure converted into interest rate units by dividing by the effectiveness  $w$  of the interest rate instrument.

It is contemporaneous pressure that matters, not expected future pressure. This marks a difference with the inclusion of, say, expected inflation in some Taylor rules. The latter are typically used to model central bank policy to control inflation between today and a year ahead, say. Such a focus on the future is not what matters in exchange rate management. The obvious example concerns the fixed rate: if today's interest rate does not offset the pressure to move away from the target today, there will be an immediate breakdown of the peg, irrespective of expected future developments. Hence, today's  $EMP_t$  is what matters for  $i_t$ .

### 2.3.3 Justifying and being supported by actual policy

As argued in the introduction, central bankers consider what they call “pressure” on their currency at the forex market when implementing exchange rate management. A high  $s_t^d$  reflects that investors intend to sell the currency. That mimics what central bankers mean by pressure. The EMP variable relates  $s_t^d$  to the lagged rate  $s_{t-1}$  and the EMP literature calls this pressure. That is in line with the phrase “downward pressure” used by Danmarks Nationalbank (2021) and Hong Kong Monetary Authority (2009), and with He et al. (2011) from the HKMA, who write that they monitor “foreign exchange market pressure.” This advocates a formalization of the word “pressure” by

$$\text{pressure} = EMP_t. \tag{15}$$

Because pressure matters in actual policy, our rule not only guides policy, but it also justifies and is supported by actual policy.

## 2.4 The rule for specific exchange rate regimes

Rule (12) can be combined with many exchange rate objectives  $s_t^o$ . The current section applies it to six regimes. Five are inspired by practice, namely the float, fixed rate, crawling peg without band, peg with possibly time-varying band, and a policy that

moderates the rate of change (called “leaning against the wind”). IMF (2020) shows that these regimes cover the majority of the countries. Examples include the United States, Bulgaria, Nicaragua, China, and Brazil, respectively, albeit that we examine only one type of policy to implement the regime, that is, interest rate policy. The other regime is a weighted combination of the fixed and floating exchange rate regimes, which we introduce because it will be convenient in theoretical analyses and we will focus on it after this section to simplify the exposition.

In the float the central bank does not try to affect the exchange rate, so any tendency for the rate to move to a particular value is ignored by its interest rate policy. More formally,  $s_t^o = s_t^d$ . Rule (12) then sets  $i_t = i_t^d$ , as expected.

For the fixed exchange rate the question at hand is what  $i_t$  the central bank should choose to ensure the exchange rate equals the target  $s^t$ . Substituting  $s_t^o = s^t$  into (12) gives the interest rate that hits this target.

The weighted fixed-floating exchange rate is a weighted average of the fixed and floating exchange rate regimes, where the weight  $\mu \in [0, 1]$  denotes the degree of exchange rate management. The regime and the interest rule implementing it are

$$\text{Policy objective: } s_t^o = (1 - \mu) s_t^d + \mu s^t \quad (16)$$

$$\text{Interest rate rule: } i_t = i_t^d + \frac{1}{w} \mu (s_t^d - s^t). \quad (17)$$

For  $\mu = 0$  this confirms the rule for the float. The higher  $\mu$ , the more  $i_t$  responds to a given  $s_t^d - s^t$ , meaning tighter exchange rate management. For  $\mu = 1$  the system represents the fixed rate. Put differently, (17) and (10) imply  $w(i_t - i_t^d) = \mu(s_t^d - s^t)$  and  $s_t - s^t = (1 - \mu)(s_t^d - s^t)$ , so that  $\mu$  captures how much of  $s_t^d - s^t$  the central bank offsets by policy, and  $1 - \mu$  shows how much ends up in the actual exchange rate gap. Note that  $\frac{1}{w}\mu$  in (17) discloses the two structural parts that underlie the coefficient for  $s_t^d$ , namely the model-determined interest rate effectiveness  $w$  and the degree of exchange rate management  $\mu$  chosen by the policymaker. So (17) disentangles a Taylor-rule type of coefficient into two structural parameters.

The crawling peg generalizes the fixed rate by having a time-varying target:  $s_t^o = s_t^t$ .

In the peg with band the exchange rate must lie in a band  $[\underline{s}_t, \bar{s}_t]$ . One example, inspired by Krugman (1991), is where  $s_t^o = s_t^d$  if  $s_t^d \in [\underline{s}_t, \bar{s}_t]$ , but once the exchange rate tends to leave the band, the central bank uses the interest rate to make sure that the exchange rate settles at the nearest boundary, so  $s_t^o = \underline{s}_t$  if  $s_t^d < \underline{s}_t$ , and  $s_t^o = \bar{s}_t$  if  $s_t^d > \bar{s}_t$ . A special case is the one-sided band, as applied in Switzerland until 2015 and in the Czech Republic until 2017, where  $\underline{s}_t$  restricts appreciation but  $\bar{s}_t$  is infinite.

In the “leaning against the wind” regime the central bank aims at mitigating the

change in the exchange rate. So it counteracts the wind  $s_t^d - s_{t-1}$ , that is,  $EMP_t$ . This regime follows from the weighted fixed-floating regime by using  $s_{t-1}$  instead of  $s^t$ . Despite the practical relevance of leaning against the wind, the rest of this paper simply focuses on the exchange rate level. Substituting  $s^t$  below by  $s_{t-1}$  gives the features of our rule for the change.

### 3 Characteristics of the rule and relation to the literature

We have derived a new interest rate rule, because our research question of finding relevant variables for exchange rate management differs from what existing rules focus on. Still, existing rules have been applied successfully in several studies, as indicated in the Introduction. In this section we thus analyze how our rule can be a useful generalization and provide additional insights.

#### 3.1 The rules to be compared

Traditional rules rely on the gap between the actual exchange rate and its target. The Monacelli (2004) rule adds this exchange rate gap to a Taylor rule, formalized by

$$i_t = i_t^d + \varphi_s (s_t - s^t), \quad (18)$$

where  $\varphi_s \geq 0$ .

Benigno et al. (2007) add the gap to the foreign interest rate instead of  $i_t^d$ ,

$$i_t = i_t^* + \varphi_s^{BBG} (s_t - s^t), \quad (19)$$

where  $\varphi_s^{BBG} > 0$  and its value does not matter for the regime.<sup>10</sup> They assume UIP.

Our rule is (17), given that we focus on the weighted fixed-floating regime. So we have  $\frac{1}{w}\mu$ , a new variable  $s_t^d$  (and thus  $EMP_t$ ), and we do not impose UIP.

#### 3.2 Encompassing both traditional rules

Traditional rules have  $s_t$  while our rule has  $s_t^d$ . By considering special cases in our framework, we can bridge that gap, as follows.

First, consider  $\mu < 1$ . Substitute the implied rule (17) for  $i_t - i_t^d$  in (10), and use

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<sup>10</sup>They also present a similar non-linear version of (19), which we do not discuss separately.

the resulting  $s_t^d$  in (17). That gives

$$i_t = i_t^d + \frac{1}{w} \frac{\mu}{1 - \mu} (s_t - s^t), \quad (20)$$

Hence, our rule becomes the Monacelli (2004) rule by choosing  $\mu = \frac{w\varphi_s}{1+w\varphi_s}$ .

Next, take  $\mu = 1$ , so the fixed rate, and assume UIP. This is the Benigno et al. (2007) setting. There our rule becomes (5) with  $s_t^d$  definition (4), that is,

$$i_t = i_t^* + \mathbb{E}_t \{s_{t+1}\} - s^t. \quad (21)$$

This implements the fixed exchange rate, so that  $\mathbb{E}_t \{s_{t+1}\} = s_t$ . Our rule thus boils down to the Benigno et al. rule, realizing that the specific value of  $\varphi_s^{BBG} > 0$ , here unity, does not matter.

We conclude that our rule encompasses both traditional rules. This holds despite the presence of different base interest rates,  $i_t^d$  and  $i_t^*$ , in those rules. The explanation is that our rule starts from  $i_t^d$  for  $\mu = 0$ , which then gradually becomes less important if  $\mu$  grows, as in the Monacelli rule, while  $i_t^*$  becomes more important and for  $\mu = 1$  in case of UIP is the only base rate. In the UIP setting, this transition can be made explicit by substituting objective (16) into rule (5), giving  $i_t = (1 - \mu) i_t^d + \mu i_t^* + \mu (\mathbb{E}_t \{s_{t+1}\} - s^t)$ .

### 3.3 Implementing the exchange rate objective

The central bank wants to implement its objective, the weighted fixed-floating regime indicated by  $\mu$ . We now examine whether the three rules fulfill this basic requirement.

Substituting the Monacelli rule (18) into (10) shows that the resulting  $s_t$  is a weighted average of  $s_t^d$  and  $s^t$ , with weight  $\frac{w\varphi_s}{1+w\varphi_s}$  on the latter. So, by not using  $w$ , the central bank does not know which regime it implements. The objective  $s_t^o$  has weight  $\mu$ , which is thus missed (unless  $\varphi_s$  happens to be  $\frac{1}{w} \frac{\mu}{1-\mu}$ ). This need not be a problem if one, like Monacelli, does not intend to implement a specific regime, but knowing the resulting regime can matter in other cases. The fixed exchange rate is not covered but is the limiting case where  $\varphi_s \rightarrow \infty$ .

Benigno et al. (2007) focus on the fixed rate. Their rule (19) says that the central bank commits to raising  $i_t$  above  $i_t^*$  if  $s_t$  tends to exceed  $s^t$ . The authors prove that in equilibrium  $s_t = s^t$ , thus implementing the objective.

Our rule (17) implements the regime for all  $\mu$ , as shown in Section 2.2.2. Zooming in on the fixed rate, we find that our rule can implement that with a finite parameter and without requiring UIP.

### 3.4 Disentangling the reduced-form coefficient $\varphi_s$

Taylor-rule parameters such as  $\varphi_s$  in the Monacelli rule (18) are typically viewed as policy-choice parameters. However, (20) reveals that

$$\varphi_s = \frac{1}{w} \frac{\mu}{1 - \mu} \quad (22)$$

not only contains a policy choice  $\mu$ , but also the effectiveness  $w$  (for  $\mu > 0$ ). Hence,  $\varphi_s$  is a reduced-form coefficient. This explains why the regime is not identified.

Our rule resolves this by giving the two underlying structural parameters. Interestingly, as  $w$  is determined by existing parameters only, disentangling  $\varphi_s$  does not increase the number of model parameters. We have simply used information in the exchange rate function, which is not exploited in the traditional rule.

### 3.5 Interest rate effectiveness and the structure of the economy

The traditional rules have  $\varphi_s$  or  $\varphi_s^{BBG}$  as the impact of the gap  $s_t - s^t$  on  $i_t$ , and the users typically choose them independently of the structure of the economy under consideration. To illustrate the consequences, consider a change in financial openness that makes the interest rate more effective for exchange rate purposes (higher  $w$ ), so that one would expect a less aggressive interest rate response for a given  $s_t - s^t$ .

Traditional rules, however, imply the same  $i_t$  as before the structural change, due to the unchanged  $\varphi_s$  and  $\varphi_s^{BBG}$ . The consequence when using the Monacelli (2004) rule is clear from (22): after the structural change, the actual regime is one of tighter exchange rate management, to an unknown extent. This need not be a problem in some studies, but it can be important in others. For the Benigno et al. (2007) rule, the impact of keeping  $\varphi_s^{BBG}$  constant is none, because the regime outcome is the same for all positive values.

In our rule, the impact of a given pressure on  $i_t$  depends on the effectiveness  $w$ , and the larger  $w$  weakens the required interest rate reaction, as expected. So the policy recommendation is that central bankers should account for the effectiveness of their instrument when determining its use. This is straightforward, but only our rule includes it.

### 3.6 Pressure instead of the actual gap

Our rule is the result of a general derivation. From that derivation  $s_t^d$ , and thus  $EMP_t$ , emerges as the key determinant. The relevance of pressure is confirmed by actual central bank policy, as shown in the Introduction.

The merits of using  $s_t^d$  instead of the usual  $s_t$  can be illustrated as follows. If investors' supply weakens the currency in the sense that there is a tendency for the currency to depreciate to  $s_t^d > s^t$ , then our rule (17) says that the central bank has to set  $i_t > i_t^d$  to ward off the pressure. If the outcome is an actual gap of  $s_t - s^t = 0$ , our rule can still have  $i_t > i_t^d$ . In contrast, by relying on the actual gap, traditional approaches imply that there is no need for using the interest rate. Because in reality exchange rates close to target can come together with substantial use of the interest rate, having pressure in the rule can be attractive.<sup>11</sup>

### 3.7 Observability and a simple rule

In theoretical models one can compute all elements needed for our rule. But what about practice? In settings where the traditional rules apply, practical implementation of our rule is equally doable as for them, because there our rule can replicate them, as shown in Section 3.2.

In more general settings, however, unobservability of  $s_t^d$  and thus pressure may well make our rule hard to implement. This matches reality, as central bankers are confronted with unobservable pressure. They use indicators to monitor pressure. For example, the Hong Kong Monetary Authority uses forward exchange rates, prices of currency options, balance of payment statistics on capital flows, and market surveys; see He et al. (2011). Section 4.3 illustrates how our rule can give inspiration for new indicators and thus assist policy.

In the literature on monetary policy rules in general, authors have introduced “simple” rules, that is, rules that are functions of observable variables only. Now that we have a rule that exactly implements the exchange rate objective, we have the possibility to learn what parts of the rule are essential for that, and from there get inspiration for a new simple rule.

First, consider the fixed exchange rate regime. There our rule is (14) with  $s_t^o = s^t$ . This can be rewritten as  $i_t = i_t^* + \frac{1}{w} (EMP_t + w (i_t^d - i_t^*) + s_{t-1} - s^t)$ . It is well known that using just the first part,  $i_t = i_t^*$ , fails to implement  $s_t = s^t$ ; see Benigno et al. (2007). For one thing, the level of the exchange rate is not determined. Adding

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<sup>11</sup>For example, consider the Annual Reports of the central banks of Denmark and Hong Kong. In February 1993 (ERM turbulence) Denmark's Nationalbank increased interest rates and succeeded in keeping the krone exchange rate stable. In 2000 (Danish referendum on euro participation) and 2008 (global financial crisis) it also defended the krone by interest rate hikes. In 2015 it set a negative interest rate to fend off appreciation pressure. Hong Kong experienced four speculative attacks in 1997-8 (East Asian financial turmoil). The central bank acted to increase market interest rates; see footnote 3 for the mechanism. It also increased the savings deposit rate. The exchange rate remained stable. In 2007 (US sub-prime mortgage problems), 2008 (Lehman collapse), and 2009 a lower interest rate counteracted appreciation pressures.

$EMP_t + w(i_t^d - i_t^*)$  does not help, because  $EMP_t$  is in depreciation units. Because the complete rule does implement  $s_t = s^t$ , we conclude that the last part,  $s_{t-1} - s^t$ , is crucial for that. This suggests  $i_t = i_t^* + s_{t-1} - s^t$  as a simple rule.

To also cover other regimes, we introduce the simple rule

$$i_t = i_t^d + \mu \left( -i_t^d + i_t^* + s_{t-1} - s^t \right). \quad (23)$$

For  $\mu = 0$  this implements the float. For  $\mu = 1$  it delivers the fixed rate, at least for all parameter combinations of the extensive grid for the model used in Section 5.1. For  $0 < \mu < 1$  the rule is a linear combination of the rules for the fixed and floating rates. As this rule differs from our baseline rule (17), the resulting intermediate regime differs from the weighted fixed-floating regime (16). This is the price paid for using  $-i_t^d + i_t^* + s_{t-1}$  as a proxy of  $s_t^d$  and not accounting for  $w$ .

Although the simple rule does not reflect the broad set of pressure indicators central bankers consider, it matches some aspects of actual policy. As Spange and Wagner Toftdahl (2014) document, the Danish central bank typically responds to ECB interest rate changes by similar changes on the same day. This supports the presence of the contemporaneous  $i_t^*$  in the rule. Moreover, they write that after a persistent weakening of the Danish krone, the bank may unilaterally increase the interest rate to boost demand for its currency and therefore strengthen it. Our rule reflects that chronology by having the lagged gap  $s_{t-1} - s^t$  instead of the actual one. This also facilitates implementation.

### 3.8 Tinbergen Rule

For simplicity, this paper focuses on one policy instrument,  $i_t$ . Still, our rule (17) typically contains two targets, the exchange rate target,  $s^t$ , and the target inside  $i_t^d$ , say an inflation target. To show this does not violate the Tinbergen Rule, let's rewrite our rule by substituting (11) using (7), which gives

$$i_t = (1 - \mu) i_t^d + \mu \frac{1}{w} (s(0, E_t) - s^t). \quad (24)$$

So the interest rate is a weighted average of a rate that typically depends on the inflation gap and a multiple of a pressure gap.

The trade-off between the targets is driven by  $\mu$ . In a float ( $\mu = 0$ ), only the inflation target matters. In intermediate regimes ( $0 < \mu < 1$ ), both targets matter, in a restricted manner. For a fixed exchange rate ( $\mu = 1$ ), only  $s^t$  matters. The intuition for the latter is that a one percentage point lower  $i_t^d$  by itself motivates an equally lower  $i_t$ , but implementing that would cause pressure and thereby a  $w$  %-points higher  $s_t$ ,

which would have to be offset by a one %-point higher  $i_t$  to maintain the peg, making  $i_t^d$  and thereby the inflation target on balance irrelevant for the actual interest rate.

We thus have one instrument and a one-dimensional (combined) target.<sup>12</sup> Hence, our approach accounts for the Tinbergen Rule. In the Monacelli (2004) rule, one has to fine-tune  $\varphi_s$  and the inflation parameter in  $i_t^d$  to achieve this. The Benigno et al. (2007) rule automatically account for the Rule, because it concerns only one target,  $s^t$ .

### 3.9 Reality check: estimating the de facto regime

Because representation (20) implies

$$\frac{1}{w} \frac{\mu}{1 - \mu} = \frac{\text{stdev}\{i_t - i_t^d\}}{\text{stdev}\{s_t\}}, \quad (25)$$

we can use data on  $i_t - i_t^d$  and  $s_t$  to estimate the left-hand side by the ratio of sample standard deviations and then, for a given  $w$ , estimate the de facto degree of exchange rate management  $\mu$ .<sup>13</sup> This adds to the existing approaches, which do not distinguish  $\mu$ . Estimating  $\mu$  also offers a simple check of the realism of our approach.

To operationalize this, we assume that  $i_t^d$  is a linear function of domestic producer price inflation  $\pi_{Ht}$  with coefficient 1.5, following Monacelli (2004). One way to obtain a value of  $w$  is by specifying a model and compute it from the model parameters. We will do that in the subsequent sections. Given the illustrative purpose of this exercise, we simply set  $w = 1.62$ , the value of the baseline economy in simulation Section 5.

We examine five countries, Australia, Canada, New Zealand, Denmark, and Hong Kong, the countries taken from the simulation section and the Introduction. The first three have an official inflation targeting policy, while the latter two pursue an exchange rate target. We use 15 years of quarterly data, from 2000 through 2014. This sample is for illustration only, and we leave a broad empirical study for future research.<sup>14</sup>

The estimates of  $\text{stdev}\{i_t - i_t^d\} / \text{stdev}\{s_t\}$  are 0.02, 0.02, 0.02, 4.03, and 5.52 for the respective countries. Figure 1 illustrates the implied  $\mu$ . For Australia, Canada, New Zealand the estimated  $\mu$  is 0.03. For Denmark we obtain 0.87, and for Hong

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<sup>12</sup>This simplicity makes the interest rate an attractive instrument for introducing our idea. In contrast, taking official forex intervention as instrument would practically require  $i_t$  as a second instrument, to account for the inflation target, which would unnecessarily complicate this paper.

<sup>13</sup>The large literature on de facto regime estimation includes Frankel and Xie (2010) and IMF (2020).

<sup>14</sup>The variables for quarter  $t$  are measured as follows. For  $i_t$  we take the three-month interbank interest rate, calculated as the period average of the daily rates in the quarter. Given period-average PPI values, we use year-on-year inflation for  $\pi_{Ht}$  and thus  $i_t^d$ . Then we express  $i_t$  and  $i_t^d$  at a quarterly basis; all interest rates in the paper are at this basis, so not annualized. The rate  $s_t$  is the log of the average daily price of one dollar (euro for Denmark). All data have been obtained from Datastream.

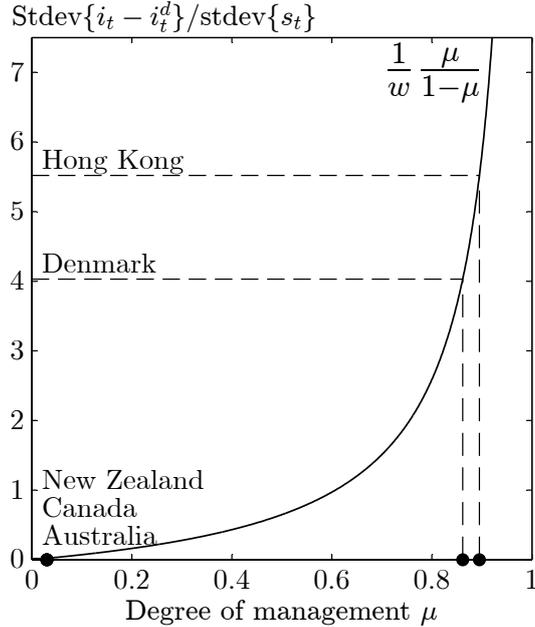


Figure 1: Estimating the degree of exchange rate management.

Kong 0.90, meaning that their regimes can be characterized as an about 90% fixed and 10% floating exchange rate regime.<sup>15</sup> All five are in line with the IMF (2020) de facto classification. We conclude from this simple analysis that our pressure-based interest rate rule can deliver useful insights into structural parameters such as  $\mu$ .

## 4 Illustration in a log-linearized DSGE model

In Section 2 we have derived the key role for EMP in policy and this has resulted in a new interest rate rule. The approach is generally applicable, and the resulting insights are valid in many models. The current section presents one specific model to illustrate the computation of EMP in a model, the rule, and some of their properties. Hence, we keep the model simple.<sup>16</sup> Various extensions are possible to make the model

<sup>15</sup>The value of  $w$  that underlies the  $\mu$  estimates is based on the core parameter values in Table 1. These are estimates. Moreover, in reality, countries may have different  $w$ . To quantify the reliability of the  $\mu$  values, we first consider the  $w = 1$  that applies under UIP. Then the resulting  $\mu$  are 0.02 for Australia, Canada, and New Zealand, 0.80 for Denmark, and 0.85 for Hong Kong, which are similar to the fractions in the main text. Next, we use the information on the posterior distributions of the core parameters, as reported by Justiniano and Preston (2010), to estimate the posterior distribution of  $w$ . The resulting 95% credible interval for  $w$  is [1.39, 2.19]. The implied intervals for  $\mu$  are [0.03, 0.04] for Australia, Canada, and New Zealand, [0.85, 0.90] for Denmark, and [0.88, 0.92] for Hong Kong. These are narrow, so that we simply focus on the point estimates.

<sup>16</sup>Several model assumptions we will make, such as producer currency pricing, are relevant for the optimality of the exchange rate regime. However, recall that we take the regime as given, so from this point of view those model assumptions are not restrictive. In fact, our rule has been derived in the

more realistic, such as a more complete description of the role of financial markets in exchange rate determination, for example based on Gabaix and Maggiori (2015). These are left for future study, but do not affect the key role for EMP in policy, our focus.

We take a two-country rational expectations New Keynesian model where the home country is a small open economy, in the spirit of De Paoli (2009). Section 4.1 presents its non-policy block, where many elements and derivations will be standard, as described by Galí (2008). The sections thereafter contain our innovations.

#### 4.1 The model: non-policy block

Web Appendix A, available from our homepages, specifies the non-policy part of the model, and derives the zero-inflation and zero-depreciation, symmetric and efficient steady state. We log-linearize the equations around that steady state and use the log-linearized version from now on. The relevant equations are (26)-(36), derived in Web Appendix B, and the foreign equivalents of (26)-(30). Lowercase Latin letters denote the logarithm of variables, except for the interest rate, and an asterisk refers to the foreign country or currency. Table 1 defines all parameters, gives their ranges, and shows the values used in simulation Section 5.

$$\text{Labor supply} : \gamma \ell_t + \sigma c_t = w_t - p_t \quad (26)$$

$$\text{Consumption Euler} : c_t = \mathbb{E}_t \{c_{t+1}\} - 1/\sigma (i_t - \mathbb{E}_t \{\pi_{t+1}\} - \delta) \quad (27)$$

$$\text{Real marginal cost} : mc_{Ht} = \log(1 - 1/\theta) + w_t - a_t - p_{Ht} \quad (28)$$

$$\text{Calvo-based pricing} : \pi_{Ht} = \beta \mathbb{E}_t \{\pi_{H,t+1}\} + \kappa_{mc}(mc_{Ht} - \log(1 - 1/\theta)) \quad (29)$$

$$\text{Labor market equilibrium} : \ell_t = y_t - a_t \quad (30)$$

$$\text{International risk sharing} : \sigma (c_t - c_t^*) = s_t + p_t^* - p_t \quad (31)$$

$$\text{Law of one price} : p_{Ft} = p_{Ft}^* + s_t \quad (32)$$

$$\text{Goods market equilibrium} : y_t = \nu c_t + (1 - \nu) c_t^* + (1 - \nu^2) \eta (p_{Ft} - p_{Ht}) \quad (33)$$

$$\text{Goods market eq. abroad} : y_t^* = c_t^* \quad (34)$$

$$\text{CPI} : p_t = \nu p_{Ht} + (1 - \nu) p_{Ft} \quad (35)$$

$$\text{CPI abroad} : p_t^* = p_{Ft}^*. \quad (36)$$

The world is populated with a continuum of households, where the population in the home country  $H$  lies in the segment  $[0, n)$ , while that of the rest of the world  $F$  is in  $[n, 1]$ . Households live forever and have identical preferences, both within and across

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general setup of Section 2, so we can safely impose the assumptions to simplify the exposition.

Table 1: Model parameters

Par.	Range	Value	Description
Core parameters			
$\beta$	(0, 1)	0.99	subjective discount factor
$\gamma$	$> 0$	1.17	inverse of Frisch elasticity of labor supply
$\sigma$	$> 0$	1.20	inverse of elasticity of intertemporal substitution for consumption
$\eta$	$> 0$	0.68	elasticity of subst. between home & foreign goods
$\theta$	$> 1$	8.00	elasticity of subst. between varieties produced within a country
$\omega$	(0, 1)	0.72	Calvo fraction of firms not allowed to change prices (stickiness)
$n$	(0, 1)	$\rightarrow 0$	size of the home economy
$\nu$	(0, 1]	0.75	home bias in preferences
$\varphi_\pi$	$\geq 0$	2.06	inflation impact on interest rate in Taylor rule
$\mu$	[0, 1]	—	degree of exchange rate management
Additional parameters for simulation			
$\rho_a$	(-1, 1)	0.81	AR(1) coefficient in labor productivity process
$\sigma_a$	$\geq 0$	0.52	standard deviation of labor productivity shock (in %)
$\sigma_i^*$	$\geq 0$	0.12	standard deviation of foreign monetary policy shock (in %)
Derived parameters			
$\delta$	(0, 1)	0.01	$= -\log(\beta)$ : subjective discount rate
$\alpha$	[0, 1]	$\rightarrow .75$	$= 1 - (1 - n)(1 - \nu)$ : share of home goods in home consumption
$\alpha^*$	[0, 1]	$\rightarrow 0$	$= n(1 - \nu)$ : share of home goods in foreign consumption
$\tau$	[0, 1]	0.13	$= 1 - \frac{\theta-1}{\theta}$ : employment subsidy
$\kappa_{mc}$	$> 0$	0.11	$= \frac{(1-\omega)(1-\omega\beta)}{\omega}$ : marginal cost impact on PPI inflation in (29)
$\varpi_c$	$> 0$	2.08	$= \sigma + \gamma\nu$ : consumption effect on product wage
$\varpi_{tot}$	$\geq 0$	0.60	$= 1 - \nu + \gamma(1 - \nu^2)\eta$ : terms-of-trade effect on product wage
$w$	$> 1$	1.62	$= \frac{\kappa_{mc}\varpi_c}{\sigma} + \frac{1+\kappa_{mc}\varpi_{tot}}{\nu}$ : effectiveness of $i_t$ to counteract deprec.

Foreign parameters  $\beta^*, \gamma^*, \sigma^*, \theta^*, \omega^*, \varphi_\pi^*, \rho_a^*$ , and  $\sigma_a^*$  equal their home counterparts.

The values of the core and additional parameters for simulation have been taken from Justiniano and Preston (2010). The authors estimate a small open-economy model for three countries vis-à-vis the United States, namely for Australia, Canada, and New Zealand, using Bayesian techniques, though they calibrate the values for  $\beta$ ,  $\theta$ , and  $\nu$ . We take the average of their three posterior medians.

countries. They derive utility from the consumption of domestic and foreign goods, with home bias in preferences, and disutility from supplying labor to firms. They live in cashless economies. For simplicity, capital markets are complete, both domestically and internationally, with frictionless trade in assets.

Households maximize expected lifetime utility. Optimization yields labor supply equation (26) and consumption Euler equation (27), where  $\ell_t$  is labor supply in period  $t$ ,  $c_t$  is consumption,  $w_t$  is the wage rate,  $p_t$  is the consumer price index (CPI), and  $\pi_t = p_t - p_{t-1}$  is CPI inflation.

Firms specialize in the production of one firm-specific good. Domestic firms produce the varieties in  $[0, n)$  and foreign firms those in  $[n, 1]$ . Each firm uses labor supplied by the households and a linear technology, where  $a_t$  is labor productivity, which is common across home firms and evolves exogenously according to some stationary stochastic process. The firm receives an employment subsidy that renders the steady state efficient. Real marginal cost  $mc_{Ht}$ , expressed in terms of the producer price index (PPI)  $p_{Ht}$ , thus depends on the product wage  $w_t - p_{Ht}$  by (28).

The firm sells its good under monopolistic competition. It sells at home and abroad without trade frictions. Prices are set in the producer's currency, and they are sticky a la Calvo (1983). Hence,  $p_{Ht}$  depends on its lag and the price chosen by firms that are allowed to reset the price. Profit maximization then yields PPI inflation  $\pi_{Ht} = p_{Ht} - p_{H,t-1}$  based on (29), showing the importance of real marginal cost  $mc_{Ht}$ , which enters the formula in deviation from its steady-state value.

Equilibrium concerns three markets. First, labor market equilibrium is (30), where  $y_t$  is domestic output. Second, the asset market is in equilibrium if the perfect international risk sharing relation (31) holds, given symmetric initial conditions, where  $s_t + p_t^* - p_t$  is the real exchange rate. Third, for the goods market, free international trade implies the law of one price, so the import price index  $p_{Ft}$  follows from (32), where  $p_{Ft}^*$  is foreign PPI in foreign currency. The goods market also clears for all varieties.

To mimic that the domestic country is small, we now take the limit  $n \rightarrow 0$ . That gives goods market clearing at home (33) and abroad (34). The former captures that higher prices for imports relative to domestically produced goods (higher terms of trade  $tot_t = p_{Ft} - p_{Ht}$ ) cause substitution towards domestic goods, stimulating domestic production. The limit also implies that home CPI in (35) follows from home PPI and the import price index, and that foreign CPI  $p_t^*$  is simply foreign PPI, as (36) shows.

## 4.2 Exchange rate function

Our interest rate rule requires the intermedial exchange rate  $s_t^d$ , so we first derive the exchange rate function (6) the model implies. Web Appendix B presents a streamlined derivation, starting from the fact that the exchange rate  $s_t$  clears the asset market and making sure the function is in the  $(i_t, E_t)$ -form defined in Section 2.2.1. This gives

$$s_t = -wi_t + v'E_t, \quad (37)$$

where

$$w = \frac{\kappa_{mc}\varpi_c}{\sigma} + \frac{1 + \kappa_{mc}\varpi_{tot}}{\nu} \quad (38)$$

and

$$v = \begin{bmatrix} w \\ w \\ w\sigma \\ 1 \\ -1 \\ \beta \\ -\kappa_{mc}(\gamma + 1) \end{bmatrix} \quad \text{and} \quad E_t = \begin{bmatrix} i_t^* \\ \mathbb{E}_t\pi_{t+1} - \mathbb{E}_t\pi_{t+1}^* \\ \mathbb{E}_tc_{t+1} - \mathbb{E}_tc_{t+1}^* \\ s_{t-1} \\ tot_{t-1} \\ \mathbb{E}_t\pi_{H,t+1} - \mathbb{E}_t\pi_{F,t+1}^* \\ a_t - a_t^* \end{bmatrix}. \quad (39)$$

Formula (38) is the model-implied version of (8), the effectiveness of the interest rate to counteract depreciation while keeping  $E_t$  constant. It is fully determined by the structural parameters of the model. We get  $w > 0$ , so an interest rate increase strengthens the home currency.

Expression (39) for  $E_t$  discloses what else matters for the exchange rate according to the model. Most determinants occur in a simple relative form, an attractive consequence of our streamlined derivation of the  $s$ -function. Because  $s_{t-1}$  has a unit coefficient in  $v$ , one could also write (37) in terms of  $\Delta s_t$  and the adjusted  $E_t$  would then consist of stationary variables only. However, that does not imply that  $s_t$  is non-stationary, because to implement an exchange rate regime the interest rate  $i_t$  may depend on the exchange rate level so as to counteract deviations from target, and that can result in a stationary  $s_t$ , similar to an error-correction specification.

## 4.3 Interest rate rule

Suppose the domestic central bank pursues the weighted fixed-floating exchange rate regime (16). To implement this, the central bank should use interest rate rule (17).

The ingredients of the rule are as follows. For the domestically-oriented rate we take

$$i_t^d = \delta + \varphi_\pi \pi_{Ht}, \quad (40)$$

though one could also use a Taylor rule with CPI inflation and output gap. The effectiveness  $w$  follows from (38), and the degree of exchange rate management  $\mu$  and the target rate  $s^t$  are both taken as given.

The other ingredient, intermedial exchange rate  $s_t^d$ , follows directly from definition (11) and the model-implied  $s$ -function (37), so that

$$s_t^d = -w i_t^d + v' E_t. \quad (41)$$

Note that this also shows what variables determine  $EMP_t = s_t^d - s_{t-1}$  in this economy, namely  $\pi_{Ht}$  via (40) and  $E_t$  in (39).

Our rule can now be expressed as

$$i_t = (1 - \mu) i_t^d + \mu \frac{1}{w} (v' E_t - s^t). \quad (42)$$

So the interest rate is a weighted average of  $i_t^d$  and  $\frac{1}{w} (v' E_t - s^t)$ . For example, in a float ( $\mu = 0$ ), the interest rate is the domestically-oriented rule  $i_t^d$ , as usual. To implement the fixed rate ( $\mu = 1$ ),  $i_t^d$  drops out, in line with the incompatible trinity. As expected, the rule then starts with  $i_t^*$ , given the top row in (39).

#### 4.4 Exogenous processes

So far, the foreign interest rate  $i_t^*$ , (the log of) home labor productivity  $a_t$ , and  $a_t^*$  have been treated as given. To close the model, as of now we assume the foreign central bank uses the following interest rate rule with a zero-mean i.i.d. monetary policy shock,

$$i_t^* = \delta + \varphi_\pi \pi_t^* + \varepsilon_{it}^*. \quad (43)$$

Home labor productivity  $a_t$  follows an AR(1) process with autoregressive coefficient  $\rho_a$  and that the i.i.d. shock involved has mean zero and standard deviation  $\sigma_a$ . The same holds for foreign productivity  $a_t^*$ .

## 5 Simulations from the model

To further illustrate EMP and our interest rate rule, we now simulate from the model just developed. The main insights from these simulations are not specific to the model,

parameter values, or draws of the shocks.

## 5.1 Model calibration, solution, and simulation

One period in the model is one quarter. We set the target  $s^t = 0$ . All parameter values are based on Justiniano and Preston (2010). Table 1 presents them, and its note provides further motivation.

We solve the model numerically using the Sims (2002) algorithm. The solution can be cast as a reduced-form VAR model of the  $20 \times 1$ -vector with elements  $c_t, \mathbb{E}_t c_{t+1}, c_t^*, \mathbb{E}_t c_{t+1}^*, \pi_{Ht}, \mathbb{E}_t \pi_{H,t+1}, \pi_t, \mathbb{E}_t \pi_{t+1}, \pi_t^*, \mathbb{E}_t \pi_{t+1}^*, y_t, y_t^*, i_t, i_t^*, s_t, \mathbb{E}_t s_{t+1}, tot_t, EMP_t, a_t, a_t^*$ . We focus on unique stationary solutions, abstracting thus from sunspot equilibria.

The necessary and sufficient condition for equilibrium determinacy is as follows, given our parameter space and thus the symmetry assumption  $\varphi_\pi = \varphi_\pi^*$ . Under the float, the condition follows from Bullard and Mitra (2002), and it here reduces to satisfying the Taylor principle  $\varphi_\pi > 1$ . For the other regimes, from the managed float to the fixed rate, we verify determinacy for a grid of parameter values, using the Sims (2002) algorithm. Here the condition also turns out to be  $\varphi_\pi > 1$ . So the same condition applies whatever the regime. It holds for all parameter values we study.

We set  $s_0 = 0$  and initialize other variables at their steady-state values. The three shocks are drawn from independent normal distributions. We draw them for 60 periods (15 years), from which we compute the paths of the variables of interest. For ease of comparison we keep the realized shocks the same across the plotted paths.

## 5.2 Implementing multiple regimes

Our rule (42) implements many exchange rate objectives and does so exactly, with a key role for  $EMP_t$ , determined by (41). To illustrate their properties, we simulate paths of the economy in three different regimes, the float ( $\mu = 0$ ), an intermediate regime (say  $\mu = 0.5$ ), and the peg ( $\mu = 1$ ). A representative set of paths is depicted in Figure 2. The variables  $EMP_t, i_t, i_t^d, i_t^*, s_t$  in the graphs are in percentage terms.

Under the float ( $\mu = 0$ ), the interest rate  $i_t$  equals the domestically-oriented rate  $i_t^d$ , visualized by the horizontal line in the second panel. As  $i_t^d$  is driven by inflation, the interest rate does not stabilize the exchange rate  $s_t$ . That is, the exchange market pressures  $EMP_t$  indicated by the gray line in the top panel are not offset by policy and they equal the actual depreciations. This makes that the gray line for  $s_t$  in the bottom panel does not revert to zero.

The stronger the exchange rate management, the more the central bank has to account for exchange rate fundamentals when setting the interest rate, making  $i_t^d$  less

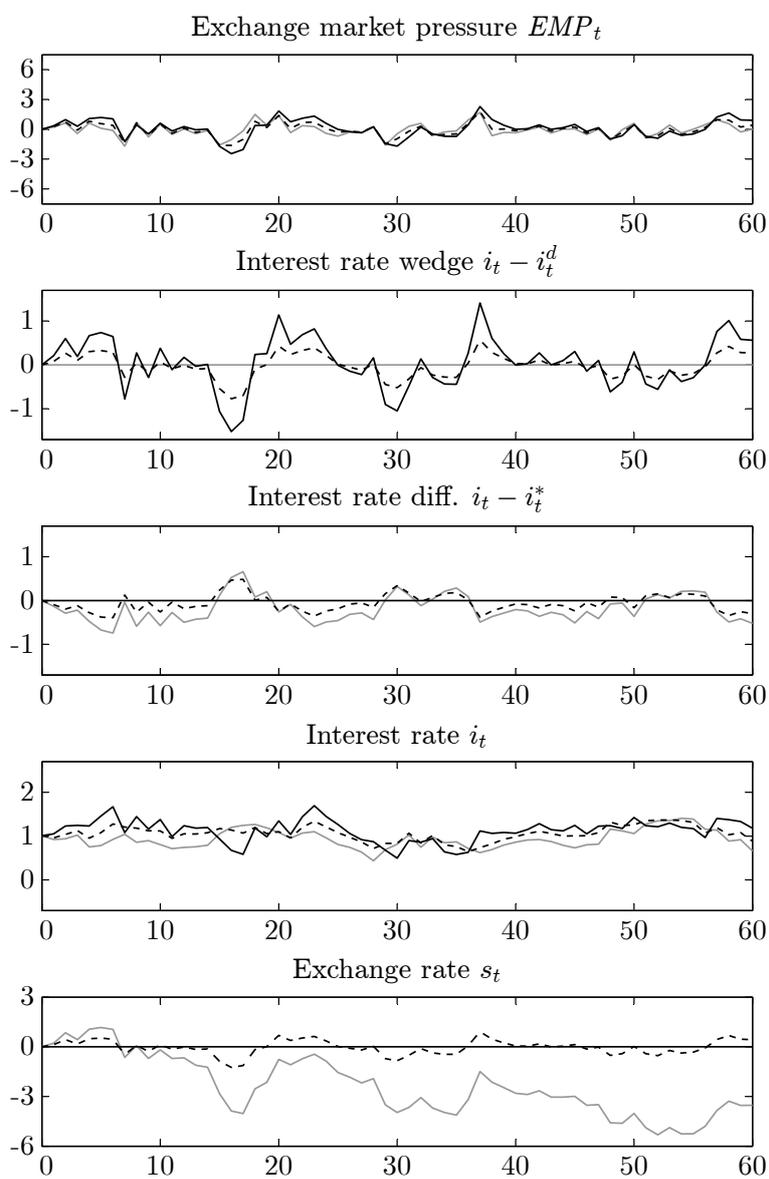


Figure 2: Paths implied by our rule (42) in various exchange rate regimes: from float ( $\mu = 0$ , gray) to intermediate ( $\mu = 0.5$ , dashed) to fixed ( $\mu = 1$ , black).

and  $E_t$  in our rule more relevant. The dashed line in the bottom panel visualizes that  $\mu = 0.5$  here already stabilizes the exchange rate considerably. The line also suggests that the weighted fixed-floating regime can be a practical linear approximation of various other exchange rate policies, such as the peg with band.

If the central bank pursues a fixed exchange rate ( $\mu = 1$ ), then the black line in the middle panel visualizes that  $i_t = i_t^*$  in equilibrium, and the bottom panel shows that the exchange rate stays on target continuously, as expected. This corroborates that the model contains UIP, by virtue of (31). Still, the top panel reveals that shocks cause periods of noticeable pressure  $EMP_t$  on the peg. There the central bank has to accept an interest rate that differs substantially from the domestically-oriented rate (second panel), which in practice may induce policymakers to give up the peg.

Finally, let us discuss the traditional rules. Section 3 gives reasons that make our rules a useful alternative. The advantages do not depend on specific simulations. Still, it may be instructive to compare the simulated paths. In this special case of the weighted fixed-floating regime, one can replicate our simulated paths of  $i_t$  and  $s_t$  for  $\mu < 1$  by using the Monacelli (2004) rule and setting  $\varphi_s$  based on (22). The problem is that this requires  $w$  (except for  $\mu = 0$ ), which is not available in his approach. This makes that simulations from his and our rules generally differ, and not knowing  $w$  implies that one generally does not know the exchange rate regime underlying his simulations. For  $\mu = 1$  one can replicate our simulations with the Benigno et al. (2007) rule. Our rule is the only rule that covers all  $\mu$  in one framework.

## 6 Conclusion

This paper has derived that EMP is a key variable for a central bank when setting the interest rate to implement a given exchange rate objective. This guides policy and, vice versa, actual policy confirms the relevance of EMP. We have formalized this in a new interest rate rule that contains EMP.

We have also derived that the economic structure matters for the EMP coefficient in the rule: the more effective the interest rate, the less it should be used to offset a given pressure. The feature helps to get that our rule exactly implements the exchange rate objective, it does so for many regimes and models, and our rule automatically accounts for changes in the economic structure. All these features make our rule a useful alternative to traditional rules, which it can replicate as special cases.

We have introduced the weighted fixed-floating regime, with weight  $\mu$  on the fixed regime. Our rule can be conveniently combined with this regime. This leads to a coefficient of EMP in the rule that discloses two structural parts, namely the model-

determined interest rate effectiveness  $w$  and the degree of exchange rate management  $\mu$  chosen by the policymaker. We have thus disentangled a Taylor-rule type of coefficient into two underlying structural parameters. We have exploited this to show how the de facto exchange rate regime can be estimated in a novel way.

We have extended the EMP literature by refining the EMP formalization and computing EMP in a modern sticky-price model, as the EMP literature typically relies on some variant of the flexible-price monetary model. We have also formalized how EMP is an ingredient for policy, and how the sticky-price model helps the policymaker to learn the determinants of EMP. All this may stimulate further research on EMP.

The general applicability of our rule and the inherent consistency with the regime and model can facilitate future research. Think of studies on the optimal degree of exchange rate management, further eased by our new structural parameter  $\mu$ , and research on models with incomplete markets and risk premia. For example, in another project we apply our idea to analyze both interest rate and foreign exchange interventions by the central bank under capital controls. This could then facilitate studies on emerging markets where central banks use forex intervention to pursue leaning-against-the-wind exchange rate management. This is left for future research.

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## Web appendix to

### “Exchange market pressure in interest rate rules”

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## A DSGE model

### A.1 Households

The world is populated with a continuum of households, where the population in the home country  $H$  lies in the segment  $[0, n)$ , while that of the rest of the world  $F$  is in  $[n, 1]$ . Domestic households maximize expected lifetime utility

$$\max \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left( \frac{C_{t+k}^{1-\sigma}}{1-\sigma} - \frac{L_{t+k}^{1+\gamma}}{1+\gamma} \right), \quad (44)$$

subject to a budget constraint (specified later), by choosing a path  $\{C_{t+k}, L_{t+k}\}_{k=0, \dots, \infty}$ , where  $C_{t+k}$  is household consumption and  $L_{t+k}$  is labor supply at time  $t+k$ .

Consider period  $t$ .<sup>17</sup> Consumption enters the domestic household’s utility as an index  $C_t$ , which is the CES aggregate of the indices of domestic consumption of home and foreign (imported) goods,  $C_{Ht}$  and  $C_{Ft}$ , respectively:

$$C_t = \left( \alpha^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \quad (45)$$

The parameter  $\alpha$ , determining the preference for home-produced goods, increases with the size of the home country,  $n$ , and with home bias  $\nu$ . We model  $1-\alpha = (1-n)(1-\nu)$ . Hence,  $\nu > 0$  means that domestic households consume fewer foreign-produced goods than the size of the foreign country implies, reflecting home bias.<sup>18</sup>

The index of domestic consumption of home goods,  $C_{Ht}$ , is the CES aggregate of the consumption of all varieties produced in country  $H$ . These are varieties  $j \in [0, n)$ . The index of domestic consumption of foreign goods,  $C_{Ft}$ , is a similar CES aggregate, but concerning all varieties produced in  $F$ , which are  $j \in [n, 1]$ . Domestic consumption

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<sup>17</sup>Results for  $t+k \geq t$  follow by substituting  $t$  by  $t+k$ , while keeping expectations conditional on  $t$ .

<sup>18</sup>Because foreign households have identical preferences, their consumption index  $C_t^*$  equals the right-hand side of (45) with  $\alpha$  substituted by  $\alpha^*$ ,  $C_{Ht}$  by  $C_{Ht}^*$ , and  $C_{Ft}$  by  $C_{Ft}^*$ . Moreover,  $\alpha^* = n(1-\nu)$ .

of variety  $j$  is denoted by  $C_t(j)$ . In formula,

$$\begin{cases} C_{Ht} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \\ C_{Ft} = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \end{cases} \quad (46)$$

So  $\theta$  concerns the substitutability between varieties produced within a country, whereas  $\eta$  in (45) is about the substitution between home and foreign goods.

As usual, utility maximization requires that within period  $t$  households maximize  $C_t$  for a given expenditure on home and foreign indices and they maximize  $C_{Ht}$  ( $C_{Ft}$ ) for a given level of expenditure on home (foreign) varieties. Let  $P_t(j)$  denote the price of variety  $j$  in domestic currency. The resulting demand function for each variety is

$$C_t(j) = \begin{cases} \alpha \left( \frac{P_t(j)}{P_{Ht}} \right)^{-\theta} \left( \frac{P_{Ht}}{P_t} \right)^{-\eta} C_t, & \text{for home varieties } j \in [0, n] \\ (1 - \alpha) \left( \frac{P_t(j)}{P_{Ft}} \right)^{-\theta} \left( \frac{P_{Ft}}{P_t} \right)^{-\eta} C_t, & \text{for foreign varieties } j \in [n, 1], \end{cases} \quad (47)$$

where

$$\begin{cases} P_{Ht} = \left[ \frac{1}{n} \int_0^n P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \\ P_{Ft} = \left[ \frac{1}{1-n} \int_n^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \end{cases} \quad (48)$$

are the home producer price index and the foreign producer price index expressed in domestic currency, respectively, and

$$P_t = \left( \alpha P_{Ht}^{1-\eta} + (1 - \alpha) P_{Ft}^{1-\eta} \right)^{\frac{1}{1-\eta}} \quad (49)$$

is the consumer price index in the home country. This implies that total consumption expenditure by domestic households is  $P_t C_t$ .<sup>19</sup>

We can now specify the period budget constraint

$$P_t C_t + \mathbb{E}_t \{ \Lambda_{t,t+1} S_{t+1} B_{t+1} \} \leq W_t L_t + S_t B_t + \Pi_t - T_t, \quad (50)$$

where we rule out Ponzi schemes. Here  $B_t$  is the value in foreign currency of a portfolio of a full set of state-contingent assets held at the beginning of period  $t$ , reflecting our

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<sup>19</sup>Similar expressions hold for the foreign country, for both demand and prices. Foreign demand follows from (47) by substituting  $\alpha, C$ , and the four  $P$  symbols by  $\alpha^*, C^*$ , and  $P^*$ , respectively. The home producer price index in foreign currency  $P_{Ht}^*$  and the foreign producer price index (in foreign currency)  $P_{Ft}^*$  follow from the right-hand sides of (48) by substituting  $P_t(j)$  by  $P_t^*(j)$ . The foreign consumer price index  $P_t^*$  (in foreign currency) equals the right-hand side of (49) with  $\alpha$  substituted by  $\alpha^*$ ,  $P_{Ht}$  by  $P_{Ht}^*$ , and  $P_{Ft}$  by  $P_{Ft}^*$ .

complete markets assumption,  $S_t = \exp(s_t)$  is the nominal exchange rate in level form,  $\Lambda_{t,t+1}$  is the stochastic discount factor making  $\mathbb{E}_t \{\Lambda_{t,t+1} S_{t+1} B_{t+1}\}$  the home-currency value at time  $t$  of the portfolio that yields a payoff in  $t+1$ ,  $W_t$  is the nominal wage,  $\Pi_t$  is nominal firm profits transferred to households, and  $T_t$  is lump-sum taxes.

As usual, the first-order conditions consist of the optimality condition regarding the intratemporal consumption-leisure trade off

$$C_t^\sigma L_t^\gamma = \frac{W_t}{P_t} \quad (51)$$

and the intertemporal optimality relation linking the stochastic discount factor to the intertemporal marginal rate of substitution in Euler equation

$$\Lambda_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}, \quad (52)$$

for all possible states of nature at times  $t$  and  $t+1$ . Note that  $\mathbb{E}_t \Lambda_{t,t+1}$  is the value of a portfolio that yields one unit of the domestic currency in  $t+1$  (mimicking a riskless domestic bond), so that the interest rate is  $i_t = -\log(\mathbb{E}_t \{\Lambda_{t,t+1}\})$ . Given  $i_t$ , prices, and the budget constraint, the (expectational) Euler equation determines  $C_t$ .

## A.2 Firms

Firms use labor supplied by the households and a linear technology. Hence, output is

$$Y_t(j) = A_t L_t(j), \quad (53)$$

where  $A_t$  is exogenous labor productivity, which is common across firms (within a country). Because of a labor subsidy  $\tau$ , financed by taxes  $T_t$ , marginal cost is

$$MC_t = (1 - \tau) W_t / A_t, \quad (54)$$

which is independent of output and thus common across firms. The firm sells its good in a monopolistically competitive market with free international trade. Profits are

$$\Pi_t(j) = (P_t(j) - MC_t) Y_t(j). \quad (55)$$

The firm sets the price in a sticky fashion a la Calvo (1983). That is, each date with probability  $\omega$  the firm is not allowed to change its price. When the firm is allowed to set a new price  $P_t^{opt}(j)$ , it will do so optimally, that is, by maximizing the current market value of the profits resulting while that price remains in place. Suppose the new

price holds until  $t + k \geq t$ . Let  $Y_{t+k|t}(j)$  denote total demand  $C_{t+k}(j) + \frac{1-n}{n}C_{t+k}^*(j)$  evaluated at  $P_t^{opt}(j)$ . The firm's objective function is therefore

$$\max \sum_{k=0}^{\infty} \omega^k \mathbb{E}_t \left\{ \Lambda_{t,t+k} \left( P_t^{opt}(j) - MC_{t+k} \right) Y_{t+k|t}(j) \right\}. \quad (56)$$

To derive the first-order condition, first note that (47) and its foreign counterpart imply that  $\partial Y_{t+k|t}(j) / \partial P_t^{opt}(j) = -\theta Y_{t+k|t}(j) / P_t^{opt}(j)$ . Moreover, other home firms face the same optimization problem, so that all domestic firms will choose the same new price  $P_{Ht}^{opt} = P_t^{opt}(j)$ . The price can be solved from the first-order condition

$$\sum_{k=0}^{\infty} \omega^k \mathbb{E}_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} \left[ P_{Ht}^{opt} - \frac{\theta}{\theta-1} MC_{t+k} \right] \right\} = 0. \quad (57)$$

### A.3 Equilibrium

World equilibrium requires that labor, asset, and goods markets are in equilibrium.

#### A.3.1 Labor market

Labor market equilibrium at home and abroad requires

$$\begin{cases} L_t &= \frac{1}{n} \int_0^n L_t(j) dj \\ L_t^* &= \frac{1}{1-n} \int_n^1 L_t^*(j) dj. \end{cases} \quad (58)$$

#### A.3.2 Asset market

As for the home country, market completeness implies there is also a unique stochastic discount factor for foreign-currency payoffs, which is  $\Lambda_{t,t+1}^* = \beta (C_{t+1}^*/C_t^*)^{-\sigma} P_t^*/P_{t+1}^*$  for all possible states of nature at times  $t$  and  $t+1$ . Given free international trade in assets, arbitrage yields the asset market equilibrium relation

$$\Lambda_{t,t+1} = \Lambda_{t,t+1}^* \frac{S_t}{S_{t+1}}, \quad (59)$$

which is a stochastic version of uncovered interest parity.

Substituting the expressions for  $\Lambda_{t,t+1}$  and  $\Lambda_{t,t+1}^*$  shows the model has the familiar perfect risk sharing relation between home and foreign households

$$C_t^\sigma = C_t^{*\sigma} Q_t, \quad (60)$$

assuming symmetric initial conditions, where  $Q_t = S_t P_t^*/P_t$  is the real exchange rate.

### A.3.3 Goods market

Goods market equilibrium consists of two parts. First, frictionless trade results in the law of one price. So for each variety  $j \in [0, 1]$  the price set by the producer in its currency implies that the price in the other currency fulfills

$$P_t(j) = S_t P_t^*(j). \quad (61)$$

For the producer price indices this yields  $P_{Ht} = P_{Ht}^* S_t$  and  $P_{Ft} = P_{Ft}^* S_t$ . Still, home bias implies  $\alpha > \alpha^*$ , so that in general for the consumer price index  $P_t \neq P_t^* S_t$ , meaning a deviation from purchasing power parity.

The second part of goods market equilibrium is the markets for all varieties clear:

$$\begin{cases} Y_t(j) &= C_t(j) + \frac{1-n}{n} C_t^*(j), \text{ for home varieties} \\ Y_t^*(j) &= \frac{n}{1-n} C_t(j) + C_t^*(j), \text{ for foreign varieties.} \end{cases} \quad (62)$$

For the home-varieties line, substitute the top demand function of (47) for  $C_t(j)$  and its foreign counterpart (as explained in footnote 19) for  $C_t^*(j)$ . For the foreign-varieties line, we do the same, but now using the bottom demand function of (47). This yields

$$\begin{cases} Y_t(j) &= \left(\frac{P_t(j)}{P_{Ht}}\right)^{-\theta} \left[ \alpha \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t + \frac{1-n}{n} \alpha^* \left(\frac{P_{Ht}/S_t}{P_t^*}\right)^{-\eta} C_t^* \right], & \text{home v.} \\ Y_t^*(j) &= \left(\frac{P_t^*(j)}{P_{Ft}^*}\right)^{-\theta} \left[ \frac{n}{1-n} (1-\alpha) \left(\frac{P_{Ft}^* S_t}{P_t}\right)^{-\eta} C_t + (1-\alpha^*) \left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta} C_t^* \right], & \text{foreign v.} \end{cases} \quad (63)$$

Substituting these into the definitions of aggregate output

$$\begin{cases} Y_t &= \left[ \frac{1}{n} \int_0^n Y_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \\ Y_t^* &= \left[ \frac{1}{1-n} \int_n^1 Y_t^*(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \end{cases} \quad (64)$$

gives

$$\begin{cases} Y_t &= \alpha \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t + \frac{1-n}{n} \alpha^* \left(\frac{P_{Ht}/S_t}{P_t^*}\right)^{-\eta} C_t^* \\ Y_t^* &= \frac{n}{1-n} (1-\alpha) \left(\frac{P_{Ft}^* S_t}{P_t}\right)^{-\eta} C_t + (1-\alpha^*) \left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta} C_t^* \end{cases}. \quad (65)$$

### A.4 Taking the limit $n \rightarrow 0$ to obtain the small economy

To mimic the small open economy we take the limit  $n \rightarrow 0$ . This implies  $\alpha \rightarrow \nu$  and  $\alpha^* \rightarrow 0$ . The limiting CPIs resulting from (49) become

$$\begin{cases} P_t = \left( \nu P_{Ht}^{1-\eta} + (1-\nu) P_{Ft}^{1-\eta} \right)^{\frac{1}{1-\eta}} \\ P_t^* = P_{Ft}^*, \end{cases} \quad (66)$$

and the limiting values of aggregate output in (65) are

$$\begin{cases} Y_t = \nu \left( \frac{P_{Ht}}{P_t} \right)^{-\eta} C_t + (1-\nu) \left( \frac{P_{Ht}/S_t}{P_t^*} \right)^{-\eta} C_t^* \\ Y_t^* = C_t^*. \end{cases} \quad (67)$$

## A.5 Steady state

Here we compute the symmetric zero-inflation and zero-depreciation efficient steady state of the model. All variables refer to the values in that steady state. Similar values apply to the foreign country, unless explicitly stated otherwise.

The constancy of  $P_H$  implies that firms choose  $P_H^{opt} = P_H$ . From (57) we obtain that  $MC$  is constant. Given that all shocks are set to zero, productivity is constant over time, denoted by  $A$ . Then (54) gives that the wage is constant  $W$ . Note that the labor subsidy  $\tau$  given in Table 1 implies  $P_H = W/A$  and thus renders the steady state efficient, and real marginal cost  $MC/P_H = (\theta - 1)/\theta$ . Similarly,  $P_F^* = W^*/A^*$ . The constancy of  $S$  then gives  $P_F = SP_F^*$ . For simplicity, we assume  $P_H = P_F$ . So  $P = P_H$ . As  $P^* = P_F^*$ , the real exchange rate  $Q = 1$ , so that PPP holds in the steady state.

Because all firms  $j$  charge the same price, (63) implies that output per firm is the same across varieties. First, consider the foreign country. Combining (63), (65) with the foreign version of (53), (58), (67), and the foreign version of (51), where  $W^*/P^* = A^*$ , implies that consumption is constant  $C^* = A^{*(1+\gamma)/(\sigma+\gamma)}$ . Similarly, combining the home equivalents of the formulas in the previous sentence and using the constancy of  $C^*$  shows that also home consumption is constant, where  $C$  is the unique solution from  $C^\sigma ([\nu C + (1-\nu) C^*]/A)^\gamma = A$ . Assuming  $A = A^*$  and using the value of  $C^*$  yields as unique solution  $C = C^*$ . From (67) we obtain  $Y = C$ , and (63), (65), (53), and (58) then yield  $L = Y/A$ . Finally, (52) gives  $\Lambda = \beta$ .

## B Derivations of equations (26)-(37)

Equations (26), (31), (32), (34), and (36) follow directly from (51), (60), (61), (67), and (66), respectively. This appendix derives the remaining equations.

- (27)

Start from (52). By definition,  $\mathbb{E}_t \{\Lambda_{t,t+1}\} = \exp(-i_t)$  and  $\beta = \exp(-\delta)$ . Substitution

into (52) gives

$$1 = \mathbb{E}_t \{ \exp (i_t - \delta - \sigma (c_{t+1} - c_t) - \pi_{t+1}) \}, \quad (68)$$

so that log-linearization yields (27):

$$1 = \mathbb{E}_t \{ 1 + i_t - \delta - \sigma (c_{t+1} - c_t) - \pi_{t+1} \}. \quad (69)$$

• (28)

This results from the log of real marginal cost (54) and employment subsidy  $\tau = 1/\theta$ .

• (29)

Producer prices are set by firms based on the Calvo structure, so that

$$P_{Ht} = \left[ \omega P_{H,t-1}^{1-\theta} + (1-\omega) P_{Ht}^{opt1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (70)$$

Log-linearization yields

$$\pi_{Ht} = (1-\omega) \left( p_{Ht}^{opt} - p_{H,t-1} \right). \quad (71)$$

The optimal price  $p_{Ht}^{opt}$  is the solution from the firm's first-order condition. First, rewrite first-order condition (57) as

$$\sum_{k=0}^{\infty} \omega^k \mathbb{E}_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} \left[ \frac{P_{Ht}^{opt}}{P_{H,t-1}} - \frac{\theta}{\theta-1} \frac{P_{H,t+k}}{P_{H,t-1}} MC_{H,t+k} \right] \right\} = 0, \quad (72)$$

where we have scaled some variables to obtain ratios that have a well-defined value in the steady state, and  $MC_{H,t+k} = MC_{t+k}/P_{H,t+k}$  is the real marginal cost. Using that  $\Lambda_{t,t+k} = \beta^k$  in the steady state, log-linearization yields

$$p_{Ht}^{opt} - p_{H,t-1} = (1-\omega\beta) \sum_{k=0}^{\infty} (\omega\beta)^k \mathbb{E}_t \{ p_{H,t+k} - p_{H,t-1} + \widehat{mc}_{H,t+k} \}, \quad (73)$$

where  $\widehat{mc}_{H,t+k}$  denotes the deviation of log real marginal cost  $mc_{t+k} - p_{H,t+k}$  from its steady state  $\log(\frac{\theta-1}{\theta})$ . Writing this equation recursively yields

$$p_{Ht}^{opt} - p_{H,t-1} = \omega\beta \mathbb{E}_t \{ p_{H,t+1}^{opt} - p_{Ht} \} + \pi_{Ht} + (1-\omega\beta) \widehat{mc}_{Ht}. \quad (74)$$

Substitution into (71) gives (29), where  $\kappa_{mc}$  is defined in Table 1.

• (30)

Start from the domestic labor market equilibrium formula in (58) and substitute (53). Also substitute  $Y_t(j) = \left(\frac{P_t(j)}{P_{Ht}}\right)^{-\theta} Y_t$ , which is implied by (63) and (65), and use that  $A_t(j) = A_t$ . That gives

$$L_t = \frac{1}{n} \int_0^n \left(\frac{P_t(j)}{P_{Ht}}\right)^{-\theta} dj \frac{Y_t}{A_t}. \quad (75)$$

Finally, we take the logarithm and use Galí (2008, p.162) to motivate why the integral part is approximately zero.

• (33)

This results from log-linearizing home aggregate output in (67), which yields

$$y_t = \nu [c_t - \eta (p_{Ht} - p_t)] + (1 - \nu) [c_t^* - \eta (p_{Ht} - s_t - p_t^*)], \quad (76)$$

and then substituting (35) and the law of one price (32).

• (35)

For  $\eta \neq 1$ , this follows by log-linearizing the top equation in (66). For  $\eta = 1$ , (66) becomes the Cobb-Douglas combination  $P_t = P_{Ht}^\nu P_{Ft}^{1-\nu}$ , which also yields (35).

• (37)

The exchange rate  $s_t$  clears the asset market, so we start from risk sharing (31). This is the core equation. After substitution of (35) and (36), it is

$$\sigma (c_t - c_t^*) = s_t + p_{Ft}^* - [\nu p_{Ht} + (1 - \nu) p_{Ft}]. \quad (77)$$

This is not yet an  $s$ -function in the  $(i_t, E_t)$ -form defined in Section 2.2.1, where the  $i_t$  argument captures the interest rate impact on the exchange rate via all contemporaneous channels, and the vector  $E_t$  accounts for everything else. Therefore, we now substitute out contemporaneous variables affected by  $i_t$ . We do so in a streamlined manner, where first-order conditions and equilibrium relations are used only once, and they substitute out the choice and equilibrating variables based on the underlying economic mechanisms.

We first focus on  $p_{Ht}$ . Calvo pricing (29) and marginal cost (28) imply

$$p_{Ht} = p_{H,t-1} + \beta \mathbb{E}_t \{\pi_{H,t+1}\} + \kappa_{mc} (w_t - a_t - p_{Ht}), \quad (78)$$

reflecting that  $p_{Ht}$  is driven by wage  $w_t$ . The latter is such that labor demand equals supply, reflected by labor market equilibrium (30). Households' labor supply  $\ell_t$  satisfies

(26). Combining them and using (35) gives the equilibrium wage

$$w_t = \nu p_{Ht} + (1 - \nu) p_{Ft} + \gamma (y_t - a_t) + \sigma c_t, \quad (79)$$

so that  $w_t$  depends on output  $y_t$ . The latter equilibrates the goods market. Substituting goods market equilibrium (33) for  $y_t$  in the equilibrium wage yields product wage

$$w_t - p_{Ht} = \varpi_{tot} (p_{Ft} - p_{Ht}) + \varpi_c c_t + \gamma [(1 - \nu) c_t^* - a_t], \quad (80)$$

where  $\varpi_c$  ( $\varpi_{tot}$ ) is the full impact of  $c_t$  ( $tot_t$ ) on the product wage for given terms of trade (consumption), as defined in Table 1. Substitution into (78) yields

$$p_{Ht} = \frac{1}{1 + \kappa_{mc} \varpi_{tot}} \left[ \begin{array}{l} p_{H,t-1} + \beta \mathbb{E}_t \{ \pi_{H,t+1} \} \\ + \kappa_{mc} (\varpi_{tot} p_{Ft} + \varpi_c c_t + \gamma (1 - \nu) c_t^* - (\gamma + 1) a_t) \end{array} \right]. \quad (81)$$

Substituting this for  $p_{Ht}$  and (32) for  $p_{Ft}$  in core equation (77) gives

$$\begin{aligned} \sigma (c_t - c_t^*) &= s_t + p_{Ft}^* - \nu \frac{1}{1 + \kappa_{mc} \varpi_{tot}} \left[ \begin{array}{l} p_{H,t-1} + \beta \mathbb{E}_t \{ \pi_{H,t+1} \} \\ + \kappa_{mc} (\varpi_{tot} (p_{Ft}^* + s_t) + \varpi_c c_t) \\ + \kappa_{mc} (\gamma (1 - \nu) c_t^* - (\gamma + 1) a_t) \end{array} \right] \\ &\quad - (1 - \nu) (p_{Ft}^* + s_t). \end{aligned} \quad (82)$$

Next, focus on the foreign price  $p_{Ft}^*$ . It follows similarly as  $p_{Ht}$ , using the foreign equivalents of (29), (28), (30), (26), and using (36) and (34) instead of (35) and (33). This gives

$$p_{Ft}^* = p_{F,t-1}^* + \beta \mathbb{E}_t \{ \pi_{F,t+1}^* \} + \kappa_{mc} ((\sigma + \gamma) c_t^* - (\gamma + 1) a_t^*). \quad (83)$$

Substituting (83) for  $p_{Ft}^*$  in (82) and then using the lag of (32) for  $p_{F,t-1}^*$  and Euler equation (27) and its foreign equivalent to remove  $c_t - c_t^*$  yields (37).

To understand that this is in  $(i_t, E_t)$ -form, realize that all predetermined, exogenous, and foreign variables are unaffected by  $i_t$ , so they have to be put in  $E_t$ . The recursive nature of (27) implies that  $\mathbb{E}_t \{ c_{t+1} \}$  is determined by expectations of future variables, so there is no contemporaneous effect of  $i_t$ , making  $\mathbb{E}_t \{ c_{t+1} \}$  part of  $E_t$ . Similarly, (29) implies that  $\mathbb{E}_t \{ \pi_{H,t+1} \}$  is part of  $E_t$ . For  $\mathbb{E}_t \{ \pi_{t+1} \}$  one should realize that households base their consumption decision on  $\mathbb{E}_t \{ \pi_{t+1} \}$  as a whole, not on just the  $p_t$  part within it. Hence,  $i_t$  can only affect  $c_t$  via  $\mathbb{E}_t \{ \pi_{t+1} \}$  if the latter as a whole changes, so that  $\mathbb{E}_t \{ \pi_{t+1} \}$  does not contain a contemporaneous channel and is thus part of  $E_t$ .