Social Network Analysis, Graph Theoretical Approaches to

Wouter de Nooy
Erasmus University Rotterdam, The Netherlands

Article Outline

Glossary

Adjacent	Two vertices are adjacent if they are connected by a line.
Arc	An arc is a directed line. Formally, an arc is an ordered pair of vertices.
Attribute	An attribute is a characteristic of a vertex measured independently of the network.
Bipartite network	See: two-mode network.
Clique	A clique is a maximal complete subnetwork containing three vertices or more.
Complete	A (sub)network is complete if it has maximum density: all possible lines occur.
Component	A (weak) component is a maximal (weakly) connected subnetwork.
Degree	The degree of a vertex is the number of lines incident with it.
Density	Density of a simple network is the number of lines, expressed as a proportion of the maximum possible number of lines.
Digraph	A digraph or directed graph is a graph containing one or more arcs.
Distance	The distance from vertex *u* to vertex *v* is the length of the geodesic from *u* to *v*.
Edge	An edge is an undirected line. Formally, an edge is an unordered pair of vertices.
Ego-network	The ego-network of a vertex contains this vertex, its neighbors and all lines among the selected vertices.
Geodesic	A geodesic is the shortest path between two vertices.
Graph	A graph is a set of vertices and a set of lines between pairs of vertices.

Incident	A line is defined by its two endpoints, which are the two vertices that are incident with the line.
Indegree	The indegree of a vertex is the number of arcs it receives.
Line	A line is a tie between two vertices in a network: either an arc or an edge.
Loop	A loop is a line that connects a vertex to itself.
Neighbor	A vertex that is adjacent to another vertex is its neighbor.
Network	A network consists of a graph and additional information on the vertices or the lines of the graph.
One-mode network	In a one-mode network, each vertex can be related to each other vertex.
Outdegree	The outdegree of a vertex is the number of arcs it sends.
Path	A path is a semipath with the additional condition that none of its lines is an arc of which the end vertex is the arc's tail.
Reachable	We say that a vertex is reachable from another vertex if there is a path from the latter to the former.
Semicycle	A semicycle is a closed semipath ending at the vertex at which it starts.
Semipath	A path is a closed sequence of lines such that the end vertex of one line is the starting vertex of the next line and no vertex in between the first and last vertex of the sequence occurs more than once.
Signed graph	A signed graph is a graph in which each line carries either a positive or a negative sign.
Simple graph	A simple undirected graph contains neither multiple edges nor loops. A simple directed graph does not contain multiple arcs.
Star-network	A star-network is a network in which one vertex is connected to all other vertices but these vertices are not connected among themselves.
Strong component	A strong component is a maximal subnetwork in which each pair of vertices is connected by a path.
Strongly connected	A (sub)network is strongly connected if each pair of vertices is connected by a path.
Structural property	A structural property is a characteristic (value) of a vertex that is a result of network analysis.
Triad	A triad is a (sub)network consisting of three vertices.
Two-mode network	In a two-mode network, vertices are divided into two sets and vertices can only be related to vertices in the other set.
Undirected graph	An undirected graph does not contain arcs: all of its lines are edges.
Vertex (vertices)	A vertex (singular of vertices) is the smallest unit in a network.
Weakly connected	A (sub)network is weakly connected if each pair of vertices is connected by a semipath.

I. Definition of the Subject and Its Importance

Social network analysis (SNA) focuses on the structure of ties within a set of social entities or actors, e.g., persons, groups, organizations, and nations, or the products of human activity or cognition such as semantic concepts, web sites, and so on. In a graph theoretical approach, a social network is conceptualized as a graph, that is, a set of vertices (or nodes, units, points) representing social actors and a set of lines representing one or more social relations among them.

A network, however, is more than a graph because it contains additional information on the vertices and lines. Characteristics of the social actors, for instance a person's sex, age, or income, are represented by discrete or continuous attributes of the vertices in the network, and the intensity, frequency, valence, or type of social relation are represented by line weight or value, line sign, or line type. Formally [1: 94-95, 127-128], a network **N** can be defined as **N** = (U, L, F_U, F_L) containing a graph **G** = (U, L), which is an ordered pair of a unit or vertex set U and a line set L, extended with a function F_U specifying a property of the units ($f : U \rightarrow X$) and a function F_L specifying a property of the lines ($f : L \rightarrow Y$). The set of lines L may be regarded as the union of a set of undirected edges E and a set of directed arcs A ($L = E \cup A$). Each element e of E is an unordered pair of units u and v (vertices) from U, that is, $e(u: v)$, and each element a of A is an ordered pair of units u and v (vertices) from U, that is, $a(u: v)$.

The application of graph theory to social relations can be traced back to at least the 1940s [2: 69-72] when the mathematician R. Duncan Luce and the engineer Albert Perry teamed up with the social psychologist Leon Festinger [3] and when the mathematician Frank Harary started his collaboration with Leon Festinger and afterwards with Dorwin Cartwright [4]. They extended pioneering work in SNA that had been done notably in sociometry [5, 6] and anthropology [7-9]. In the 1960s, advances in graph theoretical approaches to SNA such as the contributions by Øystein Ore [10], Claude Flament [11], Frank Harary [12], and innovative applications such as Everett M. Rogers' work on the diffusion of innovations [13], prepared the ground for the rise of SNA in both the USA [14] and Europe [15] as a new set of methods or a new methodology [16] in the 1970s.

II. Introduction

The conceptualization of social systems as graphs and networks offered the opportunity for systematic investigation and theorizing of the structure of ties among social actors beyond the pair. Whereas classical sociology tended to make a quantum leap from the individual and the pair to the triple, group, or society [17], graph theory offered the tools to formally describe and visualize social structure consisting of three and more actors. This led to a new awareness of social structure as a system of ties that is both the product of human action and the context and condition for human action. Because this point of view is very relevant to the issue of complexity in social networks, it is briefly presented in the next paragraph.

The prevalent action theory in SNA conceptualizes collective behavior as the socially 'orchestrated' behavior of individuals or other actors. Actors adjust their behavior and attitudes, opinions, and beliefs to the behavior (etc.) of other members of the social system in which they participate. The system itself is not supposed to behave but it constrains actor behavior: it is the social context within which actors operate. As a network of ties, the system defines to whom an actor is exposed. The immediate contacts – the neighbors in graph theory – of an actor are usually most important to its behavior, but indirect contacts such as the neighbors' neighbors may be taken into account as well. In other words, an actor’s local context or ego-network is likely to affect its behavior. At the same time, however, by ending ties or creating new ones, the individual changes both local network structure and overall network structure, that is, the system. Thus, individual action changes the local context for its neighbors’ (neighbors’ etc.) action. Complexity arises in the interplay between individual behavior and the system both as the overall structure of the network of social ties and as the local context for each actor within the system. To the actors, the change of network structure is not necessarily predictable, so the interplay between individual action and network structure may offer surprising results.

Let us turn to an example now, which is one of the earliest applications of graph theory to social networks. This example nicely illustrates both the transition from a focus on the tie within a single pair to the study of group structure in the social sciences and the interplay between local action and overall network structure. In 1946, the psychologist Fritz Heider formulated the theory of psychological balance [18], which stated among other things that a person (P) feels uncomfortable when he or she disagrees with his or her friend (O) on a particular topic (X). Person P is hypothesized to be stressed and to try to change this situation either by adapting its opinion on topic X, so it matches O’s opinion, or by changing its opinion on O, regarding him or her no longer as a friend. Figure 1 represents both a situation of imbalance and a situation of balance as a signed digraph. The circles and arrows represent the vertices and arcs of the graph and the valence of the opinions or affections are shown both by the labels and the style of the arrows: solid arcs show positive opinions or sentiments, dotted arcs show negative opinions or sentiments.

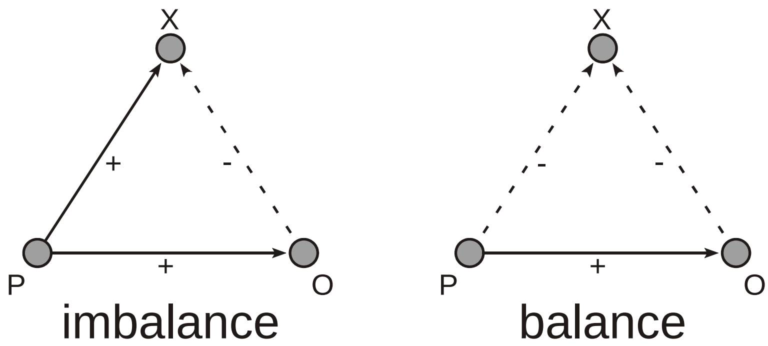


Figure 1 - The principle of imbalance and balance conceptualized as signed digraphs.

In 1956, the mathematician Frank Harary and the psychologist Dorwin Cartwright realized that psychological balance in this triad (three vertices and the lines among them) may be conceptualized as a specific pattern of arcs in a signed graph, viz., in a balanced triad, the P-O-X semicycle (a closed semipath) always contains zero or an even number of negative arcs, whereas an imbalanced triad is characterized by a semicycle with an uneven number of negative arcs [4]. As a next step, replacing the topic by a third person and perceptions of liking or disliking (by the focal person P) by expressed liking or disliking as ties, they easily generalized this idea to a network of arbitrary size. They proved that a signed network is balanced if and only if all semicycles contain no or an even number of negative arcs.

In addition, they proved that a balanced network either contains one set of vertices with just positive arcs among them, or two sets of vertices with all positive arcs within the sets and all negative arcs between the sets, which is a polarized network. In 1967, this result was generalized to polarization among three or more groups by James A. Davis, who showed that a network can be partitioned into an arbitrary number of subsets of vertices such that all positive ties are within the subsets and all negative arcs are among the subsets if the network does not contain semicycles with exactly one negative line [19]. Figure 2 shows an example from Samuel F. Sampson's [20] study of a network of sentiments among novices in a monastery. It depicts the situation at the fourth measurement wave, which was highly polarized at that time. Vertex color indicates whether the novice had previously attended one particular seminary (black: yes, white: no).

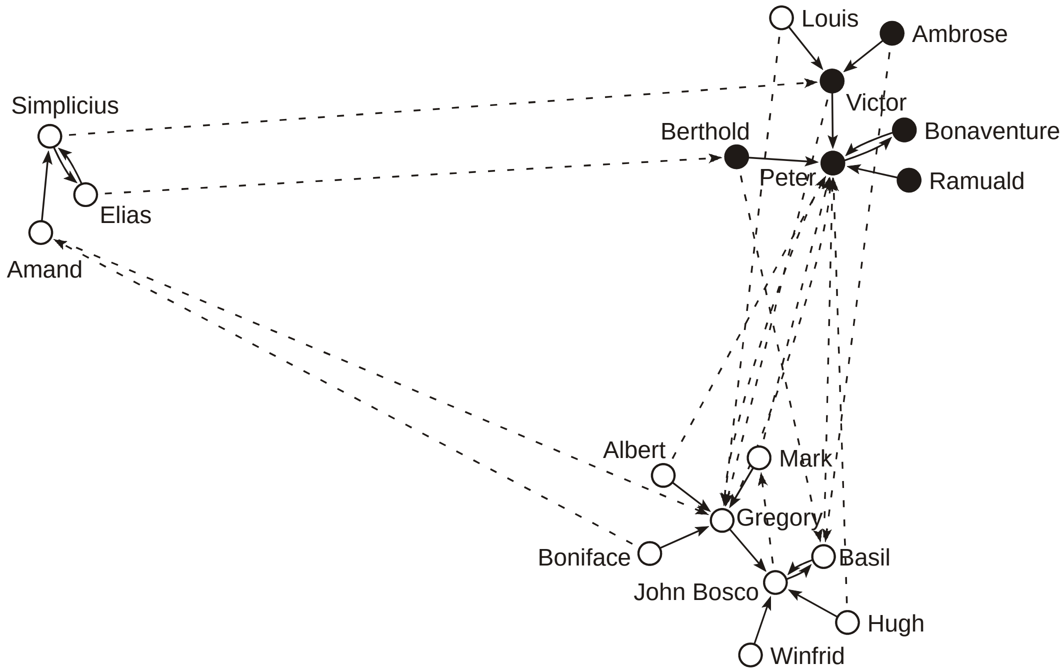


Figure 2 - Almost perfect polarization in the network of sentiments among novices.

In this case, a mathematical relation was established between a behavioral hypothesis at the level of the individual, viz., adjusting one's affect relations such that one's situation is balanced, and overall network structure, viz., polarization. Individual behavior proved to have an unexpected outcome at the level of overall network structure and graph theoretical concepts, in this case semicycles, provided the link. In many cases, however, the link between individual behavior and overall network structure has not been established formally and is sometimes even hard to predict intuitively.

This entry of the encyclopedia aims to present an overview of graph theoretical approaches to SNA, highlighting the complex relations between individual action and overall network structure. It intends to explain why current developments focus on local structure rather than overall network structure to unravel the complexity of social networks. For each of the main topics in SNA (cohesion, brokerage, and prestige), behavioral hypotheses are presented stating why actors create, maintain or dissolve ties. The typical local structure of ties produced by this behavior is sketched in combination with graph theoretical indices for measuring it, and finally the expected consequences to the overall structure of the social network are discussed in combination with the graph theoretical measures developed for measuring them. Note that this approach more or less reverses the historical development of SNA, which focused on overall network structure first and gradually became more interested in the behavior of actors that created, maintained, or changed network structure.

III. Cohesion

One of the first intuitions in SNA concerns the tendency of human beings to form cohesive subgroups. This is a classical topic in the social sciences, see, for instance, George C. Homans' book *The Human Group* [21], and it was central to the sociometry tradition [22]. But where do cohesive subgroups come from and what do they do?

The first and most general behavioral hypothesis merely states that similar people tend to interact more easily and people who interact tend to become or perceive themselves as more similar provided that the interaction is characterized as positive, friendly, and so on. In SNA, this tendency is mainly known as homophily, a concept coined by Paul F. Lazarsfeld and Robert K. Merton [23, 24], but it is known under other names in several scientific disciplines, e.g., the phenomenon of attribution [25] and affect control [26, 27] in social-psychology, assortative or selective mixing in epidemiology and ecology [28, 29: 2], and assortative mating in genetics with efforts at statistical modeling at least as early as 1985 [30].

It is important to note that there are two sides to this behavioral hypothesis, a selection effect, that is, the impact of similarities on the ties that are created, sustained, or broken [31], and an influence effect [32], which hypothesizes that perceived or actual similarities such as the socially constructed identities or opinions [33] result from the ties among actors. According to these hypotheses, people who are directly linked are or become more similar because of their interaction, so they become more likely to engage in ties and maintain ties among them. Thus, social groups form and persist as tightly linked sets of people that tend to share social and psychological characteristics, producing solidarity.

If we concentrate on the graph theoretical aspects of this behavioral hypothesis, that is, the structure of ties, and ignore the (dis)similarities among the actors for the moment, we find several characteristics of local structures that measure cohesive subgroup formation. At the level of a pair of actors, reciprocity of ties in directed networks signals subgroup formation: both actors are hypothesized to choose each other when they are similar. At the level of the triple, transitivity results from tendencies toward cohesion. If actor u establishes a tie with actor v because they are similar, and actor v establishes a tie to actor w for the same reason, actors u and w are also similar, so actor u is expected to establish a tie with w as well, creating a so-called transitive triad (Figure 3). Stated differently, the path from u via v to w is closed by an arc from u to w. In general, the closure of paths or semipaths both in directed and undirected networks signals cohesive subgroup formation at the local level. Closure within an ego-network may be calculated as the percentage of all possible ties among a vertex' neighbors that are present, which is one of the definitions of the clustering coefficient [34: 32-33] but the concept of closure can be extended beyond a vertex' immediate neighbors, e.g., the number of semicycles of length 4 or larger in which a vertex is involved, e.g., balanced semicycles in signed networks.

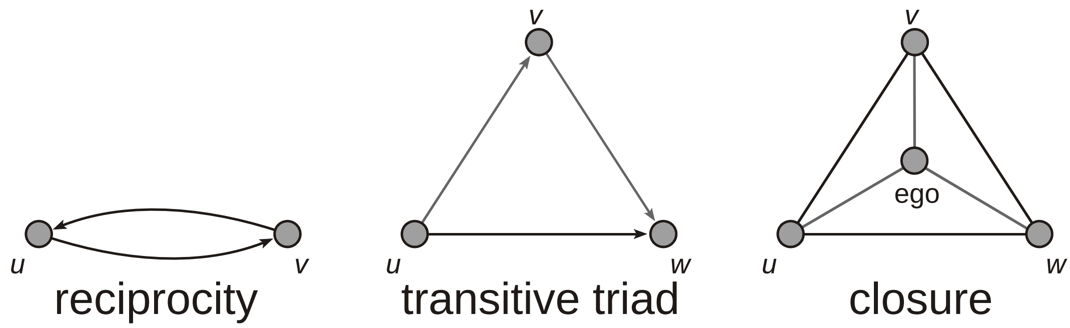


Figure 3 - Reciprocity, transitivity, and closure.

If we include vertex attributes in our measures of cohesive subgroup formation, we can calculate homophily quite simply as the probability or ratio of ties between vertices that share a particular characteristic to ties between vertices that do not. Extending this idea to the ego-network, the homogeneity of actors involved in an ego-network may be taken as a measure of tendencies toward homophily. For qualitative attributes of the actors, Blau's index of variability or heterogeneity [35] can be used ($1 - \sum p_i^2$ where p is the proportion of group members in a particular category and i is the number of different categories), which is conceptually related to the Herfindahl-Hirschman Index in economics measuring the extent of monopoly within an industry. It is interesting to note that Blau's theory hypothesizes that heterogeneity rather than homogeneity of actors within a group enhances the operation and efficiency of the group. If improving group efficiency is the aim of actors, we would have to use a behavioral hypothesis that is quite the opposite of the homophily hypothesis.

Tendencies toward cohesive subgroup formation at the level of actor behavior are most likely to produce sets of densely connected vertices in the overall network and, if we add attributes of the actors, the vertices within sets tend to have similar characteristics. In the most extreme case, the sets are disconnected, so the network consists of several weak components, that is, maximal weakly connected subnetworks, or they are connected only by ties with a negative social meaning, as in the example of polarization presented in the Introduction.

In the history of SNA, the concept of relatively densely connected subnetworks has yielded a large number of graph theoretical ways for identifying cohesive subgroups at the level of overall network structure. Limiting the discussion to one-mode networks, that is networks in which there can be a tie between any pair of vertices (for two-mode or bipartite networks, see the entry 'Social Network Analysis, Two-mode Concepts in' in this encyclopedia), Wasserman & Faust [36: 251-2] distinguish 4 approaches to defining cohesive subgroups.

In the first and strictest approach, a cohesive subgroup is defined as a set of vertices in which all vertices are adjacent, that is, directly linked, to one another. In other words, cohesive subgroups are maximal complete subgraphs, which are called cliques [3].

The second approach is based on the notion of reachability and closeness of members within a subgroup. Members of a subgroup must be reachable in the sense that there are paths between them, i.e., a sequence of lines such that the end vertex of one line is the starting vertex of the next line (following the direction of the lines if they are arcs) such that no vertex in between the first and last vertex occurs more than once. In addition, the shorter the geodesics (shortest paths) between them, the closer the vertices are in a graph theoretical sense, so the more cohesive the subgroup is supposed to be.

The criterion of reachability does not necessarily yield very dense subgroups. In sparse networks, any maximal connected subgraph (strong component) may represent a cohesive subgroup: there is a path between each pair of vertices within a component. Increasing the required number of independent paths between any pair of vertices within a cohesive subgroup yields slightly denser subgroups, e.g., requiring at least two independent paths produces bi-components, which may be generalized to *k*-components for higher minimum numbers of independent (vertex-disjoint) paths between all vertices within a subgroup [37]).

Focusing on graph-theoretical distance between vertices usually yields denser subgroups. One may, for instance, set a maximum *n* to the distance between any two vertices within a subgroup, which is the concept of an *n*-clique [38, 39]. Adding the restriction that the diameter of an *n*-clique is *n* or less, one obtains *n*-clans [38, 40]. Alternatively, one may define a cohesive subgroup as a maximal subgraph of diameter *n*, which is called an *n*-club [40].

The third approach focuses on the minimum number, strength, or multiplicity of ties among subgroup members. Subgraphs that are maximal with respect to the minimum number of neighbors within the subgraph are called *k*-cores [41] and a maximal subgraph with respect to the maximum number of vertices in the subgraph that are not adjacent are known as *k*-plex [42]. In a similar vein, restrictions can be imposed on the minimum strength or multiplicity of ties among members of a cohesive subgroup, generalizing Seidman's concept of a *k*-core to a valued core, which is called an *m*-core [43: 115-6] or *m*-slice [44: 109-10]: maximal connected subgraphs considering only lines with minimum value (or multiplicity) *m*.

In the fourth approach, cohesive subgroups are based on the relative frequency of ties among subgroup members in comparison to non-members: cohesive subgroups are relatively dense sections within the network, that is, relative to the sections outside (and between) subgroups. An *LS* set [45] is a maximal subgraph such that any of its subsets has more ties to its complement within the *LS* set than to vertices outside the *LS* set. Borgatti, Everett and Shirey [46] generalized this idea to the concept of the lambda set, which requires the number of edge-disjoint paths between any pair of vertices within the lambda set to be larger than between any vertex within and any vertex outside the lambda set. A probabilistic version of plus-clusters in signed networks, discussed in the Introduction, can also be subsumed under this approach as it requires relatively many positive lines within cohesive subgroups and relatively many negative lines among cohesive subgroups [47]. Finally, clustering techniques and some types of blockmodels [14: 741-742] also detect clusters of vertices that have relatively many ties within clusters and few among clusters. These mdoels offer an alternative way for finding cohesive subgroups [1: 133-246], see the entry 'Positional Analysis and Blockmodelling' in this encyclopedia.

The large number of alternative measures for cohesive subgroups attests to the fact that behavioral tendencies at the actor level do not play out into nicely structured overall networks in a standard way. Especially the density of the social relation under investigation has an impact on the extent and ways in which cohesive subgroups can be found in the overall structure of the network. As a consequence, measures of network cohesion such as the clustering coefficient (averaged over all vertices in the network) can be quite uninformative about overall network structure: loosely knit cohesive subgroups which are clearly identified by some of the techniques presented above, yield low clustering coefficients in a sparse network. Cohesion in a network is better summarized by calculating the percentage of vertices that are part of identified subgroups, the number and sizes of cohesive subgroups, and so on [48].

In addition to homophily, there is a second behavioral hypothesis related to cohesion in SNA. This hypothesis is based on the idea that social action is embedded in networks [49-51]. Named after the sociologist Georg Simmel, Simmelian ties are ties that are embedded in other ties, e.g., business ties are embedded in family ties, or in complete triads and cliques. They are hypothesized to enforce group norms and enhance trust, hence pressure people into the same behavior because the two actors involved in a tie share common neighbors who supervise their behavior.

Just as with the homophily hypothesis, the embeddedness hypothesis predicts that tightly connected actors will be more similar in their behavior and attitudes. In addition, it predicts that embedded ties are more stable and new ties are more likely to be established when they are embedded in existing cliques or existing ties. Closure again is an important indicator of tendencies to establish and maintain embedded ties but so is the multiplicity of relations: the extent to which a tie on one social relation duplicates a tie on another social relation. At the level of overall network structure, we should expect relatively dense sections, especially cliques, and in a multirelational network, that is, a network containing ties on two or more social relations, we should find that the same subsets of vertices are clustered on each relation. Graph-theoretical measures of the latter are rare. The stochastic blockmodeling technique developed by Krysztof Nowicki and Tom A.B. Snijders [52] is an example. See the entry 'Algebraic Models for Social Networks' in this encyclopedia.

If data on vertex attributes are available, especially if they concern public behavior, that is, behavior that is easily noticed by third parties such as publicly expressed opinions and statements, Simmelian ties are hypothesized to produce a special effect. Involvement in different groups (cliques) then exposes actors to possibly conflicting sets of norms and loyalties, which may urge them to cut their ties with some or all of these groups [53]. In this case, actors are hypothesized to withdraw from stressful relations, so they discontinue ties that incorporate them into cliques (with a preference for cutting a minimum number of ties) or they discontinue ties such that they are no longer connected to actors voicing different opinions or norms. At the macro level, this would produce disconnected sets of cliques instead of overlapping cliques.

IV. Centrality and Brokerage

The notion of centrality in social networks has a long history in SNA. It is attributed to Alex Bavelas [54]. In discussions of centrality, network ties are usually regarded as channels for the exchange of information, goods, services, and so on. Being central in this exchange system has always been hypothesized to be related to influence and satisfaction. Centralization, as a characteristic of a network, has been linked to the efficiency of a network as an exchange system. More centralized groups, for example, have often been shown to be more efficient.

Linton C. Freeman [55] argued that the approaches to centrality are based on three ideas about what being central means: (1) being active within the network, that is, maintaining many ties, (2) being efficient or independent of go-betweens by having short distances to other vertices in the network, and (3) being an important go-between, that is, being part of many paths between other vertices in the network. Although alternative classifications and approaches exist, for instance, Noah E. Friedkin's alternative classification [56] and the formal graph theoretical approach to centrality by Stephen P. Borgatti and Martin G. Everett [57], Freeman's classification is used here, adding concepts of brokerage that have been developed elsewhere in SNA.

Activity

Being active or prominent in the network means that an actor has many ties, hence access to many sources of information (etc.). As a consequence, this actor is more attractive as a neighbor for other actors, which translates to the behavioral hypothesis that actors have a preference for ties with vertices that already have many ties. The degree of a vertex (the number of lines incident with a vertex), then, is the relevant graph theoretical measure of local structure, which is also known as degree centrality. Note that the concept of centrality expresses a structural property of a vertex.

Centralization is the corresponding structural property of a network and it is defined as the variation in the centrality scores of the vertices in the network because this variation shows the extent to which there is a centre (very central vertices) and a periphery (vertices with very low centrality scores). The star and ring networks are defined as respectively the most and least centralized networks and they are known to exhibit the highest and lowest variation in centrality scores in simple networks, that is, networks without multiple lines (and loops in the case of a directed network). See Figure 4 for an illustration, showing a star and a ring network labeling the vertices with their degree centrality scores.

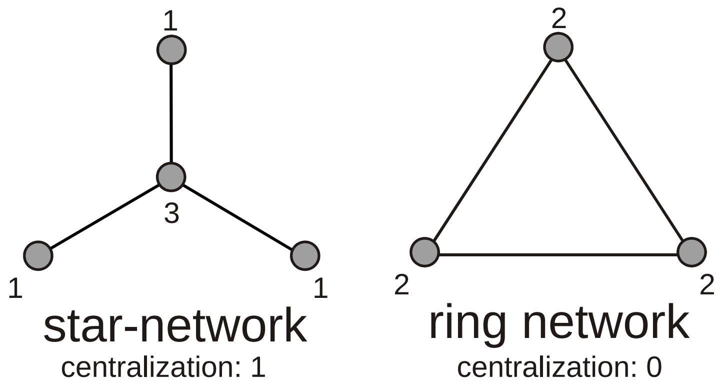


Figure 4 - Centrality and centralization in a star network and a ring network.

From research on the power law in networks [58, 59], discussed in another entry of this encyclopedia, it is known that preferential attachment to degree creates networks with a peculiar degree distribution, including many vertices with low or modest degree and few vertices with high degree. According to the definition of centralization in network analysis, this implies large variation in degree centrality scores, hence high degree centralization. In other words, the behavioral hypothesis of preference for high degree actors produces centralized overall network structure.

Note, however, that the way in which the 'power law networks' are assembled – growing from an initial seed without context – is hardly ever applicable to social networks, which usually have no discernable starting point (networks originate from networks) and are always constrained by the historical context. It remains to be seen whether power law distributions in empirical social networks are created by preferential attachment. Some results indicate that even though the degree distributions of cross-sectional snapshots of a large social network follow the power law, there is hardly any continuity in degree centrality of vertices over time, which does not suggest that the actors are driven by preferential attachment [60].

Efficiency and weak ties

The second approach to centrality focuses on graph theoretical distances between vertices. The central idea here is that actors try to improve access to information and efficient spreading of information by minimizing the number of go-betweens needed to reach or be reached by all other actors in the network. The behavioral hypothesis states that a vertex prefers to link to actors that give access to parts of the network that are presently remote to this vertex and that can only be reached through some or many go-betweens that may withhold or distort information. A minimum number of go-betweens yields maximum independence and maximum efficiency in the exchange network.

Graph theoretical measures of local structure focus on graph theoretical distance, that is, the minimum length of paths between vertices because path length equals the number of go-betweens in the network plus one. Linton Freeman's closeness centrality [55] is a straightforward implementation of this idea because it merely normalizes the average graph theoretical distance between a vertex and all other reachable vertices in the network. In addition, paths can be weighted by the centrality of the vertices on them, which is done by Phillip Bonacich's eigenvector centrality [61, 62]. Finally, the difference between incoming paths and outgoing paths may be added, which is implemented in the version of closeness centrality developed by Thomas W. Valente and Robert K. Foreman [63].

Again, the normalized variation of closeness centrality scores of the vertices in the network yields the appropriate measure of centralization of overall network structure. It is not known yet, however, whether and how tendencies to reduce distances to other vertices at the level of individual actors in the network play out into the closeness centralization of the overall network. On the one hand, if the network contains some vertices with high closeness centrality, they offer rather short paths toward many vertices so they should attract a lot of new ties. This would enhance their centrality and possibly the variation in centrality scores although a general rise in closeness centrality scores of all vertices may also decrease the variation. On the other hand, if the network has low centralization, it is more likely that vertices connect directly to remote parts, reducing path lengths among remote parts, which would not raise the variation in closeness centrality scores and yield or sustain low closeness centralization of overall network structure.

The strength of weak ties argument proposed by Mark Granovetter [64, 65] may be regarded as a special application of the notion of efficiency. In his research on finding a job, Granovetter noticed that relatively superficial ties, ties with infrequent contact, give access to new information because they are more likely to link you to someone with whom you are not linked directly or at a short distance. Strong or intense ties tend to be situated within cohesive subgroups, so they are more likely to offer redundant information already received through other ties. Granovetter is only interested in the effects of having weak ties, but if we turn his idea into a behavioral hypothesis, it suggests a preference to relate to distant vertices that are neither connected to yourself nor to your neighbors (or your neighbors' neighbors, and so on).

A tendency to connect to the most remote parts of the network means that actors tend to establish links to vertices at a large or maximum distance in the network. As in the case of maximizing closeness centrality, it is not clear whether this leads to identifiable patterns in overall network structure. It is quite obvious, however, that this tendency acts as a counterforce against tendencies toward cohesion as densely connected subnetworks: actors are hypothesized to span gaps rather than to close local configurations. Different hypotheses must be developed for strong ties, which are hypothesized to contribute to subgroup formation, and weak ties linking remote parts. The weak ties will probably increase the number of links between dense parts of the network, increasing the *k*-connectivity (minimum number of node-independent paths between any two vertices) of the network. If so, should expect high *k*-connectivity of the network if the weak ties hypothesis is true [37].

Control and structural holes

The third approach to centrality focuses on control over flows within the network: the more you are in between other vertices in the network, the more they depend on you to pass on information, the more you are able to control exchange within the network and profit from your control. Using this type of control is called brokerage.

The notion of being in between other vertices has a straightforward translation to graph theory as being part of a path between two other vertices. Limiting paths to the shortest paths between vertices both Linton C. Freeman's [55] betweenness centrality and Jac M. Anthonisse's [66] rush of a vertex are based on the proportion of all geodesics between other vertices that include this vertex. This measure has been extended to handle directed ties [67, 68]. Information centrality [69 , 70] takes into account all paths between vertices, not just the geodesics and flow betweenness [71] or entropy [72] also consider the values of lines. See Stephen P. Borgatti's [73] classification for more details.

Betweenness centralization as the normalized variation of betweenness centrality of the vertices in the network offers a measure of centralization at the network level and so do centre-periphery blockmodels [14: 741-742], e.g., in the world trade system, central countries are able to profit from the lack of trade among countries in the periphery [74-77]. The link between strategies for maximizing betweenness centrality by actors and the overall structure of the resulting network is unclear. It is not to be expected that control behavior will produce a highly centralized structure because no actor will be satisfied with low betweenness centrality and as a consequence high betweenness centrality for some vertices is unlikely. Therefore, the variation in betweenness centrality scores will not be high.

While betweenness centrality situates an actor with respect to all other actors in the network, Ronald S. Burt [78, 79] proposed a local variant, focusing on control within the ego-network of an actor. His behavioral hypothesis rests on the *tertius gaudens* principle: the benefits that accrue to an actor that is in between two actors that are not directly linked because of the opportunity to broker information between them or, in a more malicious variant, to divide and conquer.

This hypothesis translates quite easily into graph theoretical structure. The absence of a tie between two neighbors of an actor is called a structural hole (Figure 5a). The behavioral hypothesis states that actors try to increase the structural holes that they can exploit. At the same time, however, they try to minimize the structural holes through which they can be exploited. This means, among other things, that an actor will not end a tie to one of its neighbors if the two neighbors are directly linked (see Figure 5b): that would create an opportunity to broker at the expense of the actor. In this situation, the actor is constrained in its opportunities to change ties.

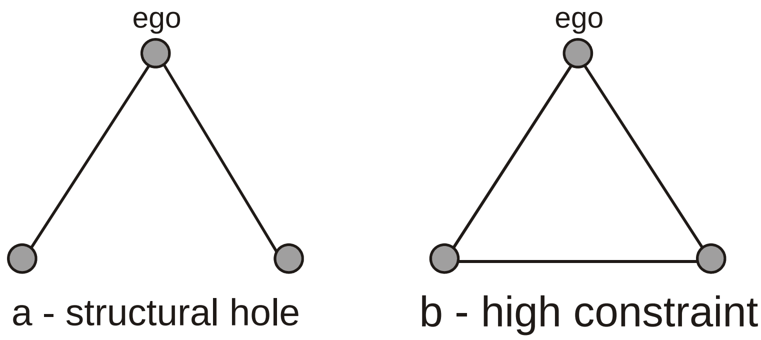


Figure 5 - A structural hole (a) and a triad with high constraint (b).

Structural holes and constraint are the flip sides of the same coin. A tie with low constraint indicates that the tie is involved in (many) incomplete triads (such as Figure 5a), so there are (many) structural holes offering the actor options for brokering. High constraint on a tie means that it is part of (many) complete triads (as in Figure 5b), so there are few or no possibilities for brokerage. Because the presence or absence of ties among an actor's neighbors is key to the argument, network analysts have also used the density of the ego-network without the ego as a proxy of constraint: the higher the density, the higher the constraint on the ego. Alternatively, betweenness centrality for ego-networks [80, 81] can be used.

If the structural holes hypothesis governs actors' behavior, what overall network structure should we expect to find? A strong tendency toward brokerage at the micro level is not likely to produce a centralized network because each actor would try to maximize the number of structural holes around itself, which would yield a bipartite graph in the extreme case (no ties among any vertex' neighbors), or it would minimize its constraint by ending all ties to neighbors that have contacts outside ego's immediate neighborhood, which would produce a highly clustered network consisting of isolated cliques or isolated vertices in the extreme case.

In summary, the relation between local action and overall network structure is simple and clear only in the case of preferential attachment to high-degree vertices. Tendencies to maximize centrality that looks beyond the immediate neighbors such as closeness and betweenness centrality, do not necessarily yield centralized networks. Even for local structures, alternative hypotheses for actor's behavior are available that are unlikely to produce centralized networks, e.g., a preference to avoid constraint. The interplay between local action and overall structure is quite ore complex.

V. Prestige and Ranking

The preceding sections have not distinguished between directed and undirected networks. For prestige and ranking, however, the direction of ties is crucial because asymmetry in networks is assumed to be linked to social prestige [82]. The general idea here is that social inequalities are reflected and possibly created by asymmetric ties, e.g., everyone invites the most popular boy or girl in class but s/he doesn't return each invitation. Of course, the nature of the social relation determines the direction of choices; ties like "reports to" or "pays respect to" point toward higher levels in a hierarchy while "beating up" points in the opposite direction.

A central behavioral hypothesis concerns the popularity or attractiveness of actors. Actors tend to (want to) relate to actors with attractive structural properties or attributes, so attributes related to power or social status increase the probability that an actor will be chosen. From a constructivist point of view, however, being chosen often is also interpreted as a sign of importance and prestige, so receiving many choices (ties) increases the probability of receiving even more. In this way, networks may produce informal status hierarchies. The Matthew Effect, proposed by Robert K. Merton [83], comes to mind here: "For unto every one that hath shall be given, and he shall have abundance: but for him that hath not shall be taken away even that which he hath" (gospel of Matthew XXV, 29).

In graph theoretical terms, the structural attractiveness of an actor refers to the number of incoming arcs on vertices, which is simply the indegree of a vertex. This is called the popularity of a vertex and, of course, we must replace it by the vertex' outdegree if the relation is negative, e.g., submission, beating up, criticizing. If indirect choices must be taken into account as well, attractiveness is measured by proximity prestige [84], which is based on the average distance from all other vertices in the network – a directed variant of closeness centrality. Proximity prestige captures the idea that nominations or choices by actors who are themselves popular, contribute more to one's structural prestige. Bonacich's measure of power [62, 85] adds the idea that power may also be derived from being connected to powerless actors rather than to other powerful people.

The popularity hypothesis is another example of preferential attachment to degree with the restriction that we focus either on indegree or outdegree. Therefore, the indegree (or outdegree) distribution of the overall network is expected to follow the power law and network structure will be characterized by high degree centralization.

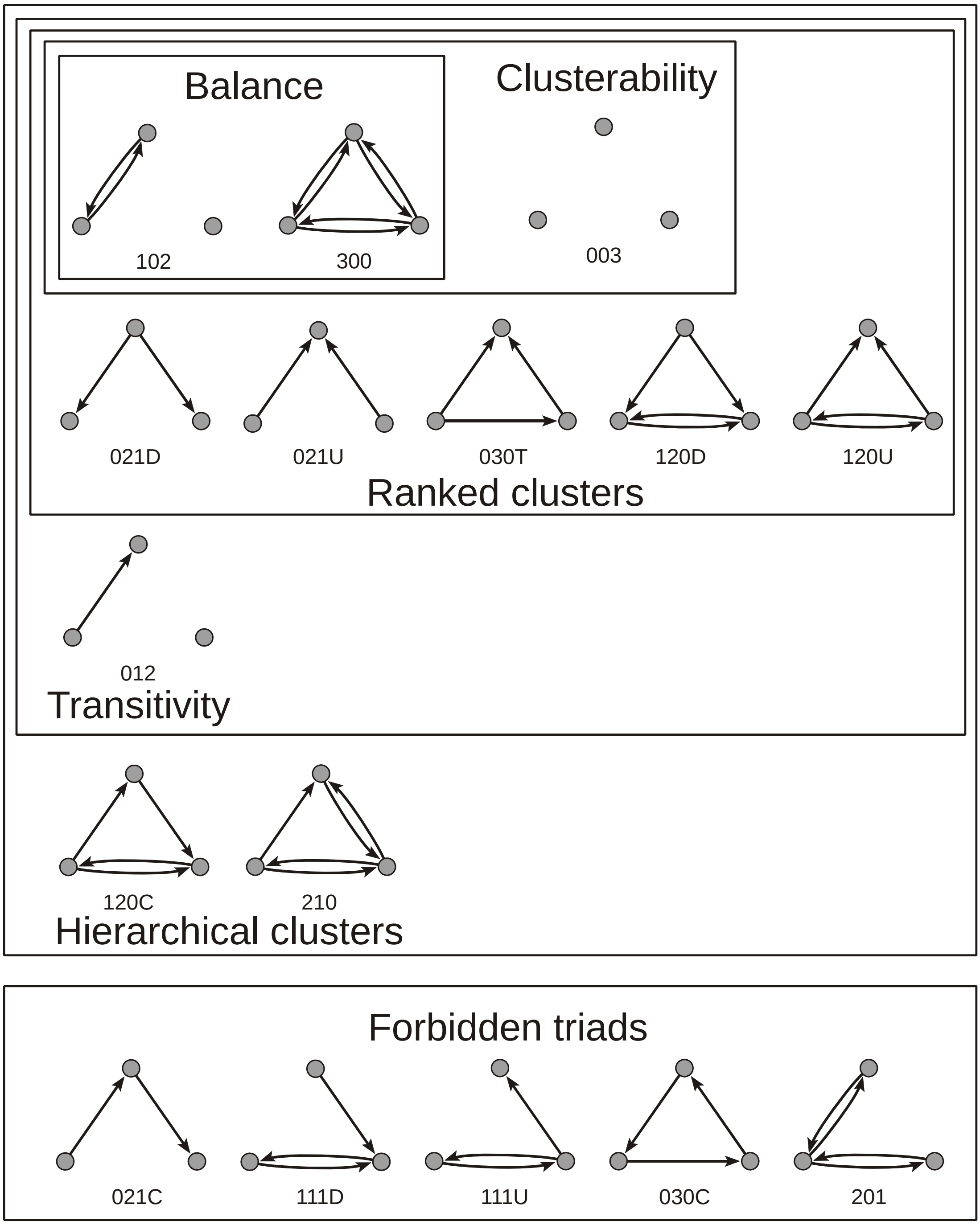
Adding data on social attributes that make some actors more prestigious such as wealth, social class, beauty, and so on, we should expect preferential attachment to vertices that score high on these attributes. Note that vertex attributes play a slightly different role here than in the case of cohesion. Now we are concerned with attributes of (at least) ordinal level, expressing prestige that an actor possesses to a higher or lower degree. In the case of cohesion, we deal with nominal attributes, which merely express an identity. In contrast to homophily, the attributes of the actor who initiates the directed tie (the tail of the arc) does not matter here because the effect is solely related to structural characteristics or attributes of the actor at the receiving end of the tie (the arc's head).

There is a second, slightly different behavioral hypothesis relating to deference or submission rather than attractiveness. The idea is that actors mainly tend to create positive ties to other actors in their own status group or to actors in a higher status group – the people they are looking up to – to consolidate and improve their social position. Similarly, they tend to direct negative ties to actors in lower status groups. Note the difference with attractiveness: it is hypothesized that actors choose upwardly but they need not prefer the most attractive (top) actors in the network as they are supposed to do according to the attractiveness hypothesis.

The main difference between attractiveness and deference is that the former only takes into account structural properties or attributes of the tie's receiver, whereas the characteristics of both the sender and receiver of the tie matter to the latter. This distinction has important consequences to the structure of the overall network. Whereas the attractiveness hypothesis yields centralized networks, the deference hypothesis yields layered networks. The layers consist of sets of vertices that are symmetrically linked, e.g., by reciprocal ties, while the ties between layers are asymmetric, all pointing in the same direction. This behavior may be both a consequence of a formal hierarchy, e.g., positions within an organization with formalized relations such as reports to, or actually show an informal social hierarchy, e.g., status differences between men and women in a particular social setting.

In simple directed networks, triads, that is, three vertices and the lines among them, are the key to measuring tendencies toward ranking at the local level. Translating the concepts of balance and clusterability (see the Introduction) from signed digraphs to unsigned digraphs, James A. Davis and Samuel Leinhardt [86, 87] replaced positive ties by symmetric ties within cohesive subgroups and negative ties by no ties among subgroups. Thus, ties within a layer, either within clusters or among clusters, are symmetric. They assumed that asymmetric ties represent the ranking of clusters into a hierarchy, introducing the model of ranked clusters. Moreover, they showed that a network with a perfectly fitting ranked clusters model contains only certain types of triads, whereas other types do not occur (Figure 6).

The ranked clusters model requires arcs from each vertex to all vertices on higher ranks. This requirement is usually too strict for empirical social networks and it is relaxed in the transitivity model proposed by Paul W. Holland and Samuel Leinhardt [88], which requires that clusters of vertices on different ranks are either completely linked or not linked at all, yielding a partial order, by simply adding one type of triad to the set of allowed triads (Figure 6). Later, Eugene Johnsen [89] proposed the model of hierarchical clusters to account for asymmetries within clusters. Note the nesting of the models for overall network structure, which is why the sets of permitted triads is extended for more general models. Finally, it was shown that the models developed for unsigned digraphs could also be detected in incomplete signed digraphs using types of semicycles [90].



Triad code: The first digit shows the number of mutual (reciprocal) ties, the second digit shows the number of asymmetric ties, and the third digit is the number of null (absent) ties. Letter D stands for Down, U for Up, C for Cyclic, and T for Transitive.

Figure 6 - Triad types and balance-theoretic models.

The triads characterizing ranked structures serve as models for tie creation, maintaining, and breaking behavior of actors under the deference hypothesis. For instance, triad 120U (Figure 6) predicts that a member of a cluster is likely to establish or maintain a tie to an actor at a higher rank if its neighbors within the cluster have such a tie. In the perfect case, there is a one-to-one relation between sets of occurring types of triads and the overall structure of the network. Therefore, triad census [91, 92], which is the frequency distribution of the sixteen types of triads in a directed network, offers an indication of overall network structure. In the imperfect case, the triad census of a network may be compared to the frequency distributions of triad types in randomly generated networks to test the tendency toward ranking.

The triad census does not show the composition of the clusters and ranks; it does not identify the vertices belonging to particular clusters and ranks. This can be done in several ways. Realizing that ranks should be connected asymmetrically in directed networks, strong components cannot include more than one rank because vertices within strong components are by definition mutually reachable. Ties between strong components, then, are asymmetric, so it is easy to establish the ranking among strong components. Strong components, however, do not require a lot of symmetry in the ties; actually, no tie need to be reciprocated. The symmetric-acyclic decomposition proposed by Patrick Doreian, Vladimir Batagelj and Anuška Ferligoj [93] does require at least some symmetric ties (mutual choices) within clusters because they define a symmetric clusters as a maximum subset of vertices that are directly or indirectly linked by symmetric ties. Generalized blockmodels [1] that are asymmetric with respect to off-diagonal blocks offer another way to identify hierarchical relations (see the entry 'Positional Analysis and Blockmodelling' in this encyclopedia).

VI. Analyzing complexity in social networks

The preceding sections presented behavioral hypotheses that have similar, different, or even opposite consequences for overall network structure. It is not plausible that one particular type of behavior dominates network formation. Therefore, it is not likely that overall structure of empirical social networks will display one particular form that can be hypothesized in advance or that behavioral tendencies can be adequately tested on particular characteristics of overall network structure. It has been shown, for example, that the degree distribution of a network does not reveal tendencies toward cohesive subgroup formation that are operative during network evolution [94].

Even if overall network structure displays certain characteristics, they may be produced by different types of behavior. Centralization in a social network, for instance, may arise from a tendency of actors to minimize paths to all other actors or from a preference for prestigious actors. Alternatively, it may be a by-product of cohesive subgroup formation: actors that are marginal to the cohesive subgroups may directly or indirectly connect different subgroups, which gives them a central position with respect to betweenness. Furthermore, pronounced overall network patterns may occur only temporarily in empirical social networks when they create socially unstable situations. The polarization predicted by balance theory in Sampson's network of novices, used as an example in the Introduction, was only temporary. After this polarization and most likely due to it, many novices left the monastery. The network, so to speak, fell apart.

For these reasons, SNA increasingly focuses on local structure using overall network structure merely as a collection of (overlapping) local structures. Behavioral hypotheses translate much more directly to local structure, that is, to the ties of the actor and those of its neighbors (and possibly their neighbors), as we have seen in the preceding sections. Local structure is the part of the network that an actor can easily survey and actually change.

The latest developments in techniques for modeling network structure and evolution apply this actor-oriented approach either in statistical models, see the entries 'Longitudinal Methods of Network Analysis' and 'Exponential Random Graph (p*) Models for Social

Networks,' or in simulation models, see the entry 'Agent-based Computational Approaches to Social Networks' in this encyclopedia. The techniques test behavioral tendencies by relating the creation, maintenance, and ending of ties by individual actors to the local configuration of ties, to previous ties between the actor and the alter or to present ties on another social relation, and to characteristics of both the actor and the alter.

In principle, the actor-oriented approach is able to test all behavioral hypotheses presented in the preceding sections on characteristics of local structure in which the actors are embedded and properties of the actors themselves. If hypothesized local configurations appear more often than expected by chance, the underlying behavioral hypothesis is assumed to guide individual behavior at least to some extent. If the behavior in a set of actors or individual actor's behavior are in line with several behavioral hypotheses at the same time, the effect of each behavioral tendency can be separated. Thus, it is possible to link complex overall network structure to compound behavior of the actors in the network.

VII. Future Directions

The techniques for analyzing local structure are in development. Models for the co-evolution of relations and quantitative attributes of vertices over time have just been introduced [95]. Not all behavioral hypotheses have been included yet, for instance, because they involve non-standard types of networks such as signed relations, and new ones are bound to be proposed. Incomplete data and external constraints on data collection or conceptual constraints on network structure such as two-mode networks (see the entry 'Social Network Analysis, Two-mode Concepts in' in this encyclopedia) may limit the applicability of current models and spur the development of new ones.

If the actor-oriented approach is successful, will overall network structure become completely redundant? Will it only serve network exploration – looking for behavioral hypotheses rather than testing them – and for analyzing the consequences of network position on behavior, attitudes, or esteem, for instance, does the subgroup to which an actor belongs or its centrality correlate with its subsequent behavior or attitudes?

Let us return to the balance example, presented in the Introduction. The high degree of polarization among Sampson's novices was followed by the voluntary or forced exit of several novices. This suggests that people are able to survey overall network structure and draw conclusions from it, rather than just react to their local network environment. A highly polarized social group does not seem to be the kind of situation that we like to be living in. An actor's perception of overall network structure may also affect its behavior.

In this respect, the balance example is interesting because the original theory by Heider referred to perceptions of affect relations and unit relations (having characteristics or possessing items) rather than objectively measured relations. The importance of perception to network analysis is stressed in a more general way by Harrison White, who argues that social ties are stories [96]. According to him, people are linked into social networks by the stories that they tell about their ties. Thus, we should expect people to react to the ties as they remember and tell them rather than to the ties as they are observed by the researcher or registered in, for instance, membership lists.

A similar argument was made by David Krackhardt when he reconsidered balance theory [97]. According to him, we should measure each actor's perception of network structure, for which he proposed the concept of Cognitive Social Structures, compare these perceptions, and use them to explain why actors behave as they do. In his approach, actors are assumed to be able and active in forming impressions of overall network structure.

If humans are capable of imagining overall network structure, then communication of these perceptions may also play a role in network formation, including the accidental and deliberate distortions or simplifications that are likely to happen. This brings us to the link between network structure and mental categories such as social classifications or culture in a more general sense as argued by Ronald L. Breiger [98, 99]. If, for example, members of the network perceive and discuss cohesive subgroups, they assign names and meanings to social configurations. Thus, social meanings and identities are created in the process of establishing social ties and interpreting them. The duality of social structure on the one hand and the structure of symbolic categories on the other hand as proposed by John W. Mohr [100], may be the essential condition for cultural meanings that are social in the sense that they affect actors' sense of identity and behavior.

These discrete and qualitative rather than continuous and quantitative classifications are very likely to affect the network behavior of actors: they define the categories that are experienced as being similar or dissimilar in the case of homophily and group formation, or superior versus inferior in the case of prestige and ranking. As a consequence, it is to be expected that the present focus on local structure will be complemented by a focus on overall network structure, especially perceived and communicated network structure.

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