Part II
Hedging in Interval Models

Authors:
Jacob Engwerda
Department of Econometrics and Operations Research,
Tilburg University,
The Netherlands

Berend Roorda
School of Management and Governance,
University of Twente,
The Netherlands

Hans Schumacher
Department of Econometrics and Operations Research,
Tilburg University,
The Netherlands

Trading strategies designed to reduce risk (i.e., hedging strategies) are a widely studied topic of research in finance. Usually this design is based on stochastic models for the underlying assets. In this part, we introduce in a discrete-time setting the deterministic modeling framework that will be used, in various forms including continuous time versions, in this volume.

After a short introduction on hedging we introduce the basic underlying concept of this framework, namely, the interval model. The interval model assumes that prices at the next time instant can fluctuate between an upper and lower bound, which are given.

We discuss the pricing of derivatives in interval models and optimal hedging under robust-control constraints. Numerical algorithms are provided to calculate the corresponding hedging strategies.

The first chapter of this part, Chap. 3, is introductory and contains well-known material that can be found, for instance, in Hull [88] and Neftci [121]. The second chapter, Chap. 4, discusses pricing in interval models, and the third chapter, Chap. 5, deals with optimal hedging under robust-control constraints. Most of the material presented in Chap. 4 has appeared in a paper published in 2005 in Kybernetika [132]; the material presented in Chap. 5 has not been published before. The work of Berend Roorda on the topics discussed here was mainly done while he was at Tilburg University, supported by a grant from the Netherlands Organization for Scientific Research (NOW) through MaGW/ESR Project 510-01-0025.

Notation

Universal constants

- $\mathbb{R}$: Set of real numbers
- $\mathbb{R}^+$: Set of nonnegative real numbers

Main variables and parameters

- $\mathbb{B}^{u,d}$: Binomial tree model with proportional jump factors u and d
- BC^g: Best-case costs under strategy g
- $\mathrm{BC}^*(S_0, V)$: Best-case costs under RCC limit V on worst-case cost
- $\mathrm{co}\, I$: Smallest convex subset containing subset I
- $E_j[S]$: Expectation of S conditional on the information available at time t_j
- $F(.)$: Payoff function of option
- $f_j(S_j)$: Option price at time t_j if the price of the asset at time t_j is S_j
- FPI($\mathbb{M}$, F, S): Fair price interval
- $\mathbb{G}$: Set of admissible hedging strategies
- $\mathbb{G}^V$: Set of admissible hedging strategies under RCC limit V
- $g_i(S_0, \ldots, S_i)$: Amount of underlying asset held at time t_j
- H_j: Realized hedge costs
- $\mathbb{I}^{u,d}$: Interval model with maximal and minimal growth factor over each time step u and d, respectively
- $I^g(\mathbb{M}, F, S)$: Cost range of strategy g
- $\mathrm{LPR}(f, V)$: Loss-profit ratio with option premium f and RCC limit V
- $\mathbb{M}$: Model, i.e., sequence of $N+1$ numbers in $\mathbb{R}^+$
- $N(\mu, \sigma^2)$: Normal distribution with expectation μ and variance σ^2
- $\Phi(d)$: Cumulative standard normal distribution evaluated at d
- $Q^g(F, \mathscr{S})$: Total cost of hedging and closure
- RCC: Robust-cost constraint
- S: Asset price path $\{S_0, \ldots, S_N\}$
- S_j: Price of asset at time t_j
- σ: Volatility (standard deviation) of stock price
- T: Expiration time
- VaR: Value-at-Risk condition

- WC^g: Worst-case costs under strategy g
- X: Strike price
- x^T: Transpose of vector x
- $[Z]^+$: Maximum of values Z and 0