The semantic anatomy of conditional sentences

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Abstract

This paper offers a compositional semantics for English conditional sentences. The proposal is worked out within the GB/Minimalist approach to the syntax/semantics interface and uses for the interpretation Plural Compositional DRT as introduced by Brasoveanu [2008]. The approach is based on the idea of Blatt and Pancheva [2006] (among others) that if-clauses are free relatives of possible worlds, and, consequently, that they have to be treated as referential expressions. The approach is well related to the philosophical literature on conditionals in that it incorporates the similarity approach to conditionals of Lewis [1973] and Stalnaker [1968]. It is also agrees with the central claims of the restrictor approach of Kratzer [1979, 1991]. In contrast to the standard spell-out of the latter the present approach does not need a covert modal operator at the level of LF to account for bare conditionals.

1 Introduction

The meaning of conditional sentences always has fascinated many thinkers. What does such a sentence mean? Can it be true? When does it count as true? The charm conditionals exert can be explained by their intrinsic relation to a number of central scientific problems, like the nature of reasoning, the possibility of knowledge, laws of nature, causality and many more. But while this fascination has resulted in an enormous body of expertise in philosophy on the meaning of conditional sentences in abstracto, for a linguist the theories proposed are in an important respect deficient. They do not explain how the meaning of these sentences is related to the concrete form conditionals take in the languages of the world. The goal of the present paper is to focus exactly on this less studied aspect of conditional semantics and address the relation between their form and meaning. With this work we hope not only to close some gaps in the landscape of research on conditional sentences. The underlying motivation is rather that studying the form of conditional sentences can provide important tools for evaluate different proposals made for the semantics of these sentences.

Our investigations will be guided by the principle of compositionality. Hence, we will assume that the meaning of conditional sentences results from the mean-
ing of its parts and the way they are syntactically combined [Frege, 1892, Partee, 1984]. Applied to conditionals the principle of compositionality faces a number of very interesting challenges, concerning the interpretation of tense, aspect and modality [Comrie, 1986, Crouch, 1993, Fintel, 2005, Iatridou, 2000]. These problems have recently attracted a lot of attention among semanticists [Ippolito, 2003, 2006, Kaufmann, 2005, Schulz, 2008, Stechow, 2005, Stechow and Grønn, 2008]. The present paper will contribute to this discussion by providing a basic framework for the compositional semantics of conditional sentences. The approach is meant as a simple module that can be used to extend or combine with approaches to related subjects, like the semantics of tense, modality, the treatment of presuppositions, etc. The reasons behind developing such a basic framework are not account for a number of concrete observations. Rather we hope that this framework will provide the right foundation for successful work on the well-known puzzles of tense, modality etc. in the context of conditional sentences.

The approach that will be presented in the present paper builds on two basic ideas/claims about the compositional semantics of conditionals. The central claim on the side of syntax is that the if-clause of a bare conditional is a free relative for possible worlds. This idea immediately gives rise to a particular view on the semantics of conditional sentences. The central idea underlying the semantics is to adopt a referential analysis of the if-clause of conditionals. The antecedent will be interpreted as a definite description of a plurality of possible worlds. The consequent is treated as an ordinary main clause. It makes a simple statement about its reference context - only that this reference context has been modified by the if-clause.

Both ideas are not new. A detailed proposal for a referential analysis of conditionals is given in Schlenker [2004], who in turn works out an idea of Lewis [1973]. The idea that if-clauses should be analyzed as free relatives has been defended very firmly in Bhatt and Pancheva [2006]. The contribution of the present paper lies in making these ideas precise and spelling out the consequences that the two basic assumptions give rise to. In this respect we will also go beyond Schlenker [2004], for instance, in having plurality of possible worlds in the formal system.

Given the dominant view on conditionals in the literature, maybe the most surprising aspect the analysis presented here is that it does not assume bare conditionals to come with a covert modal quantifier. Thus, on a compositional level, this approach deviates from the standard restrictor approach to conditionals. However, the result, the overall meaning assigned to conditional sentences will be basically the same. This could not be otherwise, because there is no doubt possible about the general observations the restrictor approach builds upon. As will be argued below, the restrictor approach (as spelled out in Kratzer [1979, 1991]) and the proposal made here complement each other. They approach the same problem, but from different angles. In the end, however, they should and do come to the same result.
The remainder of the paper is structured as follows. We will start in section 2 discussing the semantic and syntactic evidence for the two main ideas the present approach builds upon. In section 3 the formal approach will be developed and illustrated with a couple of examples. Section 4 discusses the some implications of the approach. Finally, section 5 summarizes the findings of the paper and outlines future research.

2 Motivating the proposal

The main objective of this section is to motivate the proposal that will be introduced in 3: a formally precise compositional analysis of the meaning of conditional sentences in English. But not any compositional approach was acceptable. The aim was to obtain an approach that complies to a number of additional restrictions. These restrictions will be the topic of the first subsection 2.1. In the second subsection 2.2 we will introduce the main ideas underlying the proposal. We will discuss how they can be motivated given empirical facts and how they relate to certain other approaches towards a compositional semantics for conditionals. In particular, in section 2.2.6, well will discuss the relation to the restrictor approach of Kratzer [1979, 1991].

2.1 The objectives

In order to make sure that we end up with an approach that is from a philosophical as well as a linguistic perspective convincing we imposed a number of additional constraints on the potential outcome of the project. More specifically, we adopted the three objectives in (1). In the following each of these objectives will be discussed in some detail.

(1) a. The approach should be well-related to the philosophical literature on conditionals.
   b. The approach should cover a wide range of conditional constructions.
   c. The approach should be based on a sound syntax-semantics interface.

The link to philosophy As already explained in the introduction, there is a huge amount of sophisticated literature on the semantics of conditionals coming from philosophers. This literature in general does not address the issue of meaning composition, but deals with the over-all interpretation of conditionals. Of course, in the end the compositional semantics developed here should give the correct output of the over-all meaning of conditionals. Therefore, this literature is very relevant for our purposes and should be well-linked to the approach developed here.

In order to relate well to the philosophical debate, the present approach is set up in a way that the description of the over-all meaning of conditions comes down to the similarity approach to conditionals [Stalnaker, 1968, Lewis, 1973],
According to this approach the truth of a conditional with antecedent $A$ and consequent $C$ is defined relative to a world $w_0$, a model $M$ and a function $\text{Rev}$. This function applies to a world $w$ and a proposition $p$ and intuitively returns the set of $p$-worlds (worlds that make $p$ true) that are most similar to $w$. In the original version of the similarity approach the exact character of the function $\text{Rev}$ is left open. It only has to obey a small number of very basic constraints.\(^1\)

1. If $p \neq \emptyset$, then $\text{Rev}(w, p)$ is always defined and $\text{Rev}(w, p) \subseteq p$.
2. If $p \subseteq p'$ and $\text{Rev}(w, p) \cap p' \neq \emptyset$, then $\text{Rev}(w, p') = \text{Rev}(w, p) \cap p'$.
3. If $p = \emptyset$, then $\text{Rev}(w, p)$ is undefined.
4. If $w \in p$, then $w \in \text{Rev}(w, p)$.

We will not make any additional assumptions on the nature of $\text{Rev}$, because the present paper is not concerned with the properties of this function, but rather with how the truth conditions predicted by the similarity approach can be derived from the form of conditionals sentences. This leaves the reader the freedom to insert his/her own favorite similarity function.

**Wide coverage.** The overall goal of this project was to develop a general basic framework for a compositional semantics of conditionals. The approach should only take into account the parts of the construction that are essential to being a conditional construction of some natural language. But such an aim has to be balanced by more practical considerations. Even though there is a surprising generality to the way the languages of the world express conditionals, and cross-linguistic observations will play an important role in our argumentation, the proposal will be specifically for English conditional sentences. We hope to remove this limitation in future work.

Also within English there is a lot of variation in the way conditional sentences are constructed and interpreted. Because there is only so much one can do within one paper, we will have to further narrow down the type of construction we will discuss. We will focus on what Haegeman calls event-conditionals (an example is given in (2-a) and exclude her relevance conditionals (an example is (2-b), see for more discussion Haegeman [2003]). The reason is mainly that we want to work with a semantically and syntactically homogenous phenomenon. It is generally accepted that the later type of conditionals differs in its semantics, and Haegeman [2003] argues convincingly that relevance conditionals also have different syntactic properties. Within the class of event-conditionals we will focus on conditionals that (i) have an explicit antecedent, (ii) start with the antecedent, and (iii) where the antecedent starts with *if*. This will further restrict the syntactic variation in the class of sentences we want to account for. In future research, hopefully, these restrictions can be removed.

\(^1\)We adopt the version of Lewis [1973] that allows $\text{Rev}$ to select sets of worlds.
a. If your back-supporting muscles tire, you will be at increased risk of lower-back pain. (event-condition)

b. If we are so short of teachers, why don’t we send our children to Germany to be educated? (relevance-condition)

However, based on the strong formal and semantic similarities between both types of conditionals we intend the approach developed here to be able to deal with indicative conditionals as well as subjunctive (or counterfactual) conditionals. This is not saying that there is no semantic difference between indicative conditional and subjunctive conditionals, but just that they are not due to the interpretation of the conditional core of these sentences. This might seem surprising in the view of examples like Adams [1975]’s (3), which led Lewis to conclude: ”Therefore, there really are two different sorts of conditional: not a single conditional that can appear as indicative or as counter-factual depending on the speaker’s opinion about the truth of the antecedent”. Adams observed that it is possible to accept (3-a), while denying (3-b) in one and the same situation.

(3) a. If Oswald didn’t kill Kennedy, someone else did.

b. If Oswald hadn’t killed Kennedy, he would still be alive.

The readings that Lewis and many others want to see distinguished here are an epistemic interpretation expressed by indicative conditionals and an ontic interpretation expressed by subjunctive conditionals. However, from a linguistic point the evidence for this classification is weak. There are many indicative conditionals that receive ontic interpretations (nearly all indicative conditionals about the future), and there are also subjunctive, even counterfactual conditionals that are interpreted epistemically.

Though this will not be discussed within this paper, in order to account for the differences between (3-a) and (3-b) we propose to follow Stalnaker [1981] and assume that the mood markings on conditionals express restrictions on similarity function Rev underlying the meaning computation. According to Stalnaker, the indicative mood conveys that the similarity function when applied to the antecedent ends up selecting worlds that are all in the common ground, i.e. the closest antecedent worlds are consistent with the shared beliefs of hearer and speaker. The subjunctive mood, on the other hand, comes without such restrictions. Because in the indicative case the interpretation of conditionals is restricted to worlds conform to the shared beliefs, these beliefs will affect the truth conditions of the conditional. This leads to the observed epistemic effects. We leave the details of such an approach to a different occasion.

The syntax-semantics interface A sound syntax/semantics interface is a necessary prerequisite for any compositional semantics. We will define the compositional semantics developed here in the GB/Minimalist approach to the syntax/semantics interface [Chomsky, 1982, 1995] and assume that the syntax proper generates a syntactic structure called Spell-Out (SO) that is translated
into a **Logical Form** (LF) which is then subject to interpretation. The syntax proper consists of a lexicon and syntactic rules. The lexicon contains lexical trees, i.e. words dominated by a syntactic category and a semantic type. The LF is formulated in a typed formal language. The translation of SO into LF happens via the rules of construal).\(^2\)

### 2.2 The main ideas and where they come from

Conditionals are very complex sentences. It is not surprising that there is still a lot of discussion about their syntax. To avoid making choices here that are not well-motivated, but also because we don’t want to spend too much time sorting out the syntactic properties of conditionals – in the end this is not the main focus of the paper – we will adopt the following strategy. We will look for constructions that in form and meaning work very similar to conditionals, but that are much better studied and understood. Then we will take convincing approaches to the compositional semantics of these constructions and try to make them work for conditional sentences as well.

#### 2.2.1 The interrogative link

Focussing on the form of conditionals cross-linguistically, the most obvious parallel between conditionals and some other construction of a language is their similarity to questions (see Bhatt and Pancheva [2006]). For instance, overt marking of the protasis appears to be the common cross-linguistic strategy to express conditionals. But many languages use to this purpose temporal *wh*-pronouns (e.g. German *wenn*). Other common lexical devices are interrogative complementizers/operators, e.g., English *if* is also used in embedded yes/no questions (the examples are taken from Bhatt and Pancheva [2006]).

\[(4) \quad \begin{align*}
\text{a.} & \quad \text{Peter wondered if Sue called the doctor.} \\
\text{b.} & \quad \text{Znae li anglijski, ste go vzemat na rabota.} \\
& \quad \text{know-3sg Q English will him take to work.} \\
& \quad \text{(Cuda se) znae lie anglijski?} \\
\text{c.} & \quad \text{wonder-1sg refl know-3sg Q English?}
\end{align*}\]

I-to-C movement is another formal mechanism for forming antecedents of conditionals, often employed by languages in the absence of any other indicator, lexical or morphological, of conditional marking (see (5), also from Bhatt and Pancheva [2006]). I-to-C movement is also a very common strategy to express questions. Iatridou and Embick [1994] have argued that languages that exhibit I-to-C movement in conditionals also have it in questions.

\[(5) \quad \begin{align*}
\text{a.} & \quad \text{Had he been here in time, we would have helped him.}
\end{align*}\]

\(^2\)It should be noticed that working within this framework automatically means that in a strict sense we will not obtain compositionality for the surface form. The translation rules from Spell-Out to Logical Form destroy surface compositionality. Only the interpretation of the Logical Form itself is truly compositional (see Jacobson [1995] for more discussion).
b. Wäre er rechtzeitig hier gewesen, dann hätten wir ihm geholfen.
c. Hast du was, dann bist du was.

Based on these observations it seems quite clear that there is a strong syntactic, and, therefore, probably also semantic link between the two types of constructions. Bhatt and Pancheva [2006] formulated the following generalization (see also Izvorski [2001]): *Interrogative adjunct clauses are interpreted as conditionals.* This has also led to the development of theories that link the way the meaning of both constructions is constructed to each other. Kayne [1991], for instance, claims that *if* in embedded polar questions and conditionals is the same item, with the same meaning. A similar claim can be found in Larson [1985]. On top of some empirical problems, these theories are not useful for our purposes. They generally claim that up to some point the semantics of the antecedent of conditionals and questions are identical (they both involve predicates over possible worlds, this is what is expressed by the overt and identical formal material). But the meaning of both constructions obviously differs. So their compositional semantics needs to differ as well, which is already a problem for the methodology adopted here. Furthermore, these authors then derive the semantic differences by assuming different covert material for both cases. But that means that these theories cannot help us to get at grip on the part of the construction that leads to the conditional interpretation, because that is the covert part they stipulate.

### 2.2.2 The link with definite descriptions

Given that so far looking on similarities in form wasn’t that successful, one might consider looking for constructions that agree with the semantic properties of conditionals instead. A very strong match appears in this case to exist between conditionals and definite descriptions. As has been already noticed in Lewis [1973], but more explicitly discussed in Schlenker [2004] definite descriptions and the antecedent of conditionals share a very specific semantic behavior. Characteristic properties of reasoning with conditionals appear to extend to reasoning with definite descriptions as well. Schlenker [2004] discusses in particular **Strengthening of the Antecedent, Contraposition, and Transitivity.**

3 These properties do not hold of natural language conditionals; they behave non-monotonic. Schlenker [2004] argues that the same properties hold of definite descriptions as well.

Based on these observations one might suggest to build on approaches to definite descriptions when trying to account for conditionals. That’s exactly what Schlenker [2004] proposes: a referential analysis of the antecedent of conditionals. *If*-clauses of conditionals are analyzed as plural definite description. Both, definite descriptions and conditionals are interpreted using similarity functions. 4 If is proposed to be a definite article for possible worlds. It introduce

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3 **Strengthening of the Antecedent:** If $f, y$, then $f \land f, y$

**Contraposition:** If $f, f, y$, then $f \land 
eg y, \neg f$

**Transitivity:** If $f, f, y$ and $f, y, c$, then $f, f, c$

4 Schlenker [2004] emphasizes that all the data support so far is that both constructions
a $\iota$-operator that given a similarity function, a world (the evaluation world of the conditional) and a proposition (the proposition denoted by the antecedent) returns the unique set of all possible worlds that make the proposition true and are maximally similar to the world. This object the if-clause refers to is then taken to be the argument to which the proposition expressed by the consequent is applied, i.e. it provides the parameter at which the consequent proposition has to be evaluated to get the truth value of the conditional.

1. meaning of if $\lambda P.\lambda w \iota w'. \text{Rev}(w, P)$
2. meaning of the argument of if (antecedent) $\lambda w. A(w)$
3. meaning of If $A$ $\lambda w. w'. \text{Rev}(w, A)$
4. meaning of the consequent $\lambda w. C(w)$
5. meaning of the conditional $\lambda w. C(ww'. \text{Rev}(w, A))$

This analysis can very nicely explain the semantic observations. It also has another advantage. The interpretation of conditionals is often described informally as follows: first, the interpretation if the antecedent introduces a hypothetical scenario in which the antecedent is true; then the consequent makes a statement about this scenario; namely that in this scenario the consequent holds as well. A referential approach according to the lines of Schlenker [2004] very nicely implements this intuition.

But the analysis also has a couple of shortcomings. First of all, the approach is not completely developed. For one thing, Schlenker [2004] does not provide a discussion and formal treatment of the potential plurality of possible worlds.\(^5\) Another thing that is missing in Schlenker [2004] is a serious syntax/semantics interface. Though the proposal provides a separate semantics for if and distinguishes separate contributions for antecedent and consequent, the semantic composition is not based on a syntactic analysis of conditional sentences. The analysis of if as definite article also leads to empirical problems when adverbs of quantification are involved. Basically, in this approach if-clauses with adverbs of quantification are analyzed in parallel with definite description in scope of a quantifier (see (6)).

(6) a. Always if it is raining, I forgot to bring my umbrella with me.
   b. All the women in the office are smokers.

Within the current framework, this means that the quantifier gets as restrictor not all the worlds for which the proposition of the antecedent holds, but only those worlds selected by the if-clause, i.e. the worlds most similar to the actual world. This seems to make the wrong predictions: in examples from Schlenker [2004] necessarily appears to take the entire domain as argument, not just the most similar cases. The approach cannot account for this observation.

\(^5\)Klinedinst [2009] addresses this problem of Schlenker [2004] and proposes an extension of the approach with pluralities of possible worlds. This proposal differs in some respects from what will be proposed here. We will point out the differences in section 3.
(7) a. *Necessarily, if the United States threw its weapons into the sea, there would be war. However (, necessarily,) if the United States and all other nuclear powers threw their weapons into the sea, there should be war.

b. Necessarily, if the United States threw its weapons into the sea, there would be war. Therefore, necessarily, if the United States and all other nuclear powers threw their weapons into the sea, there should be war.

There are different ways to go from here. We could say that compositionality doesn’t work, or that for some reason (structured meanings) the adverb of quantification can access meanings further down in the construction. A more plausible explanation is that if just is not a definite article for possible worlds. But then we still have to explain why conditionals and definite descriptions share so many semantic properties. Actually, Schlenker [2004] provides a number of additional arguments for his analysis that are of help here. He shows that conditionals also share with definite description a bunch of other properties related to the form of these constructions. In particular, he observes that as definite descriptions if-clauses can appear in left-dislocated positions, while quantifiers and simple restrictors can’t. Furthermore, if-clauses show condition C effects. But these properties are not specific to definite descriptions, these are properties of referential expressions in general. So, maybe we need to give up the idea that if is a definite article, but still can keep the referential approach to if-clauses.

2.2.3 The Free-relative link and related connections

So far, we have seen that conditionals are in form strongly related to question, but that this parallelism does not extend to their semantics. Furthermore, we saw that with respect to semantics conditionals are very similar to definite descriptions, but this observation alone does not explain how to derive the meaning from the form of conditionals. Finally, we saw that some of the similarities between if-clauses and definite descriptions appear to be similarities with referential expressions in general. Is there a way to bring all these observations together? That means, can we find a construction that looks very similar to questions, but shares the semantic behavior, and therefore also certain syntactic properties with referential expressions, in particular definite descriptions?

Indeed, such a construction exists: Free Relatives. Free relatives are relative clauses without any overt DP-head. Instead, it has been argued that the head of the phrase is the wh-element in the Spec position of the CP. Nevertheless, free relative obtain the interpretation of definite descriptions (and not questions) (see Grosu and Landman [1998] for a more detailed discussion). Thus, free relative look like questions (because the DP-head of the relative clause is missing), but are interpreted like DPs. In particular, they show all the properties of referential expressions.

This is thus the link that we will explore: we will analyze if-clauses as free
relatives for possible worlds. In the next section we will proceed as follows. We will first introduce a compositional semantics for free relative clauses in English, and then, in the second step, try to use this approach also for the compositional semantics of conditionals. We will see that in the end, we produce a very similar interpretation for if-clauses as predicted by Schlenker [2004]'s approach. But this time, the interaction with adverbs of quantification is less problematic. However, in order to spell out the details of this approach we will have to deal with the limitations of Schlenker [2004] discussed above: we will have to provide a syntax/semantics interface, and we will have to say what plurality means on the level of possible worlds. But before we go into the details of the approach we will first discuss more evidence establishing the link between if-clauses and free relatives, and we will have a short look on other theories exploring this link.

2.2.4 More empirical support

Additional support for the idea to analyze conditional sentences parallel to free relatives comes from similarities with two other constructions: topic constructions and correlative constructions. Both constructions are closely related to free relatives. A correlative is a particular type of a free relative construction: in this case the free relative clause is adjoined with the matrix clause and co-indexed with a proform inside it (cf. Srivastav [1991], Dayal [1995]).

(8) a. \( [CP[free\ relative]_{i}[CP...proform_{i}...]] \)
   b. \( [CP...[free\ relative]_{i}...] \)

In many languages conditionals are correlative structures themselves or are historically derived from correlative constructions. As Geis and Lycan [1989] point out, conditionals with then are the last remnants in English of a once productive strategy. The if-clause is the correlative clause, and then is a correlative proform. In languages where correlativization is a productive strategy, it is apparent that conditionals are correlatives (e.g. Marathi).

(9) (from Pandharipande [1997])
   a. (dzar) tyāne abhyās kelā tar to pā hōl
      if he-ag studying do.Pst.3MSg then he pass be.Fut.3S
      If he studies, he will pass (the exam).
   b. dzo mānūs tudzhya ō sēdzār rāhto to mānūs lekhak āhe
      which man your neighborhood-in live-Prs.3MSg that man writer is
      The man who lives in your neighborhood is a writer.

From this link we can get further evidence for the approach pursued here. It appears that the semantics of then is closely related to (if not identical with) the semantics of the pro-form in correlative constructions and other syntactically and semantically strongly related left-dislocation constructions, for instance in German and Dutch. For a detailed discussion of the semantics of then and the link see Izvorski [1996], Iatridou [1994], Ebert et al. [2008].
Finally, correlative constructions also share with if-clauses quantification variability effects (QVEs). It is well known that conditionals allows for quantificational variability effects.

(10) If a student works hard, she usually does well.
    QVE Reading: Most students who work hard do well.
    Non-QVE Reading: Students who work hard do well most of the time.

QVE effects are found with correlatives/free relatives in general. Dayal [1995] notes that QVE readings are also available even when the correlativized phrase is singular. The relevant factor seems to be the presence of genericity in the correlative clause. If the environment is made episodic, the QVE reading disappears. A similar sensitivity to the presence of genericity is also found in conditionals. This example from Bhatt and Pancheva [2002] is in Hindi because the verbal morphology allows us to detect the presence of genericity.

(11) episodic tense, no QVE reading
    (agar koi laRkii tez hai),[to vo aksar safal ho-tii hai]
    if some girl smart is then Dem often successful be-Hab.f is
    Non-QVE Reading: If some girl is smart, then she is successful on most occasions.
    (*QVE Reading: Most girls who are smart are successful.)

(12) generic tense, QVE reading available
    (agar koi laRkii tez ho-tii hai),[to vo aksar safal ho-tii hai]
    if some girl smart be-Hab.f is, then Dem often successful be-Hab.f is
    Non-QVE Reading: If some girl is smart, then she is successful on most occasions.
    QVE Readings: Most girls who are smart are successful.

The reason that here we build on the free relative link instead of the correlative link is that at least in English only a subgroup of the class of English conditionals we are discussing here appear to relate to correlative constructions. These are event-conditionals with then. To capture the entire class of event conditionals we have to relate to the class of free relatives in general and not just the subgroup of correlatives.

2.2.5 Approaches exploring the link to Free Relatives

To my knowledge there is only one approach that explicitly claims to build on the link to correlative constructions in order to derive a compositional semantics for conditionals. This is Alonso-Ovalle [2009]. The goal of this paper is to account for the observation that the inference pattern \((A \lor B) > C\) implies \(A > C\) and \(B > C\), often referred to as Simplification of the antecedent. To account for this pattern, Alonso-Ovalle [2009] propose an approach that according to him combines a alternative semantics for or with a correlative semantics for the if clause. The proposal is elegant and interesting. It links in a very innovative way Kratzer’s restrictor approach to conditionals with her work on indefinites: now
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the silent quantifier in bare conditionals is at the same time the silent operator
binding the variable introduced by or in the antecedent. For our purposes,
the approach is less relevant, because it is not in a strict sense a correlative
approach. In particular, the if clause is not analyzed as a correlative clause. So,
the approach does not fully explore the link to correlative constructions.

In Ebert et al. [2008] the similarities between conditionals and certain left-
dislocation constructions in German are discussed. These constructions (Ger-
man Left Dislocation and Hanging Topic Left Dislocation) show many syntactic
and semantic similarities with correlative constructions. Ebert et al. [2008] then
pursue a similar methodology as adopted in this paper and extend a semantic
analysis of these left dislocation constructions to conditional sentences. The
analysis is very interesting and has inspired the proposal developed here. How-
ever, we will not follow this specific approach here. The reason is that as far
as event-conditionals are concerned, this approach only deals with a particular
subgroup of these conditionals: event-conditionals with then in the consequent.
Still, the approach developed here will make very similar predictions, due to the
fact that both approaches essentially take if-clauses to be referential expressions
expressing non-anaphoric definite descriptions.

The leading idea of the present approach, i.e. that the if-clause of condi-
tional sentences has to be analyzed as a free relative for possible worlds, can be
found most prominently in Bhatt and Pancheva [2006]. This paper focuses on
the syntactic properties of conditionals cross-linguistically. This cross-linguistic
perspective leads the authors to follow Schein [2001] and Schlenker [2004], and
propose that if-clauses “involve abstraction over a (possible world) variable.”
[Bhatt and Pancheva [2006], p. 655], just like relative clauses in general. In
an earlier handout Bhatt and Pancheva [2002] they make this more concrete
and derive the interpretation of conditional clauses analogous to standard ap-
proaches towards the interpretation of free relatives of individuals (cf. Jacobson
[1995], Rullmann [1995]). They propose that a type-changing operation of max-
imization applies to the set of possible worlds denoted by the if-clause returning
the sum of all element in that set.

(13) what John cooked
    a. $LF : [CP \text{wh}_xC_0[IP Jim cooked } x]]$
    b. $MAX(\lambda x[Jim cooked } x]) = x[Jim cooked } x]$  

(14) if John arrives late
    a. $LF : [CP Op_wC_0[IP John arrives late in } w]]$
    b. $MAX(\lambda w[John arrives late in } w]) = w[John arrives late in } w]$  

As soon as plural objects are involved, the question of predication over these
pluralities arises. Bhatt and Pancheva [2002] observe that similar to distribu-
tivity over definite descriptions/free relatives, also in the case of conditionals
the consequent predicates distributively over the plural object introduced by
the if-clause.

(15) a. Peter will be unhappy [if John arrives late].
b. Peter will be unhappy [in the circumstances that John arrives late].
\[(\equiv \forall w \left[ w \text{ is an atom of } \text{MAX} \left( \lambda w' \left[ \text{John arrives late in } w' \right] \right) \rightarrow \text{Peter will be unhappy in } w \right])\]

The present paper can be best seen as an elaboration of the ideas introduced in Bhatt and Pancheva [2002, 2006], developing the underlying syntax/semantics interface and working through the consequences of having plural definite descriptions of possible worlds.

2.2.6 The restrictor approach and Dynamic Semantics

As has been pointed out in section 1, there has been in the recent years a growing interest in the compositional semantics of conditional sentences. The majority of this research builds on the restrictor approach to conditionals proposed by Kratzer [Kratzer, 1979, 1991]. The restrictor approach mainly consists of the three claims given in figure 1.

claim 1  The semantic function of if-clauses.
The role of the if-clause of a conditional is to provide domain restrictions for quantificational operators.

claim 2  The discourse function of if-clauses.
if-clauses affect a contextual or evaluation parameter that subsequent modals/quantifiers depend upon.

claim 3  Bare conditionals.
If a sentence contains an if-clause, then it also contains a quantificational operator restricted by the if-clause.

Figure 1: The restrictor approach

For most part (claim 1 and 2) the theory is not concerned with the compositional semantics of conditionals, but describes the over-all semantics function of the if-clause. These two things should not be confused: one can have a very proper and useful description of the semantic function of a certain constituent, without knowing how this function is derived from the structure of the expression. As descriptions of the semantic function of if-clauses the first two claims are convincingly supported by observations in Lewis [1981] and Kratzer [2012]. Any compositional approach to conditionals should predict an over-all meaning for conditionals that satisfies these claims. The fact that we want to capture these two claims did affect the proposal made in this paper. In particular, claim 2 has been decisive for the choice of the formal framework in which the semantics will be spelled out here. We will work with dynamic semantics to be able to account for the anaphoric relations if-clauses can give rise to.

Claim 3 is concerned with meaning composition. This claim is a straightforward extension of claim 1 to the case of bare conditionals. It is often read as meaning that at the level of LF bare conditionals contain a covert universal quantifier. The empirical support for this claim is rather meager compared to the other two. It mainly consists of showing that by assuming claim 3 one can
account for the data. Therefore, we will not adopt this claim as a restriction on a compositional approach to conditionals. This doesn’t mean that we might not end up with a theory that renders it true. But we will not assume it to be true from the start.\footnote{The decision to not adopt claim 3 sets the present proposal apart from many other compositional approaches to conditionals in the recent literature. Because of this difference, these alternative approaches will not be discussed here. They belong to a different line of approach.} In section 4.1 we will discuss how exactly the present approach relates to claim 3 of the restrictor approach.

3 A compositional semantics for conditionals

As said above, this paper adopts as guiding hypothesis for the syntax-semantics interface a proposal of Bhatt and Pancheva [2006] and analyzes the antecedent of conditionals as free relatives for possible worlds. The goal of the paper is to spell out the details of such an approach. More precisely, the question is whether we can take a solid analysis of the syntax and semantics for free relatives and straight forwardly apply it to conditional sentences. However, for this idea to work we first need a adequate analysis of free relatives. Thus, we will start in subsection 3.1 introducing a compositional semantics for free relatives. This part will not be innovative, we will follow the literature here. But we will have to make a motivated choice given that there exists different analysis for the semantics of free relatives. In subsection 3.2 we will transfer the syntactic part of the analysis to the case of conditional sentences. More in particular, we will define a surface structure and a Logical Form for conditional sentences based on analyzing the *if*-clause of a conditional as free relative for possible worlds. In subsection 3.3 we will then deal with the question how to transfer the semantics to the case of conditionals as well. While the syntactic part is quite straight forward, the translation of the semantics will be more demanding. In the final part of this section, subsection 3.4, the proposal will be illustrated by applying it to a couple of examples.

3.1 The syntax and semantics of free relatives

The sentences in (16-a) and (16-b) provide standard examples for what Grosu and Landman [1998] call free relative (FR).\footnote{More precisely, these are examples for what they call *realis* free relatives. Grosu and Landman [1998] also distinguish *irrealis* free relatives marked by an irrealis verb-form in the relative clause. We are interested in realis FRs.} Characteristic for FRs is that they contain no overt lexical head. Instead, semantically the head noun of the clause is the *wh*-expression in the Spec position of the CP of the relative clause.\footnote{There has been an extensive debate on the question whether the *wh*-element of free relatives is placed outside of the relative clause as head of the NP/DP, or whether it stays within the CP that forms the relative clause. Within this paper we adopt the second line of analysis. For arguments see Grosu and Landman [1998], Jacobson [1995] and their references.} FRs can be found in many languages, Caponigro [2004] mentions 29 language. The *wh*-phrase in free relatives can contain an explicit sortal. Therefore, free relatives are in the classification of Grosu and Landman [1998] sortal internal
relatives, in contrast to restrictive relative clauses and appositives, which are sortal externals relatives. An important feature that distinguishes sortal internal relatives from sortal external ones is that they do not allow stacking (Grosu and Landman [1998], p. 126). Characteristic of realis free relatives is that they show the same distribution normal DPs.\(^9\)

\[(16)\]
\begin{enumerate}
  \item What Jim gave to John was a shiny dagger. [Grosu and Landman [1998]]
  \item What few students came to the concert left before the encores began.[Grosu and Landman [1998]]
\end{enumerate}

The analysis Grosu and Landman [1998] propose for free relatives mainly builds on two claims.

- Realis free relatives contain a full DP structure with a \textit{pro} head, and
- the interpretation of realis free relatives involves a maximization operation at the level of CP.

For the first claim an extensive argumentation is given in Grosu [1994]. Main motivation is to explain the strongly DP-like behavior of free relatives. The second claim is in its basic idea not original to Grosu and Landman [1998], but can be found, for instance, already in Jacobson [1995], though the implementation differs. In Grosu and Landman [1998] maximality is an operation that applies freely to the meaning of the relative CP in case of free relatives. They introduce the operation in the context of degree relatives, but then extend it to sortal internal relatives in general. In Jacobson [1995] maximality is part of the meaning of special free-relative \textit{wh} pronouns, and thus anchored in the lexical semantics of these expressions. Before we discuss this point in more detail, let us first spell out the analysis of Grosu and Landman [1998].

\textbf{The rules of construal.} In the framework of GB the SF proposed by Grosu and Landman [1998] can be described as figure 2. This SF is then translated into an LF by the Rules of Construal. This involves in the case of FRs \textit{WH}-movement. The \textit{wh}-pronoun moves out of the VP and to the head of the CP, leaving a co-indexed trace (see figure 2, left side). At the landing position only the trace remains.

A second interface principle needed here is the \textit{Principle of Full Interpretation (FI)}: An LF tree contains only material that is relevant for interpretation. In consequence, the information about syntactic categories is deleted at the interface, together with lexical items that do not contribute to interpretation. In our case this means that the empty head of the adverbial phrase disappears. The result is given in figure 2, right side.

\(^9\)Grosu and Landman [1998] claim that they are not allowed in contexts of indefiniteness, i.e. contexts which show definiteness effects (\textit{there be}, relational \textit{have}, etc.), though this does not hold cross-linguistically, as argued in Caponigro [2004].
The grammar of LF. The LF is then interpreted. In order to define the rules of the compositional interpretation we need a clear (and if possible recursive) definition of the general formal language $L$ in which the LF is formulated. For a large part the syntax semantics of LF is defined along standard lines (see Heim and Kratzer [1998]). The basic types of $L$ are $e$ (individuals) and $t$ (truth values). The functional types are generated by the rule: if $a$ and $b$ are types, then $(ab)$ is a type. Outermost brackets are usually omitted. We have infinitely many variables for any type. Definition 1 describes the syntactic rules of $L$. We follow Heim and Kratzer [1998] and assume that world variables are indexed. For now we take the indices to be natural numbers. The $\lambda$-abstractor is represented by a bare index.

**Definition 1** *The Syntax of $L$.*

$L$. If $\alpha$ is a lexical tree of type $x$, then $\alpha$ is a tree of type $x$.

$\text{Var.}$ If $v$ is a variable, $n$ is a natural number, and $x$ is a type, then $[x \, v_n]$ is a tree of type $x$.

$\text{FA.}$ If $\alpha$ is a tree of type $xy$ and $\beta$ is a tree of type $x$, then $[y \, \alpha \beta]$ is a tree of type $y$.

$\lambda$. If $\alpha$ is an IL-tree of type $y$, $n$ is a natural number, and $x$ is a type, then $[xy \, n.\alpha]$ is a tree of type $xy$.

We also adopt the standard interpretation rules for this formal language. Meaning is assigned to the expressions of $L$ via an interpretation of the constants and variables. From this the meaning of complex expressions is computed using recursive interpretation principles for the rules of construction (definition 6). The interpretation of the constants is given in terms of a model. In our case, a model is a triple $\mathcal{M} = (E, \{0, 1\}, F)$, where $\{0, 1\}$ are the truth values and $F$ is a function that assigns appropriate meanings to the elements of the lexicon. $E$ is a structure $(E, \subseteq)$, where $E$ is a set of individuals and $\subseteq$ is the part-of relation on $D$. The system of semantic domains $D_a$ for each type $a$ is defined as follows:
\[ D_e = E \text{ (the set of (plural individuals),) } D_t = \{0, 1\}, \text{ and } D_{(ab)} \text{ is the set of (possible partial) function from } D_a \text{ into } D_b. \] 

The interpretation of the variables is given by an assignment relative to a model. An assignment \( a \) is a (partial) function from natural numbers to objects of the model. The definition of the function \([·]^M, a\) that interprets the complex expressions of \( L \) is stated below.

**Definition 2 LF-semantics**

For any LF-tree \( \alpha \) the meaning of \( \alpha \) with respect to model \( M \) and assignment \( g \), i.e. \([\alpha]^M, g\), is defined as follows:

**L.** Let \( \alpha \) be a lexical tree of type \( x \). Then \([\alpha]^M, g\) = \( F^M(\alpha) \).

**Var.** Let \( v \) be a variable of type \( x \) with index \( n \).

Then \([v_n]^M, g\) = \( g(n) \) if \( g(n) \in D_x \), undefined otherwise.

**FA.** Let \( \gamma \) be a branching tree of type \( y \) with daughters \( \alpha \) of type \( xy \) and \( \beta \) of type \( x \).

Then \([\gamma]^M, g\) = \([\alpha]^M, g([\beta]^M, g)\).

**λ.** Let \( \alpha \) be a tree of type \( xy \) of the form \([xy n.\beta]\), where \( n \) is an index and \( \beta \) a tree of type \( y \).

Then \([xy n.\beta]^M, g\) = \( \lambda u \in D_x. [\beta]^M, g[n/u] \).

The syntactic semantic rules are not sufficient to generate the LF of FRs. Problematic is the integration of the semantics of the FR with the rest of the sentence. The DP of the FR has to receive an interpretation that can combine with the meaning of the rest of the clause: it has to denote a generalized quantifier or an individual. Given the lambda abstraction introduced by the moved \( wh \) element, right now it denotes a property of individuals. Both, Grosu and Landman [1998] and Jacobson [1995] assume that the CP – they still distinguish CP and DP at the level of LF – denotes the set containing as single element the sum of all individuals that the predicate of the relative clause can be applied to. For this meaning it is claimed to be only natural to turn to a definite interpretation of the DP. According to Jacobson [1995] this is the result of a type shift: "... it is not at all surprising to find that natural language allows an inherently predicative expression to shift into an individual-denoting expression just in case the set characterized by the predicate is a singleton. Thus if the definite reading of a FR really involves shifting down in this way rather than up, it is no surprise that these end up equivalent to definite descriptions." [Jacobson [1995], p. 466]. Grosu and Landman [1998] are a bit unclear on this point. At an earlier place in the paper they claim that if there is no determiner present in a DP, the interpretation is given by an existential type shift, but in the context of free relatives they seem to assume an implicit definite or universal determiner.

We adopt a different line of approach and analyze FR-constructions on a par with bare nominals, along the lines of Dayal [2004]. With Jacobson [1995] we assume that a type shifting rule applies and shifts the \( et \) meaning into a generalized quantifier of type \( (et)t \). But in contrast to Jacobson [1995] the type shift itself involves maximization and turns a set \( P \) of individuals into the general quantifier denoting the set of properties that the sum of all individuals in \( P \) has. This is already a natural type shift according to Partee [1987]. This idea is spelled out in the present system by adding the following two rules, the first to the syntax, and the second to the semantics of LF.
Definition 3 Type-shift

Syntax S. If $\alpha$ is an IL-tree of type $\epsilon t$, then $[\epsilon t][\alpha]$ is a tree of type $(\epsilon t)t$.

Semant. S. Let $\gamma$ be a tree of type $(\epsilon t)t$ with an only daughter $\alpha$ of type $\epsilon t$.

Then: $[\gamma]_{M,g} = \lambda Q.Q(\iota x.[\alpha]_{M,g}(x) \land \forall y([\alpha]_{M,g}(y) \rightarrow y \subseteq x))$.

This finishes the analysis of free relative. The resulting interpretation of the free relative in example (16-a) is given in (17), where $P$ stands for the property of being given to John by Jim in model $M$. The FR denotes the set of sets containing the plurality of all the objects Jim gave to John.

\[\lambda Q.Q(\iota x.P(x) \land \forall y(P(y) \rightarrow y \subseteq x)).\]

3.2 The syntax of conditional sentences

With the analysis of free relatives in place we can now turn to conditionals and analyze the If-clause as free relatives of possible worlds. This subsection will deal with consequences of this idea for the syntax of conditional sentences up to the level of LF. In the next subsection we will discuss the effects for the semantics of conditionals.

Reanalyzing the if-clause as a free relative means that we will model the internal syntactic analysis of the if-clause following the previously discussed analysis of the free relative. This will work pretty straightforward. The difficult part is to localize the position of the if-clause within the syntactic tree of a conditional sentence, i.e. the external syntax of if-clauses. A completely parallel analysis of if-clauses and free relatives would mean that the if-clause fills an argument slot in the LF of the clause. Given that we want to analyze if-clauses as free relatives for possible worlds it should be the argument slot of a world argument.

Sentences certainly take a world argument: the evaluation world. Normally, this world argument is not visible at the level of the LF but only occurs in the intensional semantics assigned to LF constructions. But it makes sense to make this argument visible at the level of LF: in the end we do not talk intensions, but make claims about facts, i.e. truth values. At some point in an assertion the world argument needs to be filled.

It is a different question whether this slot is in case of conditional sentences filled with the if-clause. For some reasons such an analysis is very attractive. We could then transfer the treatment of tense by Stechow [2010] to the modal domain. Stechow [2010] and many others argue that verbs take temporal arguments. In a parallel fashion we could propose that verbs take world arguments. As with nominal and temporal arguments of verbs, also in the case of the world argument we could have quantifiers occupying the argument positions for the world argument in the VP. These quantifiers could be accompanied by relative clauses describing domain restrictions. This would then be the place of the if-clause.

However, because of the differences in the distribution of adverbs of time and modality, we will refrain from proposing a world argument within the VP domain. Instead, we will assume the following.
External syntax of if-clauses
We assume that sentences take world arguments but higher up in the tree: somewhere inside the CP but above the IP. For reasons of convenience, we assume that it is the head of the CP. But we do not propose that the if-clause fills this argument slot, but rather that it sits lower in the syntactic tree, somewhere in the IP or VP. It is an adjunct clause, function semantically as a modifier of the world argument contributed higher up in the tree. The head of the CP is filled by a silent deictic pronoun it0, which refers to the utterance world.

Internal syntax of if-clauses
For the if-clause internal analysis we follow the approach sketched in section 3.1. If-clauses are adverbial clauses. Just as nominal quantifiers modal adverbs of quantification can come with an relative clause contributing the restrictor argument of the adverbial quantifier.

If is analyzed as a wh-pronoun for worlds. It is treated on a par with standard wh-pronoun. Bare conditionals, i.e. conditionals without an adverb of quantification amount to free relatives: the antecedent is a relative clause constructions without an external head in the AdvP. If we follow exactly the analysis of free relatives from subsection 3.1, bare conditionals get as Spell-Out the syntactic structure sketched in figure 3, right side.

The rules of construal
We assume the same steps in translation into LF that we adopted in case of free relatives. If moves out of its original position within the CP and to the head (or Spec) of the CP, leaving a co-indexed trace. At the landing position only the trace remains. Consequently, the LF structure of (18) looks as given in (19).

\[ [t[x_{it0}]][st[st(st(st))] [st 1.[t[x_{x1} [st Hanna asked]]][st Simon would help]]] \]

The grammar of LF
The next step is to provide a syntax and a compositional semantics for the LF language that generates the formulas given in (19). The definitions of the LF introduced in subsection 3.1 can be easily extended to conditional sentences. We only need to make few small amendments. Instead of e we need a new basic type s for possible worlds. The syntactic rules apply just as before, we only add intensional functional application. Again, we need the type-shifting rule S to account for the transition from relative clause to adverbial phrase. For the exact definition see the appendix, subsection A.

\[ ^{10}\text{[Iatridou} [2000]\text{] argues that sentences-initial if-clauses involve IP-adjunction. According to Bhatt and Pancheva [2006] there is also evidence that at least some sentence-initial if-clauses have a source lower that their surface position. We will assume that adverbial clause origins in the IP, but the semantics analysis that will be proposed can be made work for VP adjunction as well. We assume IP-adjunction mainly because it allows us to ignore the VP-level completely within this paper.}\]

\[ ^{11}\text{In case of free relatives the head was pro. Pro was needed to account for case transmission into the relative clause. Case transmission doesn’t play any role for adverbial clauses. So, we can assume that the external head is empty. Assuming the restrictor approach, one could argue that this is the place where the covert modal quantifier sits.}\]
(18) If Hanna asked, Simon would help.

The next point on the agenda is to provide a compositional semantics for the LFs generated by this syntax. While doing so we will also solve some other problems that still need to be addressed. Firstly, essential to the approach to free relatives discussed in subsection 3.1 is that free relatives are semantically analyzed as plural definite descriptions. We want to transfer this part of the analysis to if-clauses, but that means that we have to make sense of the concept of plural definite descriptions of possible worlds. This will affect the ontology of the semantics we work with. Second, we already discussed in section 2 that in order to treat conditionals properly we need dynamic semantics as formal framework. So while the semantics for free relatives in subsection 3.1 was still defined in a static framework we will now make the switch to dynamic semantics. This point also concerns the ontology of the underlying semantics. Finally, we need to pay some more attention to the type-shifting operator used to interpret free relatives. Certainly when applied to if-clauses, but arguably already for nominal free relatives the operator needs to involve besides maximization also revision, i.e. the selection of maximally similar elements. We maximize only among the most similar elements. This issue concerns not so much the ontology of the semantics, but the definition of the type-shifting principle, i.e. the rules of meaning composition. All these issues will be addressed when defining the semantics in the next section.
3.3 The semantics of conditional sentences

The goal of the present section is to develop a compositional semantics for conditionals that follows the lines of the approach to free relatives. We will have to provide meanings for the basic expressions that can occur in the LF of sentences, and we will have to define rules for the composition of meanings that accompany the syntactic rules of composition.

As explained at the end of the last section, one of the challenges we have to face when defining the semantics for the LF of conditionals introduced above is to translate the concept of plurality from the domain of individuals to the domain of possible worlds. In order to deal with this question we will follow Brasoveanu [2010]. What makes this approach particularly interesting for the present purposes is that it is spelled out in dynamic semantics. Moving towards a dynamic approach to conditionals was the other ontological issue on our agenda. Another advantage this approach has is that it accounts for pluralities of possible worlds without expensive ontological commitments.

The semantic system developed by Brasoveanu [2010] is an example of indirect interpretation (Montague [1970, 1974]). The assignment of meaning proceeds via translation into an intermediate formal language for which a compositional semantics is given. To maintain compositionality of the primary language, the translation procedure must itself be compositional. This extra level of interpretation is in principle dispensable.\footnote{I don't want to attach any claims on cognitive reality to the intermediate representation} The main reason we have it here is that it facilitates working with the rather complex ontology of dynamic semantics. Figure ?? displays the basic structure of the interpretation. Below the different parts of the system will be discussed. We will start with the ontology and define the notion of a model in 3.3.1, then in 3.3.2 introduce the dynamic language used to talk about the model. Finally, 3.3.3 the semantics for the LF will be given, i.e. we will introduce the translation rules that translate LF expressions into the dynamic language describing meaning.

3.3.1 Ontology.

From static to dynamic meaning. Let's start by considering the semantic type of conditional sentences. Conditional sentences are, of course, sentences. What do we want sentences to denote? In a classical static system the type of a sentence is \( t \), the type of truth values. Because the current approach has to be dynamic, the meaning of a sentence is not a truth value, but a \textit{information state change potential} (see, f.i. Groenendijk et al. [1996]) i.e. a complex type corresponding to a set of functions from information states to information states. An information state is a set of objects, representing what according to an agent (or a group of agents) could be the utterance context. Let's call these objects \textit{possibilities}. In the formalization of possibilities we only need to take into account information that is relevant for the present purposes. This is the factual information about the actual world and discourse information about which hypothetical contexts have been introduced.
We work only with local update conditions, i.e. the meaning of all expressions is defined point-wise, on the level of possibilities. We will therefore not define the meaning of sentences as information state change potential, but as possibility change potential. From local update conditions global update conditions can be easily defined: just by quantifying over all possibilities of an information state. The advantage of keeping a local perspective in all conditions is that it simplifies the types and the complexity of the formal expressions needed.

Plurality via sets of assignments. In standard dynamic semantics possibilities are modeled as tuples \((w, a)\), where \(w\) is a possible world, representing what according to this possibility is the factual information, and \(a\) is an assignment function storing the discourse information, i.e. the values of the variables (discourse referents) already introduced in the context. Because we are only interested in discourse referents (drefs) referring to hypothetical scenarios, i.e. possible worlds, in our case the assignments are just functions from natural numbers to possible worlds. This allows us to integrate the factual information into the assignment function. We do this by demanding that the value an assignment associated with the distinguished discourse referent \(u_0\) represents the actual world according to this assignment.

In order to allow for pluralities we follow Brasoveanu [2008] and model possibilities not by single assignment functions, but by sets of assignment functions.\(^{13}\) Thereby, a single possibility can associate a particular dref with a set of values: the values all the assignments in the possibility assign to the dref. This is how

\(^{13}\)Brasoveanu calls our possibilities information states.
plural entities will be represented in the present approach.

**Assignments as primitives.** In order to achieve compositionality we adopt the perspective of Brasoveanu [2008] on the relation between variables and assignments and take the assignments as primitive.\(^{14}\) We introduce a primitive type for assignment functions: type \(s\). Variables (drefs) are perceived as projection functions over assignments. A dref \(u\) will thus take an assignment \(\rho\) as argument and return a possible world. With \(w\) as type of possible worlds, the type of a dref becomes \(sw\). A suitable set of axioms ensures that the atomic entities that assignments now are behave as we expect variable assignments to behave; see the appendix and Muskens [1996], Brasoveanu [2008] for discussion.

**The dummy world and proper possibilities.** With \(t\) as the type of truth values, possibilities are thus of type \(st\) and the type of a sentence, as possibility change potential, becomes \((st)(st)\).\(^ {15}\) To model the fact that assignments might be unspecified w.r.t. certain drefs we follow Brasoveanu [2008] and introduce the dummy world \(\star\). This is the world where every proposition is false. The dummy assignment \(\rho_\star\) assigns the dummy world \(\star\) to every dref. We can use this to define the discourse initial information state – an information state that contains no anaphoric information – as is the set containing only the set \(\{\rho_\star\}\).

For a possibility to be proper, we require that the designated dref \(u_0\) has to return the same value for all assignments of a particular possibility. The dref \(u_0\) stands for the actual world. Hence, the condition comes down to demanding that a possibility to have a uniquely defined value for the actual world. To illustrate the idea let us assume that we distinguish two propositions \(p\) and \(q\). To make the example more concrete lets assume that \(p\) is the proposition that Hanna will asks Simon nicely and \(q\) is the proposition that Simon will help Hanna. Based on these two propositions we can distinguish four possible worlds, as described in figure 5, left part. Figure 5, right part describes an information state \(\sigma\) that contains 4 possibilities, each containing just one assignment function. The columns represent the values the drefs associate with the assignments, i.e. the values the drefs \(u_0, u_1,\) etc receive according to the possibilities. The possibilities differ in what they take the actual world to be. They do not contain any discourse information yet, so all other values of the assignments are \(\star\).

\[
\begin{array}{cccc}
  & w_1 & w_2 & w_3 & w_4 \\
p & 0 & 0 & 1 & 1 \\
g & 0 & 1 & 0 & 1 \\
\end{array}
\]

\[
\sigma = \{ \rho_1 : w_1 \star \star \star \ldots \} \\
\{ \rho_2 : w_2 \star \star \star \ldots \} \\
\{ \rho_3 : w_3 \star \star \star \ldots \} \\
\{ \rho_4 : w_4 \star \star \star \ldots \} \\
\}
\]

Figure 5: An example for an information state

\(^ {14}\)For more information see Brasoveanu [2008] and the literature mentioned there.

\(^ {15}\)Remember that we define all update conditions locally, as possibility change potential.
Static and dynamic types. A nice feature of the formal system introduced in Brasoveanu [2008] is that the types of expressions in an intensional static semantics can be naturally translated into corresponding types of the dynamic language defined here. We can define a type-translation function $T$ from static types to dynamic types recursively as follows: $T(t) = (st)(st)$, $T(s) = (sw)$, $T((xy)) = (T(x) T(y))$. The translation of the basic types is obvious. In the static logic of the LF we have the type $t$ for sentences and $s$ for possible worlds. The corresponding types in the dynamic system are $(st)(st)$ for sentences and $sw$ for drefs of possible worlds. From this we can the easily build the translation of more complex types. A proposition is of type $st$ in the static system, and, consequently, of type $(sw)((st)(st))$ in the dynamic system. A quantifier over possible worlds has as static type $(st)t$, and the corresponding dynamic type is $((sw)((st)(st))((st)(st)))$. Throughout the text we will use the static types as shortcuts for dynamic types.

This finishes the discussion of the ontology underlying the formal semantics we will work with. Two issues that we needed to address in order to account for conditional sentences have been resolved: we switched to a dynamic semantics and we have a way to model pluralities of possible worlds. Pluralities of possible worlds are described as multiple assignments to one dref within the same possibility. This solution comes with the advantage of only weak ontological commitments: we have used what was already available in dynamic semantics; we did not need to introduce entirely new basic types or new relations between objects to account for these pluralities.

### 3.3.2 Plural Compositional DRT

The next step to take is to define a language to talk about this ontology, including interpretation for this language. This is the auxiliary language of figure 4 that we will use to interpret the LF for conditional sentences introduced in section 3.2. We start right away. For any type $\tau$ we have denumerably many constants and a denumerably infinite set of variables. Terms are build from atomic expressions in the standard way: we allow for functional application, lambda abstraction and identity. Also the interpretation of complex terms works completely as expected. There is one additional term-construction: $i[\delta]j$. It is used to express that the assignments $i$ and $j$ agree on all values except of $\delta$.

**Partial identity of assignments.**

**Synt.** $(i[\delta]j)$ is a term of type $\tau$ if $\tau = t$, $i, j$ are variables of type $s$ and $\delta$ is a term of type $\sigma$ for any dref-type $\sigma$.

**Sem.** $[i[\delta]j]_{M,\theta} = T$ iff for all drefs $u \neq \delta$ of type $\sigma$ it holds $ui = uj$ and for all drefs $u'$ of type $\tau \neq \sigma$ it holds $ui = uj$.

With the use of this auxiliary language, known as Ty2 (introduced by Gallin [1975]), we can now define a DRT-style dynamic semantics PCDRT (see Muskens [1996], Brasoveanu [2008]). All the well-known notation from DRT will now
make its reappearance as abbreviations of expressions in Ty2. We will restrict our discussions here to the minimum of formalism needed to define the lexical entries of the expressions occurring in the LF of conditionals. This basically reduces the the lexical entries for VPs. A more extensive description of PCDRT can be found in the appendix or Brasoveanu [2008].

To define the lexical meaning of VPs we need only one type of atomic condition. These are conditions of the form $R\{u_1, \ldots, u_n\}$, where $R$ is some (lexical) relation between possible worlds, and $u_1$, $\ldots$, $u_n$ are drefs for possible worlds. The condition $R\{u_1, \ldots, u_n\}$ is said to hold for a possibility $I$ if all assignments $i$ in $I$ assign worlds $w_1$, $\ldots$, $w_n$ to the drefs $u_1$, $\ldots$, $u_n$ that stand in the relation $R$ to each other. Thus, these conditions are unselectively distributive with respect to possibilities.

Atomic conditions, type $(st)t$ \footnote{We simplify here a bit. To be precise, we should have added to the $\lambda$-term the condition $I_{u_1 \ldots u_n} \neq \emptyset$, where $I_{u_1 \ldots u_n} \neq \emptyset$ stands for the set of assignments in $I$ that do not map $u_1$, $\ldots$, $u_n$ to the dummy world. This to make sure that we only test the relation $R$ on assignments that are defined for $u_1, \ldots, u_n$.}

$$R\{u_1, \ldots, u_n\} := \lambda I_{st}. \forall i \in I(R(i_{u_1}i, \ldots, u_n)),$$

for any non-logical constant $R$ of type $wnt$, where $wnt$ is defined as the smallest set such that $w_0t := t$ and $w_{m+1}t := w(w^mt)$.

Based on this definition of basic conditions we can now define atomic DRSs, i.e. DRSs containing one atomic condition. \footnote{In the present system DRSs are shortcuts for Ty2 expressions of type $(st)t$.} Given an atomic condition $C$, $[C]$ applied to a possibility $I$ will return $I$ if $I$ makes the condition $C$ true (see (20-a)). Also the introduction of new drefs has to be adapted in order to deal with the plural case. Following Brasoveanu [2008] we point-wise generalize the relation $i[\delta]j$ defined between assignments to the level of possibilities, see (20-b). As last basic ingredient of our update system, we need the operator ";" for sequential update. This finishes our short introduction of the auxiliary language Ty2.

\begin{equation}
\begin{align*}
(20) \quad & \text{a. } [R\{u_1, u_n\}] := \lambda I_{st}. \lambda J_{st}. I = J \land R\{u_1, u_n\} J \\
& \text{b. } [u] := \lambda I_{st}. \lambda J_{st}. \forall i \in I(\exists j \in J(i[u]j)) \land \forall j \in J(\exists i \in I(i[u]j)) \\
& \text{c. } D; D' := \lambda I_{st}. \lambda J_{st}. \exists H_{st}(DIH \land D'HJ), \text{ where } D \text{ and } D' \text{ are DRSs of type } (st)((st)t). 
\end{align*}
\end{equation}

### 3.3.3 Interpreting the LF

With our dynamic language in place we can now define the semantics of the LF-expressions. Remember that we will do so using an intermediate language that talks about the model. Thus, we are actually dealing with translation rules instead of interpretation rules. We will do so in two steps. Firstly we will assign meaning to atomic LF expressions, then we will define the rules of meaning composition.

We start with the translation rules for the lexicon. The translation system requires that the syntactic component of the grammar assigns indices to all
referential expressions (see the rule Var in definition 2). Indexed constants of type s are translated into drefs with the same index. Traces left in case of movement (the wh-movement of If) are translated into variables for drefs νs carrying the same index.

Besides referential expressions of type s the LF-lexicon only contains expressions of category VP with type st. In static semantics they denote propositions, i.e., properties of worlds. For instance, they denote the proposition that is true in all worlds were Hanna asks Simon nicely. In our dynamic system this is the type of dynamic properties of discourse referents for worlds. These are functions that given a dref u return a context-change potential. The function checks whether the world(s) that the dref u assigns to a possibility I make the proposition p true. They themselves don’t know which dref they have to check. This is determined by their dref argument. If the dref is u0, then they check whether the world the possibilities associate with u0 make the proposition p true. If the dref is ui for some 𝑖̸= 0 then they check some hypothetical scenario. In sum, expressions of category VP just denote plain update conditions, see below. The translation of basic LF-expressions is summarized in figure 6.

The semantic features of a VP-head
If P is the daughter of a tree of category VP, then \( F(P) = \lambda v. [p\{v\}] \), where v is a variable over discourse referents and p is a propositional constant.

<table>
<thead>
<tr>
<th>LF expression</th>
<th>Category</th>
<th>Type</th>
<th>Translation F(·)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[st Hanna asks]</td>
<td>V</td>
<td>st</td>
<td>( \lambda v. [p{v}] )</td>
</tr>
<tr>
<td>[st Simon will help]</td>
<td>V</td>
<td>st</td>
<td>( \lambda v. [q{v}] )</td>
</tr>
<tr>
<td>[it, n]</td>
<td>pronoun*</td>
<td>s</td>
<td>( ν_n )</td>
</tr>
<tr>
<td>[x, n]</td>
<td>trace</td>
<td>s</td>
<td>( ν_n )</td>
</tr>
</tbody>
</table>

Figure 6: LF expressions and their Ty2 translation

Now we turn to the rules governing the translation of complex expressions. This means that we need to address the final challenge on our way to a compositional treatment of conditionals. As already discussed in section 3.1; in order to get the output we want we need to integrate into the semantics f the type-shift operator selection functions for maximally similar worlds. Fortunately, it turns out that switching to a revision operator selecting maximally similar worlds automatically incorporates the maximality operation of the type-shifter of free relatives.

In case of free relatives, the type shifter turned a property of individuals into the plurality of objects having this property. In case of the if-clause we shift from a property of possible worlds (i.e., a proposition) to the plurality of worlds having this property (i.e., making the proposition true) that are closest to the
utterance world. This means that in order to define this type shift we need to be able to compare the similarity of sets of possible worlds. This similarity relation for sets will be defined based on a similarity relation on possible worlds and the subset relation. Because of the second part optimizing along the order will maximize along $\subseteq$ as well.

For the dynamic version of the type shifter we define an operation $\text{sim}^u$ that given an update condition $D$ introduces a new dref $u$ and maps it to the most similar set of worlds associated with $u$ after a plain update with $D$. The update condition $\text{sim}^u(D)$ is intensional, relative to a a dref $u'$ that defines the center of the similarity order.

\begin{definition}[similarity update]
1. $S \leq^1_{w_0} S'$ iff $\forall w' \in S' \exists w \in S : w \leq_{w_0} w'$ and $\forall w \in S \forall w' \in S' : w' \neq_{w_0} w$.
2. $S \leq^2_{w_0} S'$ iff $S \leq^1_{w_0} S'$ and if $S =^1_{w_0} S'$ then $S' \subseteq S$.
3. $\text{sim}^u(D) := \lambda u'. \lambda I. \lambda J. \langle ([u]; D)IJ \wedge \neg \exists K_{st}(([u]; D)IK \wedge uK_{u \neq *} <^2_{w_0} uJ_{u \neq *}) \rangle$, where $uI := \{ui : i \in I\}$.
\end{definition}

With this operation at hand we can now (re-)define the interpretation/translation rules for complex LF expressions. The translation rules together with the interpretation of Ty2 expressions finally allow us to assign meanings to the LFs generated in section 3.2.

\begin{definition}[Rules of translation]
For any IL-tree $\alpha$ the translation of $\alpha$ into PCDRT, $T(\alpha)$, is defined as follows:

L. Let $\alpha$ be a lexical tree of type $x$. Then $T(\alpha)$ is specified in figure 6.

Var. Let $t$ be a variable of type $x$ with index $i$. Then $T(t_i) = v_i$.

FA. Let $\gamma$ be a branching tree of type $y$ with daughters $\alpha$ of type $xy$ and $\beta$ of type $x$. Then $T(\gamma) = T(\alpha)(T(\beta))$.

IFA. Let $\gamma$ be a branching tree of type $sy$ with daughters $\alpha$ of type $s(sx)y$ and $\beta$ of type $sx$. Then $T(\gamma) = \lambda u.T(\alpha)(u)(T(\beta))$.

$\lambda$. Let $\alpha$ be a tree of type $xy$ of the form $[xy i. \beta]$, where $i$ is an index and $\beta$ a tree of type $y$. Then $T(i. \beta) = \lambda i.T(\beta)$.

Shift. Let $\gamma$ be a tree of type $s(st)t$ with an only daughter $\alpha$ of type $st$. Then: $T(\gamma) = \lambda v. \lambda P_{st}. \text{sim}^u(T(\alpha)(u))(v); P(u)$.
\end{definition}

\footnote{In the long term the limitation to the utterance world as center of similarity needs to be removed.}

\footnote{This definition differs from Klinedinst [2009] in the second condition of the order. We have chosen the particular definition given here, because we want to allow for partial orders. Another difference is that Klinedinst [2009] employs an independent operation of maximization, while we have build maximization into the order, but that has no effect for the outcome.}

\footnote{$I_{u \neq *} := \{i \in I \mid ui \neq *\}$.}
3.4 Bringing everything together: some examples

The purpose of the present section is to illustrate the rather complex theory introduced in this paper via the computation of some concrete examples. We start with a plain sentence without if-clause (21-a). (21-b) and (21-c) provide Spell Out and Logical Form for the sentence, (21-d) gives the translation of the LF into the dynamic language.\(^{21}\)

\[(21)\]
\[\text{a. Simon will help Hanna.}\]
\[\text{b. } [CP\![c_0t_0]\!][\!IPSimon will help Hanna\!]]
\[\text{c. } [s_1t_0][\!stSimon will help Hanna\!]]
\[\text{d. } \lambda v._q\{(v)\}(u_0) = [q\{u_0\}]\]

The result is a dynamic proposition. It selects possibility where the expressed proposition is true in the context the dref \(u_0\) refers to, i.e. the proposition is true in what according to this possibility is the actual world. In this case there is no effect of the potential plurality of a possibility. When defining the notion of possibility we demanded that all assignments of the same possibility have to associate the dref \(u_0\) with the same world. If we, for instance, update the possibility \(\sigma\) from figure 5 with the condition expressed by clause (21-a), we will end up with an information state \(\sigma'\) that only contains the possibilities \(I_2\) consisting of the assignment \(\rho_2\) and \(I_4\) only consisting of the assignment \(\rho_4\). The other two possibilities will be eliminated.

The sentence could also have made a statement about some hypothetical scenario \(u_n\) with \(n \neq 0\).\(^{22}\) In this case, we have basically the same eliminative update effect: the sentence selects possibilities \(I\) such that for all of their assignments \(i\) it holds that the world \(u_{n_i}\) makes \(q\) true. In case of a hypothetical context the values the assignments of a possibility \(I\) associate with \(u_n\) can differ. Still, all of these values have to make \(p\) true. This is where the unselective distributivity of update conditions comes into play. It introduces a universal quantifier over all atoms of some plural entity.

\[\text{Let's now have a look at the conditional sentence (22-a). (22-b) and (22-c) provide Spell Out and Logical Form for the sentence, ?? gives the translation of the LF into the dynamic language. We will use } H \text{ to shorten the expression Hanna asks Simon nicely and } S \text{ to shorten Simon will help Hanna.}\]

\[(22)\]
\[\text{a. If Hanna asks Simon nicely, he will help her.}\]
\[\text{b. } [CP[c_0t_0]\!][\!IPAdvP[Adv][CP[Xwh\!-\!it_0]\!][\!IPH]\!][\!IPS\!]]\]
\[\text{c. } [s_1t_0][\!st(s(st))\!][\!s_1s_1\!][\!stH\!][\!stS\!]]\]
\[\text{\(^{21}\)We still use the conventions for the proposition letters } p \text{ and } q \text{ of figure 5. Thus } q \text{ expresses the proposition that Simon will help Hanna.}\]
\[\text{\(^{22}\)Notice, by the way, that Stalnaker's analysis of the indicative mood we have adopted here implies that sentences in the indicative mood cannot be about hypothetical contexts that are not subsets of the common ground. Given that according to the truth conditions of the clause the result of revision has to contain the hypothetical context itself, the use of the indicative mood would come down to demanding that the hypothetical context has to be a subset of the common ground.}\]
Again, the result is a dynamic proposition. We consider the effects of updating a possibility \( I_1 \) containing a single assignment function \( i_1 \) that maps \( u_0 \) to the world \( w_3 \) and everything else to \(*\) (see \( I_1 \) in figure 8; we use the definition of \( w_1, \ldots, w_4 \) from figure 5). The semantics of the maximality operator is rather complex (see definition 4 on page 27). So lets build it up step-wise, first interpreting the update conditions \( \{u_1\} \) and \( p\{u_1\} \) and afterwards selecting optimal results out of the update. We start with considering the condition \( [u_1](I_1) \). Given the rule in (20-b) this update will lead to all the possibilities \( I_2 \) that may differ from \( I_1 \) only in the values considered for \( u_1 \). Let call the set of all possibilities \( I_2 \) for which it holds \( I_1[u_1]I_2 \) the information state \( \sigma_2 \). Because a possibility can contain more that one assignment, there will be a lot of variation in the update: for each element \( X \) of the powerset of \( \{w_1, w_2, w_3, w_4, *\} \) there is a corresponding possibility in the output of \( [u_1](I_1) \) consisting of the assignments giving for \( u_0 \) the value \( w_1 \) and for \( u_1 \) one of the elements of \( X \) (see \( \sigma_2 \) in figure 8).

Next, we consider the information state \( \sigma_3 \) that results from updating all the possibilities in \( \sigma_2 \) with the condition \( p\{u_1\} \). This condition eliminates all possibilities from \( \sigma_2 \) that contain an assignment \( i \) associating with \( u_1 \) a world \( u_1i \neq * \) that doesn’t make \( p \) true. The proposition \( p \) is false at world \( w_1 \) and world \( w_2 \). Thus the remaining possibilities \( I_3 \in \sigma_3 \) are such that \( u_1I_3 \subseteq \{w_3, w_4, *\} \) (see figure 8).

Now lets go back to the possibility \( I_1 \) and consider the information state \( \sigma_4 = \{I_4 \mid sim^{u_1}(p\{u_1\}) (u_0)I_4I_1 \} \). Given definition 4, this condition only keeps possibilities \( I_4 \in \sigma_3 \) that assign to \( u_1 \) a set of worlds maximally similar to \( u_0 \). No matter of how the similarity between worlds is defined, we did assume that \( w_3 \), the center itself, is part of the set of maximally similar worlds. So, there are only two possible similarity orders here. Either the set of worlds most similar to \( w_3 \) that makes \( p \) true just contains \( w_3 \) or it also contains \( w_4 \). Then \( \sigma_4^1 \) and \( \sigma_4^2 \) are the respective results for the two possible selection functions.
Finally, we consider the update effects of the consequent. As pointed out when discussing the plain sentence (21-a), the condition \( q\{u_1\} \) amounts to erasing all possibilities that contain at least one assignment \( i \) such that \( u_1 i \) makes \( q \) false. Because of weak centering, all possibilities that can result from the update with the antecedent have to contain the world \( w_3 \) where \( q \) is false. Hence, updating with the consequent results in the empty information state. In other words, the sentence (22-a) is predicted to be false in \( I_1 \).

Given the constraints formulated at the beginning of the journey, this is the intended results. The predicted truth conditions are exactly the truth conditions of the similarity approach. Furthermore, this condition can be used as update condition on information states. Updating an information state with a conditional would then mean to erase all those possibilities where the conditional is predicted to be false. In the present case all possibilities would be erased that take the actual world to be \( w_3 \).\(^{23}\) This gives for indicative conditionals the intended epistemic effect that the interpreter learns that it is not possible that Hanna asks Simon nicely, but he does not help her.

\[
I_1 = \{w_3, *, *, ..., \}
\]

\[
\sigma_2 = \begin{cases} 
\{w_3, *, *, ..., \} \\
..., \\
\{w_3, w_1, *, ..., \} \\
\{w_3, w_2, *, ..., \} \\
\{w_3, w_3, *, ..., \} \\
..., \\
\end{cases}
\]

\[
\sigma_4 = \begin{cases} 
\{w_3, w_3, *, ..., \} \\
..., \\
\{w_3, w_4, *, ..., \} \\
..., \\
\end{cases}
\]

\[
\sigma_3 = \begin{cases} 
\{w_3, *, *, ..., \} \\
\{w_3, w_3, *, ..., \} \\
\{w_3, w_4, *, ..., \} \\
..., \\
\end{cases}
\]

\[
\sigma_4^2 = \begin{cases} 
\{w_3, w_3, *, ..., \} \\
..., \\
\{w_3, w_4, *, ..., \} \\
..., \\
\end{cases}
\]

Figure 8: The different stages of updating with example (22-a)

As a final example we discuss the update effects in case the conditional is uttered in a world where the antecedent is false. We take as concrete conditional the sentence (23-a). The fact that the sentence is not uttered in the indicative mood has no effects on the LF and its interpretation, because this approach does not take mood into account. As a result the calculation of SO, LF and

\(^{23}\)Depending on the information state and the chosen revision function, it might erase other possibilities as well.
semantics is the same as for example (22-a). But we do see a difference in the
result when the encoded update condition is applied to a context where the
antecedent of the conditional is false.

\[(23)\]

a. If Hanna had asked Simon nicely, he would have helped her.

b. \[CP[\text{it}0][IP[AdvP[AdvCP[XP[X \text{wh} - \text{it}][IP[H]][PS]]]]]\]

c. \[\text{it}1[\text{at}][\text{at}[\text{at}][\text{at}][\text{at}]]\]

d. \(\text{sim}(\{p\}{u})\)(\{q\}{u})

We consider the effects of updating a possibility \(I_1\) containing a single assign-
ment function \(i_1\) that maps \(u_0\) to the world \(w_1\), where the antecedent is false,
and everything else to \(*\). The effects of updating the possibility with the con-
dition \([u_n][p\{u_n\}]\) are the same as previously described (see figure 8), the only
difference is that now \(u_0\) is always mapped to \(w_1\) instead of \(w_3\) in the previous
case. We consider the information state \(\sigma_3\) that results from updating \(I_1\) with
the condition \(\text{sim}^a([p\{u\}])\)(\(u_0\)). The condition \(\text{sim}^a([p\{u\}])\)(\(u_0\)) eliminates all
possibilities from \(\sigma_3\) that do not associate \(u_1\) with a set of worlds maximally
similar to the set of worlds they associate with \(u_1\), i.e. \(\{w_1\}\). The outcome now
depends on the exact similarity relation assumed. Lets assume that there is
some law-like relation between Hanna asking Simon and him helping her. Thus,
worlds in which \(p\) holds are normally also worlds in which \(q\) is the case, and,
hence, \(w_4\) is more similar to the actual world than \(w_3\). Under this assumption
the result of update is \(\sigma_4\), see figure 9.

Finally, we consider the update effects of the consequent. Updating \(\sigma_4\) with
\([q\{u_1\}]\) amounts to erasing all possibilities that contain at least one assignment
\(i\) such that \(u_1\) makes \(q\) false. However, this is the case for neither of the remaining
possibilities in \(\sigma_4\). Thus, updating \(I_1\) with (23-a) results in the information state
\(\sigma_4\). This is a non-trivial information state, so the sentence (22-a) is predicted
to be true in \(I_1\). This is exactly what we expect in the described context.

4 Discussion

4.1 Relation to the restrictor approach

The predictions the proposal of section 3 makes for the over-all meaning of condi-
tional sentences are conform with the main claims of the restrictor approach. In
particular, the approach agrees with the first two claims ascribed to the restric-
tor approach in section 2.2.6, page 13: the role of the if-clause of a conditional
is to provide domain restrictions for quantificational operators; and if-clauses
affect a contextual or evaluation parameter that subsequent modals/quantifiers
deeply upon. Crucial is that now we derive these properties from the com-
positional structure of conditionals: the first claim rolls out of the treatment

\[24\] In this case it is not a singular fact of the actual world that makes the difference for
similarity, but normalcy of a world. This is already assuming a particular view on what
similarity amounts to. But without such assumptions, we cannot test the theory. Furthermore,
the assumption that normalcy counts for similarity is accepted by many authors on this topic.
Figure 9: The different stages of updating with example (23-a)

of if-clauses as relative clauses, the second follows from taking if-clauses to be
definite descriptions and the dynamic formalization adopted here.

In a certain sense the proposal made here also agrees with the third claim
of the restrictor approach discussed in section 2.2.6: the interpretation of conditionals involves a hidden operator. It is the definite article, here encoded in
the type shift involved in the interpretation of the if-clause. But is is a different
operator than assumed by the restrictor approach, and it is not present at the
level of LF. In fact, there are even two hidden operators: we also have a universal
quantification in the consequent, because we assume that predication about
pluralities of possible worlds is necessarily distributive. So, this operation is
hard-wired into the semantics of atomic conditions for possible worlds. Again,
it is not present at the level of LF.

Given this, we can say that the proposal made here is conform with the
restrictor approach – as far as this approach should go. The restrictor approach
is essentially a description of the overall meaning of conditional sentences. The present paper adds a theory of how this output is constructed from the form of
the sentence. What we should learn from the case of conditionals is to always
carefully distinguish between theories about the overall meaning of some natural
language construction and the arguments supporting them, and theories about
meaning composition and arguments they can build upon. We should not too
easily move from claims about the first to claims about the second.

4.2 Conditionals in dynamic semantics

One of the distinctive features of the present proposal is that it uses a dynamic
semantics, though this is certainly not the first time conditionals are approached
in a dynamic setting. The standard rule for the interpretation of conditionals
in dynamic semantics is given in (1)\(^{25}\) (\(s\) is a set of possibilities \(i\), representing
the actual information state of the interpreter; each \(i\) represents a possibility

\(^{25}\)See Heim [1994], a more recent reference is Gillies [2004].
for what the actual utterance context might be according to the interpreter). According to this rule, update with a conditional $\phi > \psi$ is defined as a test condition. The test checks whether the consequent adds any information to a hypothetical update with the antecedent. If this is the case, then the test fails and the empty information state is returned. Otherwise, the common ground is returned unchanged, and the hypothetical update with the antecedent is discarded.

$$s[\phi > \psi] = \{ i \in s \mid s[\phi](\psi) = s[\phi]\} \quad (1)$$

The present approach differs in a very fundamental way from this standard approach. In contrast to the formula in (1), we do not analyze conditionals as test condition but as true update condition. Ignoring the compositionality issue and the referential aspect of if-clauses, our approach comes down to the rule in (2): worlds that when revised with the antecedent do not lead to a context that supports the consequent are erased.

$$s[\phi > \psi] = \{ w \in s \mid \text{Rev}(w, \phi) \models \psi\} \quad (2)$$

An important aspect of such a local update condition for conditionals is that without additional support it excludes epistemic effects in the semantics of conditionals: the conditional will only depend on the facts of the evaluation world. In other words, so called epistemic readings of conditionals are not supported. In a Stalnakarian framework [Stalnaker, 1981] epistemic information becomes relevant via a backdoor, because of the defeasible constraint on selection functions he formulates for indicative conditionals. For subjunctive conditionals this rear exit is closed. This does go on a par with the observation that so called epistemic readings of subjunctive conditionals are marginal if possible at all. Unfortunately, we won’t be able to discuss this issue within this paper in more detail.

An interesting alternative implementation of an dynamic approach conditionals one might want to consider is to build revision into the basic update condition (atomic boxes). At some point we need revision anyway when encountering counterfactual information that we want to accommodate. So, why not generalize the definition of update? Revision in case of update with an if-clause would then fall out as a side benefit of this new generalized update rule. Such an approach would simplify the semantics of conditionals: we wouldn’t need something like the type shifter anymore to get the revision effect.

We didn’t take this route here, because it makes empirically wrong predictions. The mode of revision involved when encountering counterfactual information and the mode of revision we need to account for conditionals is simply not the same. Frank Veltman (slides at various occasions) explains the difference as follows: "Consider for example the proposition Oswald killed Kennedy.

---

26This notion of information state differs from the one that will be employed in the present proposal see section 3.3.1.
Supposing that Oswald had not killed Kennedy might make you think ‘If Oswald had not killed Kennedy, Kennedy might still be alive’. If, however, at some point you were to find out that your belief that Oswald killed Kennedy is in fact wrong, and you had to revise your beliefs accordingly, it is very likely that after this revision you would still believe that Kennedy is dead.”

There is also another problem with this line of approach. If you build revision into the basic update, there is no way to circumvent it: as soon as interpretation hits an if-clause, it will revise the context it gets as argument. This leads to problems with sentences like (24). In this case (because of the presence of necessarily) we do not want to restrict quantification to the normal/stereotypical cases that make the if-clause true, we want to consider all cases. But because this becomes only clear when the computation hits the adverbial, i.e. at a point when the if-clause is already computed, there is no straight forward way to implement this in a compositional approach with revision in the basic update.

(24) Necessarily, if the United States threw its weapons into the sea, there would be war.

5 Conclusions

This paper offers a compositional semantics for English conditional sentences. The proposal is worked out within the GB/Minimalist approach to the syntax/semantics interface. For the interpretation of the predicted Logical Forms we used Plural Compositional DRT as introduced in Brasoveanu [2008, 2010], which, in turn, builds on Muskens [1996] and van den Berg [1996].

The approach is based on the idea that if-clauses are free relatives of possible worlds and, consequently, that they have to be treated as referential expressions. As discussed in section 2, this idea is supported by cross-linguistic considerations of the form and meaning of conditional sentence. The approach is well related to the philosophical literature on conditionals in that it incorporates the similarity approach to conditionals of Lewis [1973] and Stalnaker [1968]. It also agrees with the central claims of the restrictor approach of Kratzer [1979, 1991]. In contrast to the standard spell-out of the latter the present approach does not need a covert modal operator at the level of LF to account for bare conditionals. Still, we predict that the conditional comes out as true if in all (selected) antecedent worlds the consequent is true. The all-quantifying effects emerge out of combining a the-type shift letting the antecedent denote a plurality of possible worlds with the distributive predication of the consequent. This improves on the restrictor approach, because it can do without stipulating covert material at the level of LF, while maintaining the same semantic properties. Another advantage of this approach is that it links conditionals to other semantically and philosophically interesting constructions like bare plurals and, thereby, generic expressions. Nevertheless the approach can account for if-clauses modifying overt quantificational elements like adverbs of quantification.

The individual parts of the approach are not entirely new, Bhatt and Pancheva...
[2006] and Schlenker [2004]\textsuperscript{27} already formulate some of the central ideas underlying the approach defended here. The contribution of this paper consists above all in bringing everything together and turning it into a fully worked-out proposal. Bhatt and Pancheva [2006], for instance, clearly formulate the claim that if-clauses are free relatives of possible worlds, but not develop this idea. Schlenker [2004] goes much further, but misses a developed Syntax-Semantics interface and an ontology that can work with pluralities of possible worlds. From Brasoveanu [2008, 2010] we borrow the formal system, but we employ it in a different way.

What is the use of having such a detailed exploration of the ideas of Bhatt and Pancheva [2006]? For one thing, it shows that these ideas can actually put to work; that an approach building on them can be realized. It also helps to understand the ramifications of these ideas. Just to give an example, in view of the relation drawn here between free relatives and conditionals, one might want to think about the question whether revision plays a role in the interpretation of free relatives as well. The main motivation behind the study reported on in this paper was different. First, I hope that based on the proposal made here we will now be able to address the challenging issues of the interpretation of mood, aspect, tense and modality in conditional sentences. As explained in the introduction, there is a lot of interest in these questions at the moment, and the present proposal might provide a new basis to address them. Second, I believe that focussing on the form of conditionals and the question how to derive their semantics from this form might also help to improve on the purely semantic approaches to conditionals that fill the philosophical literature on conditionals. Given that we can make a good case for the derivation of meaning from the form of conditionals developed here, one could test semantic approaches by investigating whether what they take the over-all meaning of conditionals to be can be derived compositionally in the way described here. It is obvious that anything that can be translated into a variant/special case of similarity approach will pass this test. But what about other approaches?

There are many other issues the present approach leaves open and that need to be addressed in the future. Some of them have been already mentioned in the paper, like how to model the distinction between the indicative and subjunctive mood within the present theory. Another aspect that has not been discussed sufficiently so far are the dynamic effects of the meaning of conditionals and the relation to modal subordination. The proposal is spelled out in dynamic semantics and takes if-clauses to introduce discourse referents. So, the means are there to tackle these issues, but it still needs to be done. Just to mention one other issue, future work has to show whether the link to generics via the form of bare plurals established here can also inspire links between semantic/philosophical approaches to conditionals on the one hand and generics on the other.

\textsuperscript{27}Schlenker [2004] is taking up an idea of Lewis [1973].
A The LF of conditional sentences: Syntax

Definition 6 The Syntax of \( \mathcal{L} \).

L. If \( \alpha \) is a lexical tree of type \( x \), then \( \alpha \) is a tree of type \( x \).

Var. If \( v \) is a variable, \( n \) is a natural number, and \( x \) is a type, then \( [x \, v_n] \) is a tree of type \( x \).

FA. If \( \alpha \) is a tree of type \( xy \) and \( \beta \) is a tree of type \( x \), then \( [y \, \alpha \beta] \) is a tree of type \( y \).

IFA If \( \alpha \) is an IL-tree of type \( (sx)y \) and \( \beta \) is an IL-tree of type \( x \), then \( [y \, \alpha \beta] \) is an IL-tree of type \( y \).

\( \lambda \). If \( \alpha \) is an IL-tree of type \( y \), \( n \) is a natural number, and \( x \) is a type, then \( [x \, y \, n \, \alpha] \) is a tree of type \( xy \).

S. If \( \alpha \) is an IL-tree of type \( st \), then \( [(st)t\alpha] \) is a tree of type \( (st)t \).

B Dynamic Ty2

We start with some short remarks on the history of the formal system used. The formalization of the semantics chosen here is adopted from Brasoveanu [2008, 2010]. The system has been developed by van den Berg [1996] to deal with nominal anaphora. Brasoveanu [2008] combined the formalism with Muskens [1996] to obtain a compositional semantics. The Logic of Change from Muskens [1996] reformulates dynamic semantics in type logic, Gallin’s Two-Sorted Type Theory (Ty2, Gallin [1975]). Muskens shows here that the syntax of DRSs can then be seen as abbreviations of certain first-order terms. The only difference with the system proposed here is that the type of individuals is now replaced by the type \( s \) for possible worlds. Brasoveanu [2010] extends the system to modal quantification. This is the part that is particularly interesting for the purposes of the present paper. However, while the formalism will be directly adopted from Brasoveanu [2010], the present paper applies the formalism in a different manner to conditional sentences. Brasoveanu [2010] basically integrates the restrictor approach of Kratzer [1977, 1981] in the dynamics semantics, while the present proposal tries to extend the analysis of free relatives to conditional sentences.

To sum up, there is nothing about the formal system used that is new. Everything that has not been explicitly discussed in section 3 can be already found in the appendix of Brasoveanu [2008] or Brasoveanu [2010]. The definitions are restated mainly for reasons of convenience, and because the system has been reduced to the essence that is truly needed for the proposal made here. Therefore, this simplified version might be easier to digest than the full-blown versions in Brasoveanu [2008, 2010]. The innovation sits in how this formal system is used here.

Definition 7 Dynamic Ty2\(^{28}\)

\(^{28}\)The definition of types isolates a subset of types as the types of drefs: these are functions from assignments (type \( s \)) to static objects of arbitrary type. We restrict our drefs to functions from variable assignments to static objects of arbitrary types because, if we allow for arbitrary
The set of dref types $\text{DRefTyp}$ and the set of types $\text{Typ}$.

1. The set of basic static types $\text{BasSTyp}$: $\{t, w\}$ (truth-values and possible worlds)

2. The set of static types $\text{STyp}$: the smallest set including $\text{BasSTyp}$ and such that, if $\sigma, \tau \in \text{STyp}$, then $(\sigma \tau) \in \text{STyp}$

3. The set of dref types $\text{DRefTyp}$: the smallest set such that, if $\tau \in \text{STyp}$, then $(s \tau) \in \text{DRefTyp}$

4. The set of basic types $\text{BasTyp}$: $\text{BasSTyp} \cup \{s\}$ (variable assignments)

5. The set of types $\text{Typ}$: the smallest set including $\text{BasTyp}$ and such that, if $\sigma, \tau \in \text{Typ}$, then $(\sigma \tau) \in \text{Typ}$

Definition 8 Dynamic Ty2–terms.

1. Basic expressions: for any type $\tau \in \text{Typ}$, there is a denumerable set of $\tau$-constants $\text{Con}_\tau$ and a denumerably infinite set of $\tau$-variables $\text{Var}_\tau = \{v_{\tau,0}, v_{\tau,1}, \ldots\}$.
   i. $\text{Con}_w = \{w, w_0, w_1, \ldots\}$,
   ii. $\text{Var}_w = \{v, v_0, v_1, \ldots\}$,
   iii. $\text{Con}_w = \{\text{Hanna.laughts}, \text{Simon.dances}\}$,
   iv. $\text{Var}_\tau = \{f_0, f_1, f_2, \ldots\}$, for any $\tau \in \text{STyp}$,
   v. $\text{Con}_w = \{u, u', u''', \ldots, u_0, u_1, u_2, \ldots\}$,
   vi. $\text{Var}_\tau = \{x, y, z, \ldots, x_0, x_1, x_2, \ldots\}$, for any $\tau \in \text{Typ}$.

2. For any type $\tau \in \text{Typ}$, the set of $\tau$-terms $\text{Term}_\tau$ is the smallest set such that:
   i. $\text{Con}_\tau \cup \text{Var}_\tau \subseteq \text{Term}_\tau$.
   ii. $\alpha(\beta) \in \text{Term}_\tau$ if $a \in \text{Term}_\sigma$ and $b \in \text{Term}_\rho$ for any $\sigma, \rho \in \text{Typ}$,
   iii. $(\lambda x.\alpha) \in \text{Term}_\tau$ if $\tau = (\sigma \rho), x \in \text{Var}_\sigma$ and $\alpha \in \text{Term}_\rho$ for any $\sigma, \rho \in \text{Typ}$,
   iv. $(\alpha = \beta) \in \text{Term}_\tau$ if $\tau = t$ and $\alpha, \beta \in \text{Term}_\tau$ for any $\sigma \in \text{Typ}$,
   v. $(i|\delta j) \in \text{Term}_\tau$ if $i = t$ and $i, j \in \text{Var}_\tau$ and $\delta \in \text{Term}_\tau$ for any $\sigma \in \text{DRefTyp}$.

Definition 9 Dynamic Ty2–frames
A standard frame $F$ for Dynamic Ty2 is a set $\{D_\tau : \tau \in \text{Typ}\}$ such that $D_w$, $D_t$ and $D_s$ are pairwise disjoint sets ($D_t = \{T, F\}$) and $D_{\sigma \tau} = \{f : f$ is a total function from $D_\sigma$ to $D_\tau$, for any $\sigma, \tau \in \text{Typ}\}$.

Definition 10 Dynamic Ty2–models
A model $M$ for Dynamic Ty2 is a pair $\langle F, [\cdot]_M \rangle$ such that:

1. $F$ is a standard frame for Dynamic Ty2
2. $[\cdot]_M$ assigns an object $[\alpha]_M \in D_\tau^M$ to each $\alpha \in \text{Con}_\tau$ for any $\tau \in \text{Typ}$.

Dref types, e.g., $s(st)$, we might run into counterparts of Russells paradox see Muskens (1995, pp. 179-180, fn. 10).
3. $M$ satisfies the following axioms:

- **Axiom 1 (Unspecific drefs)**
  \[ \text{udref}(\delta), \]
  for any unspecific dref name $\delta$ of any type $(s\tau) \in \text{DRefTyp}.$

- **Axiom 2 (Drefs have unique dref names)**
  \[ \text{udref}(\delta) \land \text{udref}(\delta') \rightarrow \delta \neq \delta', \]
  for any two distinct dref names $\delta$ and $\delta'$ of type $\tau$ for any type $\tau \in \text{DRefTyp}$ (i.e., we ensure that we do not accidentally update $\delta'$ when we update $\delta$).

- **Axiom 3 (Identity of assignments)**
  \[ \forall_i \forall_j (i \neq j). \]

- **Axiom 4 (Enough assignments)**
  \[ \forall_i \forall_{\sigma,\tau} \forall_d (\text{udref}(\nu) \rightarrow \exists_j (\nu[j \land \nu j = f]), \]
  for any type $\tau \in \text{STyp}.$

**Definition 11 Assignments**

An $M$-assignment $\theta$ is a function that assigns to each variable $x \in \text{Var}_r$ an element $\theta(x) \in D^M_{\tau}$, for any $\tau \in \text{Typ}$. Given an $M$-assignment $\theta$, if $x \in \text{Var}_r$ and $d \in D^M_{\tau}$, then $\theta^x d$ is the $M$-assignment identical with $\theta$ except that it assigns $d$ to $x$.

**Definition 12 Interpretation**

The interpretation function $[\cdot]^{M,\theta}$ is defined as follows:

1. $[\alpha]^{M,\theta} = [\alpha]^M$ if $\alpha \in \text{Con}_r$ for any $\tau \in \text{Typ}$.
2. $[\alpha]^{M,\theta} = \theta(\alpha)$ if $\alpha \in \text{Var}_r$ for any $\tau \in \text{Typ}$.
3. $[\alpha(\beta)]^{M,\theta} = [\alpha]^{M,\theta}([\beta]^{M,\theta})$.
4. $[\lambda x.\alpha]^{M,\theta} = ([\alpha]^{M,\theta} \dref d : d \in D^M_{\tau})$ if $x \in \text{Var}_r$.
5. $[\alpha = \beta]^{M,\theta} = T$ if $[\alpha]^{M,\theta} = [\beta]^{M,\theta}$; $F$ otherwise.
6. $[[\delta[j]]^{M,\theta} = T$ if $\delta \in \text{Term}_r, \sigma \in \text{DRefTyp}, \forall x \in \text{Var}_r ([\text{udref}(\nu x) \land x \neq \delta \rightarrow x_i = x j)]^{M,\theta} = T$
   and $[[\forall x \in \text{Var}_r (\text{udref}(\nu x) \rightarrow x_i = x j)]^{M,\theta} = T$ for all $\tau \neq \sigma, \tau \in \text{DRefTyp}; F$ otherwise.

**Definition 13 Truth**

A formula $\phi \in \text{Term}_l$ is true in $M$ relative to $\theta$ iff $[\phi]^{M,\theta} = T$.

A formula $\phi \in \text{Term}_l$ is true in $M$ iff it is true in $M$ relative to any assignment $\theta$.

C  Plural Compositional DRTthe basic system

**Definition 14 Shortcuts**

1. $I_{u\neq*} := \{ i \in I \mid u_i \neq * \}.$
2. $I_{u_1,...,u_n \neq*} := \{ i \in I \mid u_1i \neq * \land u_2i \neq * \land ... \land u_ni \neq * \}.$
3. $uI := \{ u_i \mid i \in I_{u \neq*} \}.$

**Definition 15 Atomic conditions, type $(st)t$**

(lexical relations $R\{u_1,...,u_n\}$ are c-ideals; conditions that are interpreted collectively at the discourse-level):
1. \( R\{u_1, \ldots, u_n\} := \lambda I_{st}. I_{u_1, \ldots, u_n \neq *} \neq \emptyset \land \forall i_s \in I(R(u_1 i, \ldots, u_n i)) \), for any non-logical constant \( R \) of type \( w^{nt} \), where \( w^{nt} \) is defined as the smallest set such that \( w^0 t := t \) and \( w^{m+1} t := w(w^m t) \).

2. \( u_1 = u_2 := \lambda I_{st}. I_{u_1, u_2 \neq *} \neq \emptyset \land \forall i_s \in I(u_1 t = u_2 i) \)

**Definition 16** Atomic DRSs (DRSs containing one atomic condition)

\( \text{Type} (st)((st)t) \) (where \( \text{Dom}(C) := \{I,t : \exists J_{st}(CIJ)\} \) and \( \text{Ran}(C) := \{J_{st} : \exists I_{st}(CIJ)\} \) for any atomic DRS \( C \)):

1. \([C] := \lambda I_{st}. \lambda J_{st}. I = J \land CJ \), for any atomic condition \( C \), e.g.
   - \([R\{u_1, u_n\}] := \lambda I_{st}. \lambda J_{st}. I = J \land R\{u_1, u_n\} J\),
   - \([\lambda u_{12}] := \lambda I_{st}. \lambda J_{st}. I = J \land (u_1 = u_2) J\).

**Definition 17** Condition-level connectives

(negation, disjunction, implication), i.e., non-atomic conditions:

1. \( \neg D := \lambda I_{st}. I \neq \emptyset \land \forall H_{st}(H \neq \emptyset \land H \subseteq I \rightarrow \exists K_{st}(DHK)) \),
   i.e., \( \neg D := \lambda I_{st}. I \neq \emptyset \land \forall H_{st} \neq \emptyset (H \subseteq I \rightarrow H \not\subseteq \text{Dom}(D)) \), where \( D \) is a DRS (type \((st)((st)t))\).

2. \( D \lor D' := \lambda I_{st}. \exists K_{st}(D IK \lor D' IK) \),
   i.e., \( D \lor D' := \text{Dom}(D) \cup \text{Dom}(D') \).

3. \( D \rightarrow D' := \lambda I_{st}. \forall H_{st}(D IH \rightarrow \exists K_{st}(D'HK)) \),
   i.e., \( D \rightarrow D' := \lambda I_{st}. DI \subseteq \text{Dom}(D') \), where \( DI := \{J_{st} : DIJ\} \).

**Definition 18** DRS-level connectives (dynamic conjunction):

\( D; D' := \lambda I_{st}. \lambda J_{st}. \exists H_{st}(D IH \land D'HJ) \), where \( D \) and \( D' \) are DRSs of type \((st)((st)t))\).

**Definition 19** Random assignment of value to a dref:

\([u] := \lambda I_{st}. \lambda J_{st}. \forall i_s \in I(\exists j_s \in J(i[u]j)) \land \forall j_s \in J(\exists i_s \in I(i[u]j)) \).

**Definition 20** Selecting similar contexts:

\( \text{sim}^{w}(D) := \lambda u'. \lambda I_{st}. \lambda J_{st}. ([u]; D) II \land \exists K_{st}([u]; D) IK \land uJ_{u \neq *} \leq u' I uK_{u \neq *}) \),
where \( D \) is a DRS of type \((st)((st)t))\).

**Definition 21** Truth

A DRS \( D \) of type \((st)((st)t))\) is true with respect to an input info state \( I_{st} \) iff \( \exists J_{st}(DIJ) \), i.e., \( \text{iff} I \in \text{Dom}(D) \).

**References**


