Performative *may*

This is a sketch of a performative semantics for 'may'. The idea is very simple: it is this classical idea that *may* adds to the permission set, while *must* erases possible deontic options. We use premise semantics to describe the revision of permission sets. This approach cannot deal with inconsistent obligations.

Theory. We start with the basic definitions. The model is defined as a set of possible worlds, describing the permissible options. We define a similarity relation between possible worlds using premise semantics: given some relevant set of proposition letters we say that worlds w_1 is more similar to w than w_2 if all relevant proposition letters true in w are also true in w_1 and all relevant proposition letters true in w_1 are also true in w_2. Based on this order we define a similarity relation that returns for a world w and a sentence ϕ the worlds closest to w that make ϕ true.

Definition 1 *The model.*
Let σ be a set of possible worlds.

Definition 2 *A premise semantics order.*
Let $\mathcal{P}$ be some set of relevant proposition letters. We define an order on possible worlds as follows:

$$w_1 \leq_w w_2 \ \textit{iff}_{def} \ \forall p \in \mathcal{P} : w \models p \Rightarrow w_1 \models p \ \& \ \forall p \in \mathcal{P} : w_1 \models p \Rightarrow w_2 \models p$$

Definition 3 *The similarity function.*

$$f_{\leq}(w, \phi) = \{w' \in [[\phi]] | \neg\exists w'' \in [[\phi]] : w'' <_w w'\}$$

Now we define the update with sentence of our performative language. We start with the meaning of *may*. We define a positive and a negative update. The positive update adds for each world in the permission set the most similar worlds where the sentence in scope of *may* is true. The negative update subtracts the same set of worlds.

Definition 4 *Update with may*

$$\sigma[may\ \phi]^+ = \sigma \cup \bigcup_{w \in \sigma} f(w, \phi)$$
$$\sigma[may\ \phi]^- = \sigma - \bigcup_{w \in \sigma} f(w, \phi)$$

We also have to define the meaning of logical operators scoping over *may*. Here we use standard definitions for three-valued logics. In particular, negation switches the polarity of the update.

Definition 5 *Wide-scope logical operators.*

$$\begin{array}{l} \sigma[\neg\phi]^+ = \sigma[\phi]^- \\ \sigma[\neg\phi]^- = \sigma[\phi]^+ \\ \sigma[\phi \wedge \psi]^+ = \sigma[\phi]^+[\psi]^+ \\ \sigma[\phi \wedge \psi]^- = \sigma[\phi]^- \cup \sigma[\psi]^- \\ \sigma[\phi \vee \psi]^+ = \sigma[\phi]^+ \cup \sigma[\psi]^+ \\ \sigma[\phi \vee \psi]^+ = \sigma[\phi]^-[\psi]^- \end{array}$$

We can even define the meaning of *must* as the dual of *may*. One can show that then *must* works simply as intersection: it only keeps the options where the sentence in scope of *must* is true. If such options do not exists, then the update returns the absurd permission state. Here it is very obvious, that the approach cannot deal with inconsistent obligations.

Definition 6 *Update with must*

$$\begin{array}{l} \sigma[must\ \phi]^+ = \sigma[\neg may \neg\phi]^+ \\ \sigma[must\ \phi]^- = \sigma[\neg may \neg\phi]^- \end{array}$$

Fact 1 $\sigma[must\ \phi]^+ = \{w \in \sigma \mid w \models \phi\}$.

Applications. To see whether we get free choice permission in this setting, we need some notion of entailment. Here we just use standard entailment in update semantics.

Definition 7 *Dynamic entailment*
Let Σ be a set of permission sets, i.e. a set of set of possible worlds. We define $\phi \Rightarrow_\Sigma \psi$ iff$_{def}$ $\forall\sigma \in \Sigma : \sigma[\phi]^+[\psi]^+ = \sigma[\phi]^+$. We write $\phi \Rightarrow \psi$, if $\phi \Rightarrow_\Sigma \psi$ holds in case Σ is the class of all permission states.

The next fact shows that in the relevant contexts (i.e. those where the utterance is not trivial) free choice permission is a valid inference.

Fact 2 *Free Choice Permission*

(i) *Let Σ be the set of contexts where $may(A \vee B)$ is not trivial, i.e. $\sigma[may(A \vee B)]^+ \neq \sigma$. Then $may(A \vee B) \Rightarrow_\Sigma (may\ A \wedge may\ B)$ and $(may\ A \vee may\ B) \Rightarrow_\Sigma (may\ A \wedge may\ B)$.*

(ii) $may(A \vee B) \Leftrightarrow (may\ A \vee may\ B)$.

It is interesting to notice that we get free choice here for narrow scope disjunction as well for wide scope disjunction. Fox (2006) and also the new implicature approach of Robert and me don't get this. This is insofar a problem as (1) appears to allow for a free choice reading. But notice that (2) doesn't.

(1) You may take an apple or you may take a pear.

(2) You are allowed/permitted to take an apple or you are allowed/permitted to take a pear.

One could now propose that the reason for this difference lays in the fact that *may* has a performative reading, which allows for free choice permission of wide scope disjunction (as semantic inference). *Allowed* and *permitted* only have assertive/reportative uses and, therefore, free choice can only be derived as conversational implicature for the narrow scope reading.

Another interesting prediction of this performative semantics of *may* is that it can nevertheless correctly account for free choice constructions under negation. The sentence $\neg may(A \vee B)$ does not just exclude the option that A and B both are true, but also the options $A \wedge \neg B$ and $B \wedge \neg A$. Thus, the sentence (3) does not work as just excluding the conjunction of A and B.

(3) You may not take an apple or a pear.

Finally, this approach can also explain why *any* in scope of *may* is fine, but in scope of *must* is bad.