The Common Ground as a Dialogue Parameter

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1 Introduction

This paper tries to define a central notion in the semantics of dialogues: the common ground between the speaker and hearer and its evolvement as the dialogue proceeds. The starting point is the theory of pragmatics introduced by Stalnaker in *Stalnaker 1978*. Here implicatures arise as the preconditions of certain speech acts and presuppositions are defined as the shared assumptions of speaker and hearer. This theory makes the common ground the central notion in understanding speech acts and presuppositions and makes it the parameter which controls decisions of the speaker about his communicative course of action and of the hearer in deciding what to make of the speaker's contribution. In the theory, the common ground is therefore one of the starting points for the explanation of linguistic behaviour and for understanding interaction. In this paper, I try to apply this idea in a characterisation of speech acts by stating (epistemic) preconditions on the common ground for their use, by stating their guaranteed contribution and by indicating the moves for the other party that are available after it.

The paper is innovative in making common grounds a special kind of information states and in making these the basis of an update system. They will not only have facts, but will also have opinions about the beliefs of the speaker and the hearer. I take this to be the crucial step: without it there is not enough expressive power to define which information states are common grounds, the characterisation of the speech acts is approximative only and it is not possible to model conflict. And absence of conflict makes it impossible to apply update semantics directly in the study of conversation.

Equating information states with common grounds gives the update of an information state special logical properties. In update semantics, one of the ways to define logical consequence is by quantifying over information states σ: $\varphi_1, \ldots, \varphi_n \models \psi \Leftrightarrow \forall\sigma(\sigma \models \varphi_1, \ldots, \varphi_n \Rightarrow \sigma \models \psi)$. This cannot be maintained if σ ranges over common grounds rather than standard information states. Also, updating an information state to obtain another common ground is different from plain updating. The first half of this paper is concerned with common grounds as information states and their logic.

A distinction that is important and feasible is that between logical and pragmatic update operations. Logical updates correspond to what we are used to in logic and can be formally defined here as those operations that are eliminative and distributive over the information states that they update. They coincide with the operations that can be characterised by a Tarskian truth definition. The pragmatic ones (the speech acts and presupposition) can be defined as the ones that are not: they require properties of the information state as a whole in their definition. They typically also give rise to partiality. On this view, the presupposition operator (contra *Beaver 1992*) is a typical pragmatic operator. The second part of this paper studies some more of these operations. We end by an attempt to show that *might* is really a logical operator (contra *Veltman 1994*).

2 A basic update system

About the simplest possible update system is the one given by a language of propositional logic taking information states to be sets of models for that language.

We take all sets of models to be information states. The definition of update is given in (1). $[A]$ is the function from information states to information states, and we write $\sigma[A]$ for the result of applying $[A]$ to an information state σ. Updates are defined over this system by putting the update of an information state σ to be the intersection of the set of models in σ that satisfy φ.

(1) $\sigma[\varphi] = \{i \in \sigma : i \models \varphi\}$

We define $\sigma \models \varphi$ as an abbreviation of $\sigma[\varphi] = \sigma$, but we could equally well define it to be: $\forall i \in \sigma\ i \models \varphi$.

To this system we can add belief operators B. We assume a classical modal treatment: an operator B corresponds to an accessibility relation R_B between extended propositional models. The set $B_i = \{j : iR_Bj\}$ is the set of worlds that are accessible for B from a world i. This associates a set to every operator in every world. These sets can be thought of as the information state that B associates with i.

Kripke models could be used here, but instead I will follow *Gerbrandy & Groeneveld 1996* in thinking of the elements of the information states as possibilities. A possibility is function that maps propositional letters to truth values and the belief operators to sets of possibilities. Using Aczel's non-well-founded set theory

(*Aczel 1988*), we can show that possibilities exist. The main advantage is that we will have an easier time when discussing CG-updates later on.

We assume at least the system K for our belief operators. Assuming introspectivity ($B\varphi \to BB\varphi$) would not be problematic but adding reflexivity ($B\varphi \to \varphi$) would create problems for our common grounds[1]: we could no longer agree to disagree.

There are two different updating rules for the belief operator in the literature. One is due to Kamp[2], another due to Stalnaker[3]. The Kamp definition starts from the idea that the belief subject may have any kind of information. Some possibilities i will have $B_i \models \varphi$, others will not. So an update can be given by eliminating the possibilities where the subject does not believe the proposition attributed to her. This restricts the information state to possibilities in which the proposition holds in the belief state of the subject. The definition is given in (2).

(2) $$\sigma[B\varphi] = \{i \in \sigma : B_i \models \varphi\}$$

Stalnaker's way is to collect the belief information states in the different possibilities and collect them in one single information state by set union. This gives a single information state (what the subject believes in the information state) which is then updated by the proposition the subject is asserted to believe. We then check whether a possibility assigns to B an information state that has as least as much information as the information state that results from the update. If not, the possibility is eliminated.

(3) $$\sigma[B\varphi] = \{i \in \sigma : B_i \subseteq (\bigcup_{i\in\sigma} B_i)[\varphi]\}$$

For our basic system, both definitions coincide. For a distributive and eliminative system: $B_i \models \varphi$ iff $B_i \subseteq (\bigcup_{j\in\sigma} B_j)[\varphi]$. This does not always hold: Beaver (p.c.) shows that they diverge on Veltman's *might*-operator.

In the sequel, we will freely use both definitions[4].

[1]Some material in the common ground is known, other material is only believed. This holds in particular for the material that we acquire in the course of communication

[2]As reported by *Heim 1992*

[3]*Stalnaker 1988*

[4]Both definitions continue to be the same when we generalise to a version of FOL. A discussion falls outside the scope of this paper.

3 Common Grounds

We can now embark on a discussion of the common ground. With a common ground, there is the set of participants whose common ground it is. We can equate these with a set of belief operators P in some set CGP (common ground partners). It seems reasonable to ask that CGP is a finite non-empty set of belief operators. The case that CGP has only one partner is special. For a common ground between P and himself, $\sigma \models \varphi$ iff $\sigma \models P\varphi$. They are the information states in which P's beliefs coincide with the available information: it is introspective belief.

The case of two participants seems representative of the case of more than 1 participant and we will sometimes assume that $CGP = \{S, H\}$ to facilitate discussion. A basic intuition is that the common ground contains the information that each participant shares with the other. But this is not sufficient as parties may agree with each other in certain respects without knowing so. When this happens, the material should not be in the common ground: the parties share it but they are not aware that they do so and therefore, they cannot draw on this material in their consideration of collaborative actions and communication with each other. We must strengthen our definition to read: the common ground contains all that information about which all parties, according to the common ground, agree. This is circular, but we can still employ it to single out among the information states those states that are common grounds. Let $P_1, \ldots, P_n$ be the belief operators of the participants. Then (4) is a necessary condition under which an information state can be a common ground.

(4) $\sigma \models \varphi \Leftrightarrow \sigma \models P_1\varphi \wedge \ldots \wedge P_n\varphi$

One of the things that we can prove in general is that the state of no information 1 and the state of inconsistent information 0 fulfill this condition. For 0, notice that $0 \models \varphi$ for any φ. Notice that for 1, $1 \models \varphi$ only if φ is a tautology. But $P\varphi$ is then also a tautology and on 1, $P\varphi$ only holds if φ is tautology.

I want to introduce another idea here, which may be more controversial. Participants in a common ground know that they are dealing with a common ground. That is they believe that whatever they believe to be common ground between them is the case according to the common ground. So the common ground comes with the pretense that what is believed in common is common ground, i.e. true according to the common ground. This makes it plausible to add another condition in a definition of common ground: whatever is shared holds. Since this is a shared belief, we want it to hold in the common ground. This leads to the definition of a common ground in (5).

(5) An information state σ is a common ground if and only if
$\sigma \models P_1\varphi \wedge \ldots \wedge P_n\varphi \rightarrow \varphi$ and
$\sigma \models \varphi \Leftrightarrow \sigma \models P_1\varphi \wedge \ldots \wedge P_n\varphi$

Updates of a common ground will not in general bring us from one common ground to the next, because the conditions may cease to hold.

In the sequel we will use $\Box\varphi$ as an abbreviation of $P_1\varphi \wedge \ldots \wedge P_n\varphi$.

Notice that what a participant believes in the common ground cannot be less than what the common ground itself contains as information. But it could well be that in some or all of the possibilities of the common ground the participant believes more. Think of the case that a participant has expressed a belief that has not been accepted by the others. This leads to a structural condition on common grounds σ.

(6) $P_i \subseteq \sigma$ for P a participant and $i \in \sigma$

(This follows from the demand: $\sigma \models \varphi \Rightarrow \sigma \models P\varphi$ and the assumption that CGs are uniquely determined by their theories.)

A second fact of this kind is the fact that all possibilities in a common ground must be allowed for by at least one participant. That is, the equation (7) holds for common grounds σ.

(7) $\sigma = \bigcup_{P \in CGP} \bigcup_{i \in \sigma} P_i$

One side of the equation follows from (6) . The other follows from lemma (8),

(8) **Lemma** $\sigma = \bigcup_{i \in \sigma} \Box_i$ where $\Box_i = \bigcup_{P \in A} P_i$

a lemma for which we also have to make the extra assumption that our information states are uniquely determined by their theories.

Our lemma then follows from lemma (9).

(9) **Lemma** $\sigma \models \varphi \Leftrightarrow \bigcup_{i \in \sigma} \Box_i \models \varphi$

Proof Let σ be a common ground and assume that $\sigma \models \varphi$. By the definition of common grounds, this is equivalent to $\sigma \models \Box\varphi$. By distributivity, this is the same as demanding that $\Box_i \models \varphi$ for each $i \in \sigma$. By a second application of distributivity this is the same as demanding that for every $i \in \sigma$ and for every $j \in \Box_i\{j\} \models \varphi$. But that is equivalent by distributivity to $\bigcup_{i \in \sigma} \Box_i \models \varphi$.

So the shared beliefs of the participants are subsets of the common ground and also form a cover of the common ground.

These lemmas suggest a direct semantic definition of the common ground. Let R be the union of the accessibility relations. :σ is a common ground iff $\{i : \exists j \in \sigma : jRi\} = \sigma$, and $R \cap (\sigma \times \sigma)$ is reflexive[5].

4 Updating the Common Ground

The problem that we have to face now is adding information to an information state that is a common ground in such a way that we end up with a common ground again. This problem is a variation of what has always seemed problematic about common grounds. If we add φ we have to add $P\varphi$ for each participant P as well. And if we have done so we must do this again for the new statements as well. And so forth ad infinitum.

In the same way, we must take care when we add a statement of the form $P\varphi$. Not only do we have to take care that this new statement gets added as beliefs of all the participants, but we also have to take care that it is not suddenly the case that a new bit of common ground has emerged as it can already be the case that all the other participants agreed about φ.

The problem faces us with almost every speech act. If we have only a speaker S and a hearer H, an assertion is only proper when it is in the common ground that the hearer does not believe the content of the assertion. It is reasonable to demand that the hearer does not believe the negation of the content, it should be common ground (or be accommodatable) that the hearer does not have the opinion that φ. If all goes well, after the assertion, it should be the case that the hearer now believes that φ. The extra evidence for the content of the assertion that has made her change her mind was the fact of the assertion. But this means that addition of information by communication is not a monotonic process: we must get rid of information and replace it by new information.

Let us look at this in some more detail. A successful assertion of p can be described as a transition from a common ground σ to an information state τ such that (10).

(10) $\tau \models p$
$\tau \models \Box p$

Things go wrong if we describe the speaker as making the assertion because of her assessment of the common ground: she must take it to be the case that it is not

[5]Thanks go to Gerd Jaeger for suggesting this definition

the case that the hearer believes that p, i.e. she must believe $\neg Hp$. So whether the assertion is successful or not, it is evidence that the speaker believes p and believes $\neg Hp$. If this is so, the assent of the hearer (leading to the desired result of the assertion) has to override the conflicting determination of the speaker's assessment of the hearer's attitude with respect to p.

This is not a mistake. What goes on in communication is a change in the world: first the hearer has no evidence for p, now she has. First there was no reason for H to open the window, now the request has provided a reason. First, S was under no obligation to do X, the promise has changed this.

I am not dealing with corrections, only with assertions where the speaker is adding facts consistent with the common ground and with the expressed beliefs of the hearer. It is conceivable that the hearer believes the negation of p but that the speaker is trying by her assertion and maybe by later argument to get her to change her mind. The treatment of such corrections is however difficult within the current setting. A reasonable first step is to find a way of updating that can deal with the problem of conflict-free updates. What remains open for conflict, is the road of keeping records of earlier information states. The common ground however does not seem to offer new ways of dealing with the problems of belief revision.

Let σ be given. Let p be the content of the assertion and consider $\sigma[p]$. $\sigma[p]$ will no longer be a common ground. Let us assume that σ has a p-possibility i in which the hearer does not believe that p. That means that H_i is partly outside $\sigma[p]$, as $\sigma[p]$ only contains p-possibility and H_i must have at least one possibility that is not a p-possibility. i will survive the update with p and thereby keep $\sigma[p]$ from being a common ground.

The intuition behind the following operation of restriction is the following. We want to change the possibilities in the information state that have the offending property by changing the possibilities to which the partners have access. We start by looking at a more general case. Consider the following operation of *restriction* of one information state by another given a fixed CGP.

(11)
$$\sigma^\tau = \{i^\tau : i \in \sigma\}$$
$$i^\tau(x) = i(x) \text{ for } x \notin CGP$$
$$i^\tau(x) = (i(x) \cap \tau)^\tau \text{ for } x \in CGP$$

The operation is not recursive in set theory, but it is allright under Aczel's AFA. What should however be clear that the definition of $\sigma^\tau \models \varphi$ is recursive in the definitions of $\sigma \models \varphi$ and $\tau \models \varphi$. By our earlier assumption, it follows that we can think of σ^τ as an information state.

The operation limits the extension of the beliefs of participants in σ to the information state τ. We will use this operation to model the following situation: We update a common ground σ with some new information φ and then restrict the new information state by itself. This happens to be a common ground.

The self-application σ^σ can be written as an operation * mapping information states to information states.

So $\sigma[\varphi]* = \sigma[\varphi]^{\sigma[\varphi]}$. We can prove the following theorem (12) which is slightly more general than we require.

(12) **Theorem** If $\sigma \models \Box\varphi \to \varphi$ for all formulas φ, then $\sigma*$ is a common ground.

Proof.
We use τ for $\sigma*$ and let $\Box_j = P_{1_j} \cup \ldots \cup P_{n_j}$ and $\Box_\tau = \bigcup_{i\in\tau} \Box_i$.
We first show that $j \in \Box_j$ for $j \in \tau$.
Let $j \in \tau$. Then there is an $i \in \sigma$ such that $j = i^\sigma$. Because $i \in \Box_i$ (by $\Box\varphi \to \varphi$) and $i \in \sigma$, we have $j \in \Box_j$.
From this it follows immediately that $j \in \Box_\tau$ and so that $\tau \subseteq \Box_\tau$. It also follows that $\tau \models \Box\varphi \to \varphi$.
The construction on the other hand guarantees that $\Box_\tau \subseteq \tau$. Combining, we have $\tau = \Box_\tau$ and it follows that $\tau \models \varphi$ iff $\tau \models \Box\varphi$.

In particular, it follows from the theorem that $\sigma[\varphi]*$ is a common ground if σ is a common ground.

What do we know about our new common ground $\sigma[\varphi]*$? First of all, if a formula holds on σ and it does not contain any occurrence of P, it will continue to hold on $\sigma[\varphi]*$. Second, for such formulas it also holds that they will continue to hold when prefixed with a P: the participants' belief sets become smaller. What can stop holding are negations of $P\varphi$ and this is as it should be: the information of the participants has increased and they believe more than they used to. Also, if there is a possibility in which $P\varphi$ holds, with φ free of P's, $\sigma[\varphi]*$ will also have such a possibility.

(13) **Lemma** Let φ range over formulas in which there is no operator P, with P taken from CGP. Then:
1. $\sigma \models \varphi$ iff $\sigma^\tau \models \varphi$
2. If $\sigma \models P\varphi$ then $\sigma^\tau \models P\varphi$
3. If $\sigma \not\models \neg P\varphi$ then $\sigma^\tau \not\models \neg P\varphi$(Here and in (2) P may be a sequence of operators from CGP.
4. If $\tau \models \varphi$ then $\sigma^\tau \models \Box\varphi$

From (13) it follows that $\sigma[\varphi]* \models \varphi$ whenever φ is equivalent to a positive formula over the operators in CGP. How about the other formulas? A curious example is Moore's paradox:[6] on $\sigma[p \wedge \neg Pp]*$ the formula $p \wedge \neg Pp$ does not hold but Pp. Another example of an update that leads to its negation is given by Gerbrandy and Groeneveld using the Conway-paradox. It is the $*$-operation that is to blame here: it extends the knowledge of the conversationalists. This shows that common ground updating does not obey the principle: $\sigma[\varphi] \models \varphi$ and so can easily fail as a characterisation of logical operators, unlike the basic update system we considered before.

5 Computation

In abstracto, it is hard to deal with common grounds, in practice much easier, at least if we restrict ourselves to what is needed for a theory of communication.

In communication, a common ground can always be thought of as being a basis which is closed off under the schemes we have been discussing. The basis is the set of those facts which cannot be derived by the schemes from other facts. For the analysis of real conversations, it would (as judged by one of the participants) consist of what she has in common with the other party in knowledge of the language used and in world knowledge, what aspects of the speech situation are shared and finally of her own commitments and those of the other participants.

In practice, world knowledge and knowledge of language can be reduced to that part that has been actively used in the communication at hand. Use of a word or expression indicates knowledge of language, inferences indicate the acceptance of certain world knowledge.

Our characterisation of the common ground gives us two schemes that can be almost directly used. For the second scheme, it is just a matter of adding the formula scheme $\Box\varphi \to \varphi$ to the basis. This will also take care of one half of the first scheme: the part that says that if $\sigma \models \Box\varphi$ then $\sigma \models \varphi$.

For the rest we need closure under the rule: $T \vdash \varphi$ then $T \vdash \Box\varphi$, where T is the extended basis.

To sum up, if we assume that T is a CG-basis then $T \vdash_{CG} \varphi$ iff $\varphi \in S$ where S is the smallest set containing T, and all instances of $\Box\varphi \to \varphi$ which is closed under K and the rule $S \vdash_K \varphi \Rightarrow \Box\varphi \in S$.

But what is T? It is reasonable to allow common grounds to start from somewhere: general knowledge of the kind that is described as knowledge of language and world knowledge is one ingredient. The other components can be taken as

[6] $\sigma[p] * [\neg Pp]*$ is the inconsistent information state

consisting of two elements: a characterisation of the speech situation and the commitments of speakers and hearers. The last element directly corresponds to the commitment slates due to *Hamblin 1971* and put to action by Van Leusen in the context of corrections. It appears therefore that there is little difference between our CG-updating and maintaining commitment slates, rather the two views of maintaining conversational information are complementary. Commitment slates are a practical answer to how to maintain a common ground as a conversation unfolds, CG-updating supplies an answer to the question what the meaning of the commitment slate is and what consequences can be drawn from a given commitment slate.

Commitment slate updating corresponds to CG updating. If an assertion is made we can add to the commitments: $S\varphi$, if the assertion is accepted we add φ. If the hearer rejects it, we add $H\neg\varphi$ and so on. We will study this process more closely in the next section. Things become common ground, because both speaker and hearer believe it and there is no reason to add a special section in a commitment slate which maintains common beliefs. There is also no reason for limiting oneself to formulas of a particular logical complexity.

6 Applications

Our basic system is both eliminative and distributive. That means it is not necessary to treat it as an update semantics at all[7]. This changes as soon as we switch to the operations on common grounds in which we are really interested: speech acts, presupposition resolution, querying and epistemic modalities.

The notion of information can be understood as a test[8]. We imagine a subject who we tell that she is placed in a possible state of affairs and we want her to tell us whether the possible state of affairs is the actual world or not. There is no limit on the amount of investigation of the alternative the subject can engage in. Now the criterion is: can a conflict between (in principle discoverable) facts in the state of affairs and the information of the subject be constructed. If there is such a conflict, the subject will conclude that no, this is not the actual world, otherwise she will not be able to decide whether it is or not.

What happens of course in the test is that we keep the information constant: this is the resource for the subject to carry out the test. Now it seems that information about information is typically what speech acts are involved with, and it seems right that we separate this off from information of the kind that gives a criterion for deciding that a possible state of affairs is not the actual world. An assertion

[7]It has been shown that such semantics can always be dealt with in a static way

[8]I am indebted here to Haas-Spohn, see her 1995

is an indication that I have certain information, a query an indication that the speaker wants to have such and such information, a presupposition an indication that the speaker takes such and such information for granted, etc.

What we attempt below is to use our framework as a means for defining the basic moves in conversation. I will call such moves speech acts. This is appropriate, as they share important characteristics with actions. A speech act is essentially a way to change the common ground in a controlled way. It can take effect only under certain circumstances (the preconditions) and has both a basic effect and intended effects. The basic effect is always reached, the intended effect depends on further speech acts of the hearer. A question can only be put under certain conditions, e.g. that the speaker knows the answer cannot be common ground. It has as direct effect to make it common ground that the speaker does not know the answer. But the intended effect is that the answer will become common ground, a goal that is only reachable through the participation of the hearer.

6.1 Assertion

The most basic case is the assertion. For a proper assertion (not a correction or a self-correction or a reiteration) it must be the case that the common ground does not deny its content, that the speaker is not known to deny its content and that the hearer is not known to deny its content. In all these cases it is a correction of some kind. It should also not be the case that the content is known, known by the speaker or known to the hearer. (Here the fact that the speaker (or the hearer) does not know it or its negation entails that it is not CG.)

In our setting each of these means that there is a possibility in the CG in which the content is not true and this is not a fact that can be inspected by looking at one possibility only. So the fact that an assertion is proper given a CG (otherwise it would be undefined) cannot be seen as a distributive and eliminative update. Much the same holds for presupposition and the epistemic modalities.

It turns out that for the definedness of special updates, corresponding to speech acts, it is necessary to look at what information is not contained in the information state. This is not a distributive test, as non-satisfaction, in a distributive eliminative update semantics comes down to the existence of a carrier that does not satisfy the proposition in question.

A good case is assertion. I start from a conception of assertion where the assertion is carrying out one of the useful functions of communication: to supply information that one could in principle acquire by one's own observation but which one has not observed oneself. The asserter is here ideally the end of a chain going back to an original observation of the asserter or of someone that has transmitted, directly or indirectly, the information to the asserter by communication. The fact that someone asserts something then has a comparable status to observation itself: it is evidence for the truth of the content of the assertion. If we take assertions to be an attempt on the part of the speaker to change the common ground to contain some information it did not previously contain, by means of the evidence constituted by the speaker's assertion, we get the four demands in (14) on what the common ground should be like if the assertion is to be successful[9].

(14) $\sigma \not\models H\varphi$
$\sigma \not\models H\neg\varphi$
$\sigma \not\models S\varphi$
$\sigma \not\models \neg S\varphi$

As we are dealing with a common ground these four conditions entail the Stalnaker conditions given in (15).

(15) $\sigma \not\models \varphi$
$\sigma \not\models \neg\varphi$

But the Stalnaker conditions are weaker: they entail that two of the four conditions hold. The common ground can e.g. be as in (16). It follows from these two demands (and σ not being the absurd state) that the Stalnaker conditions hold. On the basis of this example, it seems fair to conclude that the Stalnaker conditions are too weak.

(16) $\sigma \models H\neg\varphi$
$\sigma \models S\varphi$

The four conditions can be justified as follows. If the hearer would already (be known to) know the content of the assertion, the assertion could not change the common ground in the sense of adding the content of the assertion to it. If it is possible to utter φ at all when this condition applies, we would be dealing with the assent of the speaker to a previous assertion of the hearer. This is not an assertion.

If the hearer would be known to believe the assertion is false, we are ready for conflict. This is certainly possible, but it is useful to distinguish this case from proper assertions and reserve the word correction for that. The speaker can certainly not expect by his utterance of φ alone to change the common ground in the desired direction.

[9] Jelle Gerbrandy noted two important problems in the original treatment

If the speaker is known to believe the content of the assertion already, it seems again that the assertion by itself will not be sufficient to change the common ground. The speaker repeats his previous statement and obviously the hearer did not believe him before.

Finally, if the speaker is known not to know the assertion, it is unclear by what means he hopes to change the common ground. The pretense associated with the use of an assertion is that the speaker has acceptable evidence for his belief in the truth of the assertion. If it is known he does not know it the fact that he asserts it will not be evidence for the truth of the assertion.

The assertion of φ will minimally indicate that the speaker believes that φ. So after one step, we reach (17).

(17) $\sigma[S\varphi]*$

The choice is now to the hearer: he can assent, express his disbelief or express his doubt. This brings us to the states in (18) respectively.

(18) $\sigma[S\varphi]*[\varphi]*$
$\sigma[S\varphi]*[H\neg\varphi]*$
$\sigma[S\varphi]*[\neg H\varphi]*$

In the first case the speaker reaches his goal.

In the other cases, there is now clarity about the hearer's opinion about the question whether φ.

Perhaps, I should say something about the other cases: the pseudoassertions arising by the failure of one of the conditions. First of all, if the speaker is assenting to the hearer, the assertion is automatically successful (unless the hearer has a general reason for distrusting the speaker) as the assertion at least is evidence for the speaker's belief in the proposition. The addition of the speaker's belief makes the content a part of the common ground: the evidence for φ in the speaker's assertion is not the reason for it becoming common ground.

When the speaker utters φ against the opposite view of the hearer, the strategy of the speaker must be different from just adducing evidence for φ by asserting it. The speaker can count on his position of authority, on the force of the arguments he is going to bring in later on, but perhaps his goal is also a more modest one: to bring about doubt or to bring about a deadlock in the communication.

Second, it is possible to reiterate what one has said before, and it even appears that this can carry out a useful function in the flow of communication. We can get back to earlier phases in the communication in this way or we can identify objects that have been referred to before. But it is clear as well that we do not adduce further evidence for the content by such a reiteration.

Last, it is also possible to correct oneself. This requires further justification: why one was wrong before and now is right. A normal isolated assertion will not lead to the goal.

6.2 Other Speech Acts

If one considers other speech acts like the question, the request and the promise, things are not very different.

Let us assume (for convenience) that we are dealing with a *yes-no*-question. The question is correct if the conditions in (19) are satisfied.

(19)
1. $\sigma \not\models S\varphi \vee S\neg\varphi$
2. $\sigma \not\models H\varphi$
3. $\sigma \not\models H\neg\varphi$
4. $\sigma \not\models \neg(H\varphi \vee H\neg\varphi)$

If the speaker would be known to know the answer to the question, his purpose of eliciting the answer from the hearer would be defeated (It would be a rhetorical question). If the hearer is known to know a particular answer, similarly the purpose of eliciting the information is not achievable by the question as it has already been reached before the question is asked. Last, if it is known that the hearer does not know the answer, there is again no purpose in asking the question. Together the four conditions entail (20).

(20)
5. $\sigma \not\models S\varphi$
6. $\sigma \not\models S\neg\varphi$
7. $\sigma \not\models \varphi$
8. $\sigma \not\models \neg\varphi$

In addition to the four conditions, we should be able to assume that it is consistent to assume that the speaker wants to have information from the hearer. I will stay clear from questions of desire[10], but just offer the extra demand in (21).

(21) $\sigma \not\models \neg want(s, S\varphi \vee S\neg\varphi)$

Putting the question the speaker changes the common ground to (22).

[10]It seems that it is possible to maintain a set of shared desires in the common ground.

(22) $\sigma[want(s, S\varphi \vee S\neg\varphi])]*$

The hearer can answer *yes* or *no* or can deny to know the answer or can refuse to answer, changing the common ground to respectively (23).

(23)
$$\sigma[want(s, S\varphi \vee S\neg\varphi])] * [H\varphi]*$$
$$\sigma[want(s, S\varphi \vee S\neg\varphi])] * [H\neg\varphi]*$$
$$\sigma[want(s, S\varphi \vee S\neg\varphi])] * [\neg H\varphi \wedge \neg H\neg\varphi]*$$
$$\sigma[want(s, S\varphi \vee S\neg\varphi])] * [\neg wants(h, S\varphi \vee S\neg\varphi)*$$

Assents and denials can then further bring φ or its negation in the common ground. (In case the hearer gives a positive or negative answer, the speaker's desire is fulfilled and can be eliminated. One way of achieving this is by preference semantics).

An interesting observation about questions is that their preconditions are the ones that make any answer to it a proper assertion. This supports the view that all assertions must be seen as answers to (possibly hidden) questions.

It is possible to steer completely free from the moral dimension. The common ground is an assumed object for the speaker and the hearer which is manipulated by them both according to what they think is happening. Lies are occasions where the speaker manages to insert things in the common ground he knows are false, false promises occasions where a promise is made without the intent to carry it out. This may be immoral, but it changes little as to the communication itself: as always, we build a faithful picture of what has happened between speaker and hearer and keep a list of what they want, plan or believe.

6.3 Might

One of the motivations for developing this account of common ground updating is a dissatisfaction with the semantics of the might-operator proposed by Veltman. The semantics that Veltman proposes is (24).

(24) $\sigma[might\varphi] = \sigma$ if $\sigma[\varphi] \neq 0$ and otherwise 0.

This semantics does not connect well with the standard idea that assertive updates give information, which in our context is equivalent to them eliminating at least some possibilities. Of course updates with *might* φ may eliminate all possibilities, but this would be too much: if we move to the inconsistent information state we have lost everything and it may be right to assume —with Stalnaker— that one of the principles guiding our interpretation system is to avoid landing in the absurd information state. So in both cases, there is conflict with the Stalnaker conditions.

We may of course question whether an utterance of *might* φ is indeed an assertion. This may be fruitful but runs counter to the intuition that indeed utterances of *might* φ normally supply extra information, an intuition which is the basis for wanting to classify *might* φ as an assertion.

Let us however proceed from the opposite view. We will try to analyse *might* as a speech act operator and then show that its effect can be captured by assuming it is a logical operator.

Assume then that utterances of *might* φ are no assertions but speech acts of a kind of their own and let us try to analyse this new class of speech acts in the way we did before.

It seems reasonable to assume the preconditions in (25).

(25) $\sigma \not\models \neg S \neg \varphi$
$\sigma \not\models \neg H \neg \varphi$
$\sigma \not\models S \neg \varphi$
$\sigma \not\models H \neg \varphi$

The first two make the utterance have a purpose, the second two ensure that we steer clear of conflicts.

From these it follows that also the conditions in (26) hold.

(26) $\sigma \not\models \varphi$
$\sigma \not\models \neg \varphi$
$\sigma \not\models S \varphi$
$\sigma \not\models H \varphi$

Does *might* φ have a contribution? It would appear that the least that is required is the speaker does not think that $\neg\varphi$ is the case. So the contribution of the speaker's speech act in his utterance of *might* φ is the common ground (27).

(27) $\sigma[\neg S \neg \varphi]*$

The hearer has a choice of reactions: assent, denial or doubt.

Assent would be the further change to (28). (There is not much point in assenting to the speaker's disbelief).

(28) $\sigma[\neg S\neg\varphi] * [\neg H\neg\varphi]*$

Denial would have to take the form of an assertion (!) of $\neg\varphi$ and doubt would be the impossibility for the hearer to decide between his knowing or not knowing that $\neg\varphi$. This is $\neg HH\neg\varphi \wedge \neg H\neg H\neg\varphi$ which admittedly is a somewhat sophisticated attitude to have towards a proposition. (A: Maybe John is home. B: I don't know). So far so good. Notice that *might* φ gives new information. If accepted by the hearer, it makes ignorance of $\neg\varphi$ common ground.

Suppose the above is correct. We may then represent *might* φ as $\neg S\neg\varphi \wedge \neg H\neg\varphi$ which we can abbreviate as $\Diamond\varphi$ (notice that this is not the diamond belonging to our earlier necessity operator or the diamond defined by $\neg B\neg$).

This will give us the following preconditions (29) on an utterance of *might* φ as instances of the assertion precondition discussed before.

(29)
$$\begin{array}{l} \sigma \not\models S\Diamond\varphi \\ \sigma \not\models S\neg\Diamond\varphi \\ \sigma \not\models H\Diamond\varphi \\ \sigma \not\models H\neg\Diamond\varphi \end{array}$$

This entails two of our previous conditions, i.e. (30).

(30)
$$\begin{array}{l} \sigma \not\models S\neg\varphi \\ \sigma \not\models H\neg\varphi \end{array}$$

but not the other two we had before:

(31)
$$\begin{array}{l} \sigma \not\models \neg S\neg\varphi \\ \sigma \not\models \neg H\neg\varphi \end{array}$$

Instead we only get the weaker condition (32).

(32) $\sigma \not\models \neg S\neg\varphi$ or $\sigma \not\models \neg H\neg\varphi$

It would appear though that this condition is more correct than the earlier one. Suppose it is common ground that S does not know that not φ. Our precondition then entails that it is not common ground that H does not know this. H's assent would add something to the common ground.

(33) S: I do not know that φ is false. It might be that φ.
H: Yes, it might.

Inversely, suppose that it is common ground that the hearer does not know that φ is false, e.g. because the hearer has asserted φ before. Then the condition boils down to it not being common ground that the speaker does not know that $\neg\varphi$. This seems very natural, witness (34)

(34) H: John is ill.
S: He might.

So, it seems clear that our earlier conditions are too strong and that the current ones are better.

Also, the contribution changes slightly (and I think unimportantly):

(35) $\sigma[S\Diamond\varphi]*$

The hearer can assent by making the common ground into (36).

(36) $\sigma[S\Diamond\varphi] * [H\Diamond\varphi]*$

Denial would indeed be the negation of φ and the curious declining of the speaker's proposal would be equivalent to asserting $\neg\Diamond\varphi$, which given the fact that $\neg S\neg\varphi$ has been established comes out as the sophisticated doubt about the hearer's disbelief that we found before.

I conclude that $\Diamond\varphi$ is as good an approach to *might* in the current context as the separate speech act theory. It moreover makes *might* a logical operation with a distributive and eliminative update.

One can wonder however whether we have captured the meaning of might, and, indeed, I am not convinced. Suppose John is a BSE expert to whom we ask: Can the consumption of cheese lead to BSE? We of course do not have a clue, that is one of the reasons we ask this to John. John now says: it might. John seems to speak not so much on behalf of us, the conversational partners, but on behalf of his professional group: The BSE experts have not been able to rule this out.

Consider further the embedded use of *might* in e.g. *John thinks it might rain.* It seems obvious that neither the speaker's opinion nor the hearer's opinion as to whether it rains has any bearing on the truth of this attribution. In (37), there are two examples.

(37) John is home but Bill thinks he might be at work.
John is at work but Bill only thinks that he might be home.

Both doubts point in the direction of conceiving of *might* as an epistemic operator which claims of a certain group of people that they do not have the information to rule out the complement. The group of people would be determined by the context, much like a pronoun. The group must obey one constraint: the speaker or thinker must be inside it. In a conversation, when *might* does not appear in a propositional attitude context as generated by verbs like believe, know or say (these verbs would change the identification of the group, as they may change the identity of the speaker or thinker) a very natural resolution of the group parameter is the group of the conversational partners. So the analysis we provided is only a special case.

It can also be shown that the "stability facts" from Veltman's paper around *might* are undisturbed. We can have sequences

(38) $\textit{might } \varphi.\ \varphi$
$\textit{might } \varphi. \neg\varphi$
$\textit{might } \varphi.\ \textit{might } \neg\varphi$

but not sequences like

(39) $\varphi.\ \neg\varphi.$
$\varphi.\ \textit{might } \neg\varphi$
$\varphi.\ \textit{might } \varphi.$

All of these acceptabilities and inacceptabilities can be explained from the assertion preconditions.

To sum up, we have presented a theory of might which makes it into a logical operator which exhibits both distributivity and eliminativity. We concur with Veltman in his assumption that (normally) might sentences do not affect the factual basis of the common ground and in the contention that there are certain stability facts around might. We do not think however that these observations lead to an analysis of simple might-sentences which makes them into non-assertions or even turns *might* into a pragmatical operator.

7 Conclusion

This paper was written in response to an observation and a worry by David Beaver (p.c.). The observation was that a sentence $\textit{might}\varphi$ (with the Veltman semantics) is a counterexample to the equivalence between the Kamp and the Stalnaker update rule for belief. Indeed the Stalnaker rule leads to incorrect results. The worry was that common ground updating might well be inconsistent. The observation is devastating for any theory of presupposition resolution and accommodation in update semantics which wants to treat belief contexts, as using the Kamp rule would make the choice between resolution and accommodation or the choice between different accommodations dependent on individual possibilities, whereas these choices determine the global interpretation of the sentence. For a proper treatment, we need the Stalnaker rule. The equivalence can only be maintained, if we find no operations in the sentence which are either non-distributive or non-eliminative. A presupposition operator as proposed by Beaver *Beaver 1992* would be the other candidate that I know of. And I just indicated that it coexists badly with the Kamp rule.

This work needs follow-up in three directions. The first is an obvious one: first order logic, which will also allow more questions. The second direction, is to find out more about corrections. We can now only state correctness conditions for corrections, but we want to be able to actively retract material and, importantly, to guarantee the continuation of the common ground. If the effect of retraction is not public, the new common ground is not public and therefore not a common ground. The third direction is to incorporate more than just facts in a common ground. We can have joint public goals and obligations and there are speech acts involving goals and obligations. It remains to be seen whether these can be incorporated in our model.

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