

# A Neoclassical Analysis of Belief Sentences

Henk Zeevat  
University of Amsterdam

The classical logic of belief is (Hintikka 1971). Hintikka provides a belief operator defined over possible worlds and treats quantification by quantifying over a set of individual concepts. This paper adds some modern features.

Many problems are known about possible world based belief theories and the aim of this paper is to repair a number of them. The aim is not so much to give a logic of belief, but rather an analysis of belief reports. What we are interested in, is a theory of the interpretation of belief sentences which connects to a logic containing a belief operator. The theory of interpretation makes the connection of the surface sentence to its logical representation an indirect one, determined by various contextual factors. Some of the problems that arise within the logical system in which the belief report is represented do not arise for the natural language belief reports. So what we are studying is not the logic of belief but the intuitive consequence relation between belief reports. I believe bringing in the recent advances in formal pragmatics to the study of these problems brings about a clarification of some issues around belief sentences, notably about the status of names and indexicals in opaque contexts.

What this paper adds to Hintikka's theory is first of all a new interpretation of the belief alternatives assumed by Hintikka. This analysis is Haas-Spohn's diagonal theory of belief alternatives, based on a radical indexicalism towards the meanings of natural language lexemes. Without this underpinning, the current paper could not have been written.

The second element that is added to Hintikka's theory is a different account of the individual concepts which are the basis of his account of quantifying into belief contexts. This account is in essence my earlier history-based theory of objects of belief (the most recent version is Zeevat 1994), but this time formulated in possible worlds semantics rather than in discourse representation theory. This formalisation is useful and adds to the original account.

Third, we offer an alternative account of proper indexicals. Here, we use the theory of presupposition that has been developed over the last ten years primarily by Van der Sandt (Van der Sandt 1992).

The resulting theory also offers a radically different account of existential *de dicto* belief reports. This account has an important consequence: it opens the door to a solution of the problem of closure under logical consequence, a solution that however relies on the idea that all judgements are essentially existential. This view can be found in certain versions of Davidson's approach to event semantics, in certain forms of DRT and of course in the intuitionistic tradition, especially in type theory.

The organisation of the paper is as follows. In the first section, we will introduce Haas-Spohn's theory of subjective interpretation and the consequences of this the-

ory for the analysis of belief sentences. Section 2 removes the basic obstacle to the extension of this theory to all *de dicto* belief sentences: Kaplan's theory of demonstratives. Section 3 is concerned with individual concepts and will enable us to give a full account of belief sentences. Section 4 gives a closer look at two traditional problems with belief sentences and presents a theory of names. Section 5 finally offers the tentative solution to closure under logical consequence and explores some consequences of the present theory for the theory of information and for natural language semantics.

The theory of belief that defines the truth of belief reports in terms of a set of belief alternatives for the subject suffers from a large number of problems. These problems originate from on the one hand the conception of the relation of belief and necessity and on the other from the fact that a set of possible worlds is always closed under logical consequence. Representative of the first problem is Kripke's puzzle (Kripke 1979).

Logical consequence is a problem of the connection of belief with practical reasoning. As beliefs are the basis for action, the theory commits us to actions we are not in fact prepared to carry out. Kamp (1994) gives a simple example involving an arithmetical truth.

## 1 A Diagonal Theory of Belief Alternatives

What information does a subject get from a conversation? Which belief alternatives can be eliminated?

Suppose you are availing yourself of the Blind Date Telephone Service and you are speaking to a girl who claims to be a blonde. The subjective information you are deriving from her claim (which you happen to believe) is not at all the same as the proposition expressed by the sentence "I am a blonde" which she whispered in the phone. That proposition is about that girl at the other side of the phone, Anita, a person you do not happen to know. The information you have cuts down your set of belief alternatives (the other hair colours) but the criterion by which you cut them down is not related to the girl herself, but to whoever happens to be on the phone according to the particular alternative. There are some limitations: she has that kind of voice, she is that sort of person, but within those limits anything goes. If the person on the phone in the alternative is a brunette, the alternative is rejected, if she is a blonde it stays. What is going on with Anita in the alternative is irrelevant.

The criterion for eliminating a belief alternative uses not the actual referent of "I", but the referent of "I" according to the belief alternative. If the "I" of the belief alternative is not a blonde, the alternative is eliminated. That means, we have to diagonalise the sentence's character (in terms of Kaplan's character semantics) in order to find out which belief alternatives survive the update. The subjective meaning of the sentence for a person *a* can be defined as the intersection of the diagonal of the sentence's character with the belief alternatives of *a*. This analysis of subjective information is due to Haas-Spohn (following ideas of Stalnaker 1978) and it seems correct.

The disquotation principles introduced by (Kripke 1979) immediately connect subjective information to a theory of *de dicto* belief. According to this theory of belief a sentence describes one of your beliefs if the diagonal determined by the sentence is a superset of your belief alternatives.

This is an alternative approach to the semantics of belief sentences. Certain possible worlds are consistent with all that a person believes. Let us call them belief alternatives. To employ a formulation of Haas-Spohn(1994): if you would put the person into such a possible world she could not discover that it was not the actual world because the world meets everything that she believes. There are no surprises in a belief alternative: they are what the actual world can be according to a belief subject. If we have possible worlds, characters and belief alternatives, we can define a belief operator by stipulating  $B_a\varphi$  is true iff for each belief alternative  $i$  of  $a$ , the proposition expressed by  $\varphi$  at  $i$  is true at  $i$ <sup>1</sup>. If  $\chi_\varphi$  is the character of  $\varphi$  and  $bel(a, j)$  are the belief alternatives of  $a$  in a world  $j$ , we get:  $\chi_{B_a\varphi}(i)(j) = 1$  iff  $\forall k \in bel(a, j)\chi_\varphi(k)(k) = 1$ .

The character based semantics can be transformed in a rather straightforward way into the framework of Hintikka. We define a new set of possible worlds  $W'$  from the old worlds  $W$  by putting  $W' = \{ \langle w, w' \rangle : w, w' \in W \}$ . We now define  $\llbracket \alpha \rrbracket$  as a function from  $W'$  to the appropriate  $\alpha$ -denotations, by putting  $\llbracket \alpha \rrbracket(\langle w, w' \rangle) = \chi_\alpha(w)(w')$ . The modal accessibility relation (which before was the open relation) becomes  $nec(\langle w, w' \rangle) = \{ \langle w, w'' \rangle : w'' \in W \}$  and the new set of belief alternatives is  $bel(a, \langle w, w' \rangle) = \{ \langle w'', w'' \rangle : w'' \in bel(a, w') \}$ . In this way, the belief alternatives are situated on the diagonal of the product and, by the same image, the set of modally accessible worlds from a given world, is the vertical line on which it is situated.

I will further assume that the meaning of non-logical basic expressions is determined in a way that is uniform for their category. That the denotation of John in all possible worlds is the individual John depends on the contingent fact that John bears the name “John” in the actual world. I assume that the principle from which this is an instance is true not just in the actual world, but in all worlds on the diagonal<sup>2</sup>. This principle can mutatis mutandis be extended to (kinds of) verbs, nouns and adjectives. Its full adoption is equivalent to the theory of formal character introduced by Haas-Spohn.

## 2 Demonstratives and Indexicals Revisited

The theory outlined works fine for *de dicto* belief attributions that are free of variables and indexicals or demonstratives. It is rather easy to show it fails when indexicals occur.

Consider an utterance of (1).

- (1) John believes that I am bald.

The diagonal theory forces us into the position that it would be John’s opinions as to who the speaker of the sentence is that determines the referent of *I*. Yet John most likely does not have any opinion in that matter as he is not present at the

<sup>1</sup>It may be objected here that we are neglecting an important aspect of character semantics: the fact that contexts give us in addition to possible worlds also speakers, times and places. As this is not an objection to the theory I will arrive at, which does not rely on extra structure in the context for its treatment of indexicals. Here the reader is invited to add as much structure to the first parameter as she feels is necessary and to live with the consequence of having slightly too much structure in the resulting notion of a possible world.

<sup>2</sup>On a pointed model, we might express this as a schema of MPs like  $\forall x(\text{name}(\text{“john”}, x) \leftrightarrow \Box x = \text{john}) \wedge \forall y B_y \forall x(\text{name}(\text{“john”}, x) \leftrightarrow \Box x = \text{john})$

occasion of the utterance. We are rather reporting something which John would have rendered (if he in fact believes it) as: He is bald, or That man is bald or even Bill is bald. There is no systematic way of reconstructing what John would have said and thereby it becomes hard to state what *I* contributes to the meaning of the complement.

How to deal with direct reference then? We have found a problem with assuming that direct referentials are “local” elements evaluated by “global” rules. There is a conflict between our analysis of *de dicto* beliefs as holding when they are diagonally true on the belief alternatives of the subject and what happens if I tell you (1) John’s absence. John’s opinions about who I am are irrelevant. He has a belief about me.

If we assume that belief sentences are diagonally interpreted, the context parameter is the wrong one for indexicals: it equals the context of utterance with one of the belief alternatives of the subject.

We can look for a solution in different directions. For example, we could increase the number of parameters: the fixed context of utterance, the floating context of belief (equal to the first parameter outside beliefs) and the circumstance parameter. We then divide direct referentials into two groups: a group using the first parameter and a group using the second. “Me” would be of the fixed kind, a name would belong to the second.

The problem with this approach is that neither for names nor for indexicals we can uniformly say to what group they belong. Sometimes the belief subject is familiar with the name and its bearer, sometimes he knows the bearer, sometimes he just knows the name. Classically, one would say that in the last case it is a *de dicto* belief, (which means it is non rigid by different values for the second parameter) whereas in the second case the name is interpreted external to the belief. In the first case we should get both: it is interpreted by the first parameter and shifts with the second. So in some cases the name would count as directly referential, in some cases it would be subjective and in some cases finally, it would be both. That the same holds for indexicals is a little bit harder to see. In the example (1), we obviously have a direct interpretation. But what happens when the belief subject takes part in the current conversation? Can his views about what the indexicals denote be ignored? As it turns out, they cannot but this is better explained after introducing the alternative view.

In the DRT-tradition, a more realistic account is available in the presupposition resolution and accommodation model developed by Van der Sandt. According to this model the treatment of a presupposition trigger (a definite article, a factive verb, a cleft construction etc.) proceeds as follows. We determine at the position of the trigger the content of its presupposition. We then determine if it is possible to resolve the presupposition to material that is accessible from the position of the trigger. Here there is a preference for locally available material. If it is we are nearly done, only the variables in the computed presupposition have to be unified with the corresponding variables in its antecedent. If resolution does not work, we accommodate the material with a preference for the global context, a preference that can be overridden by conflicts with the contents of the global context or with the informativeness of the current utterance.

A DRS can be understood as the structure in which we do the bookkeeping of the emerging common ground between the speaker and the hearer. Normally, this is the bookkeeping of the part that results from linguistic exchanges. Here, we also need to add basic data about the utterance situation: the identity of the speaker, the place

of the utterance, the time and other features. Also, when there is a prior common ground concerning the beliefs of the belief subject or when the belief subject is part of the utterance situation, facts concerning this person should be considered to be part of the DRS in which we are interpreting the sentence.

We also make an extra assumption about presuppositions in belief contexts: if it is not common ground that the subject believes there is nobody who bears the name, we locally accommodate that the name has bearer, also in case the name is externally resolved or accommodated<sup>3</sup>.

Let us consider the indexical *I*. In contrast to e.g. the bearer of a proper name, the speaker of the utterance will be part of the common ground between speaker and hearer and it will also be common ground whether or not the belief subject is party to the same common ground. So accommodation instead of resolution does not play any role at all. The identity of the speaker is common ground and it is common ground whether this is also so for the belief subject.

So we get the following predictions concerning an (immediate) occurrence of *I* under a belief operator:

1. If the belief subject is party in the common ground between speaker and hearer (generally this means that she is part of the utterance situation), the discourse referent introduced by *I* will be unified with the discourse referent representing the speaker as the speaker in the beliefs of the belief subject. That discourse referent is identical to the discourse referent representing the speaker in the DRS as a whole.
2. Otherwise, the discourse referent is resolved to the discourse referent that represents the current speaker in the common ground.

This makes a subtle difference in case 1: it rules out that the belief subject will represent me in his belief by a description of which he does not assume that it refers to the same person as the speaker. If I happen to be the last person to leave the building and John does not know this, the fact that John thinks the last person to leave the building is ill, is not sufficient for the truth of the preferred interpretation of (1) .

Why is this theory of indexicals superior? Consider the following situation<sup>4</sup>. Harm

---

<sup>3</sup>This assumption has been challenged in the otherwise excellent Geurts 1995. Geurts however agrees with the predictions that follow from my assumption. He proposes to derive the predictions by means of a pragmatic mechanism supplementing his own single accommodation or resolution model. This would suit me fine, if he would have such a mechanism on offer. I suspect however that such a mechanism is as hard to find as the mechanism postulated by Karttunen that would derive global accommodations from local ones.

<sup>4</sup>The example

- (2) If I would have been the wolf, I would eat you now.

(U. Haas Spohn, p.c.) is another putative counterexample to the theory of direct reference. The example is spoken by the mother goat to her baby goats as she has found the door unlocked against her express instructions. Interpreting the antecedent, we have to resolve *I*. There is in the antecedent no current speaker so the pronoun is resolved to the external speaker, who is in the common ground not just the current speaker and the mother goat but also the person who has just opened the door. The properties of being the current speaker and the mother goat are given up as they are not consistent with the assumption that this is the wolf. So we inherit in the counterfactual situation only: person who just entered the door. To my mind, we do not have genuine counterexample here. “I” still refers to the speaker of the utterance, who in the

has after work made his way to one of the many pubs on the busy street where his office is located. Due to his immodest consumption of beer, he has at one point formed the belief that he is not in the pub in which he happens to be but in the pub on the other side of the street. Looking out of the window he sees in the pub across the street his colleague Wim. I stand next to Harm and seeing him looking as he does and knowing about his state of mind I remark to you:

*Harm thinks that Wim is here.*

As Harm believes that he is in pub A, which happens to be where he sees Wim, this report is true on the premises of the direct reference theory and also on the widest scope theory about indexicals discussed and rejected by Kaplan<sup>5</sup> But according to my intuitions this interpretation is hardly available.

On our predictions, (2) is false on the preferred interpretation. As Harm shares with us (part of) the speech situation, it is his representation of the place of the utterance (in fact pub B) which prevails and he would not say “Wim is here”. After all, the beer has not impaired his vision.

### 3 What are Individual Concepts?

Let me try to spell out the consequences of these considerations for the meaning of individual terms like demonstratives, first person pronouns and names. Let’s assume Harm says *that man* pointing to a balding academic holding a glass of dark beer. What we had to say before is that since Harm is the speaker in the actual world and since he in that world points to a particular person, say the professor of French poetry, it means that person in each of the worlds in the modal hull of the actual world. What we have to say now is more: we have to say what *that man* denotes in the diagonal. The first thing that follows is that it does not always denote something. In some worlds Harm did not say *that man* and in others he omitted the pointing. If we diagonalise over those worlds we fail to get a referent, in the first case because there is nothing that has a relationship to the utterance (it did not take place) in the second case because the rule associated with the pointing does not point: the demonstration is only partially there. The second conclusion is that the meaning is non-rigid. There are certainly worlds where somebody else might have been standing there. Importantly, we may fail to see clearly whom Harm is pointing to and we have thereby belief alternatives in which “that man” has different values.

Much the same holds for the pronoun “I” and for names. The meanings we have to live with for individual terms are non-rigid incomplete individual concepts over our extended set of possible worlds.

Our acceptance of the theory of subjective information has landed us in this predicament and it is an uncomfortable position: we have lost the beautiful and simple theory of quantifying in based on Russell’s singular propositions. We are suddenly left—for the attitudes—with partial non-rigid individual concepts and have to make do with them. We have to answer the question which individual concepts there are and how we can deal with them.

---

counterfactual situation assumed would be the wolf as well (and loses the properties inconsistent with that assumption).

<sup>5</sup>Within our model we would get this theory by ruling out local resolution, i.e. abolishing clause (1).

That not every partial function from (new-style) worlds to individuals is available seems given with our intuitions about belief. We do not want to infer from: *John believes that somebody ate the cake* to *There is somebody such that John believes he ate the cake*. (We combine, for each belief alternative of John, the object that ate the cake there. ) We also do not want to give up rigidity, only with extending rigidity to belief alternatives as well. So, we must assume that for each object in the actual world we have a rigid intension over its modal hull. But this assumption offers no help for quantifying into belief contexts, as the intensions so described are insufficient to make any *de re* beliefs true.

My solution is essentially the one that I have advocated in earlier work (Zeevat 1994). That solution attributes to persons objects that are objects of belief. Objects of belief in turn can be counterparts of other objects and can themselves have other counterparts. It is when they are counterparts of real objects, that they satisfy *de re* beliefs. For finding out what objects of belief there are, we have to enquire what experience the subject has had and to what communication she has been exposed.

The difference with my earlier work is that the objects of belief here become partial individual concepts and that we can give an account of their genesis in terms of Hintikka's belief alternatives. I believe the account here improves in some ways on my earlier work. The starting point are cognitive events. These are events in which the subject acquires a new belief. Examples of such events are perceptions, acceptances of an assertion and reasoning. One thing which happens in veridical perceptions and acceptance of true assertions is the genesis of new *de re* beliefs.

Take the example of a perception of an object that the subject has not seen before, let's say Bob's new Ferrari. Before the perception, it will be true that we can describe the Ferrari uniquely by some unique description, say *bob\_ferrari*. It then holds before the perception that

$$(3) \quad \neg \exists x (bob\_ferrari(x) \wedge belief(s, x = x))$$

After the perception this has changed, the subject *s* has acquired a new object:

$$(4) \quad \exists x (bob\_ferrari(x) \wedge belief(s, x = x))$$

Of course this is not all that has changed: the belief alternatives of the subject have changed to take in the content of the perception: the beautiful color and the attractive shape will be as the subject perceived them. On the rather unconstructive picture of belief alternatives assumed in this paper the update consists of two steps. First the subject has a unique description of what he sees. The uniqueness follows from the uniqueness of the description *what I am currently seeing* but takes in any other property or relation that the subject attributes to the object in his perception. This first reduces the belief alternatives to those where the description is satisfied (a normal eliminative update.) The second step is the addition to the set of individual concepts of the individual concept formed by the Ferrari (extended to the modal hull of the actual world) and the object that in each of the new belief alternatives is the Ferrari for the subject.

This is the simple case. The object may already be there as the subject may have heard about it before. Two things can happen: the subject recognises the Ferrari from his description so that the updating is limited to a simple update of the alternatives. Alternatively, he does not, which means that the perception will form a second individual concept, which derives from the same Ferrari. The final

possibility is misrecognition: the Ferrari is wrongly recognised as another object the subject is aware of. We get a new concept (the new perception extends the actual Ferrari, coinciding within the belief worlds with the older concept) and an update over the older concept.

Finally, there are hallucinations. Here we only seem to perceive something. It follows that the earlier steps in the updating are as before but the bridge itself cannot be added. We can however add the concept consisting of the referent of the description in each of the belief alternatives.

The general rule which we can abstract for this is that after the perception an individual concept bridges the actual world (if the perception was veridical) and the subject's belief alternatives with the bridge being given by a definite description in the belief alternatives given by the perception itself. Veridical perceptions of an object not seen before can be described as a switch in the truthvalue of a sentence of the form:

$$\exists x(x = a \wedge \text{believe}(b, x = x))$$

Before  $b$ 's perception the sentence was false, as there was no bridge connecting  $b$ 's mind to  $a$ . After the perception, such a bridge has come into being.

In communication things are not very different, except for the fact that now it is the beliefs of the speaker and the concepts organising his beliefs that take the place of the actual world. I will make the assumption that even in this case it is possible to speak of the object underlying the speaker's utterance: it is the internal object that causes that particular part of the utterance. This assumption can however be eliminated<sup>6</sup>

Bridges will now consist of the speaker's object extended to the belief alternatives of the subject. If the speaker's object was already a bridge it now becomes a longer bridge bringing us to yet other people and possibly to the actual world. The process itself is the same as in the case of perception. There are however two cases where the interpretation process can lead to empty bridges: hallucinated utterances and mishearings are one case, the other is the case of the speaker saying things he does not believe. What we here characterise is the fact that typically communication can lead to a switch in the truth conditions of (5)(e.g. Mary tells Susan about her purported enemy.).

- (5) Mary believes that she has an enemy and Susan believes that he is dangerous.

These processes fill the set of individual concepts. The occurrence of cognitive events add individual concepts to the given set of individual concepts.

Let us first have a look at the set of individual concepts available in a certain possible world at a given time. This world has a history in which it has acquired new individual concepts. The principle is that either a concept derives from a cognitive event or it was there all the time. We can also say something about what was there all the time. I already mentioned the rigid intensions of the objects of the world. Other concepts that have been always there are necessary objects of belief. I take it that for each subject there is one such object: the self, identified by the subject as the thing that is having these thoughts and experiences. This is a bridge:

---

<sup>6</sup>The assumption corresponds with the difference between the classical DRT approaches to indefinite reference (no uniqueness) and the approaches based on Evans's work. Non-uniqueness could be accommodated in principle.

it bridges between the subject and what she is in each of her belief alternatives.

Among the individual concepts that exist with respect to a mind there is a distinguished concept that denotes the body of the mind in the actual world and that is defined throughout the belief alternatives of that mind. Originally, the belief alternatives coincide with all that could be known. The self ranges over all these alternatives and realises each possibility as to what it can be. Later as the subject acquires empirical information about itself, certain possibilities with respect to who it can be are eliminated as are the alternatives themselves.

The self is what accounts for *de se* beliefs (Lewis 1979). Adding a property *selve*( $x$ ) to the logical representation language offers solutions to the other problems connected with ego-centric beliefs. Reasons of space prevent any further discussion, here.

Normally, the new individual concepts are (in the set-theoretic sense) extensions of already existing individual concepts, but the idea that all such new concepts form proper extensions is wrong. What happens in a successful communication or in a perception guarantees —if we assume uniqueness— that the originating concept —restricted to the modal hull of the actual world or to the belief alternatives of the sender of the communication — is —when the communication or perception is successful— and the interpreting concept are consistent. If there is overlap between the domains, the uniqueness assumption guarantees that the concept assumes the same values within the overlap. This is however not enough for unique extendibility. A simple example may suffice. Assume that on my way home from work, I have witnessed a traffic accident. Coming home, I tell my wife about the accident in the certain opinion that she will not tell our daughter about this, as she prefers to keep our daughter from such harsh realities. Yet, the next day my daughter tells me about an accident that happened. I assume that she is speaking of another accident, and not that my wife has talked. But my wife has and she is talking about the same accident. My concept of the accident is inconsistent with the concept that causes it, which contains my concept of the original accident.

To solve this problem we could make a formal distinction between the actual world and between the belief alternatives of each subject. This can be done using the selves. Another solution is to switch to a notion of semi-extension with respect to a belief subject. A semi-extension with respect to me, can overwrite values for the concept in my belief alternatives. As the causing concept will continue to exist, they can be recovered. In the example, it holds that there is an accident of which I believe conflicting things (e.g. that my daughter knows about it and that she does not know about it) but it is surely wrong to attribute to me an internal inconsistency). I will in the sequel, ignore this problem.

## 4 Some Refinements

Let us consider the Hob-Nob-sentence (6).

- (6) Hob believes that a witch poisoned his cow and Nob believes that she killed his pig.

On the hardest interpretation of the sentence there is a rumour in the village about a witch and Hob and Nob are parties to that same rumour (due to a conflict they never talk with each other). In terms of our model, this means that there are two

chains to the originator of the rumour, but not chains such that one extends the other. The sentence is true therefore if there is a chain (of which neither Hob or Nob is the endpoint) which can be extended to both the Hob chain and the Nob chain.

We can write this in the following way:

$$(7) \quad \exists xyz(x \subseteq y \wedge x \subseteq z \wedge \text{belief}(\text{hob}, \text{witch}(y)) \wedge Py) \wedge \text{belief}(\text{nob}, Qy))$$

There are two problems with this representation. First, it seems we have a *de re*-version of an obviously *de dicto*-sentence. and, second, the sentence has one quantification and not three.

Let us start with the second problem, which is less serious anyway. We can build into the semantics of the belief operator the possibility to extend individual concepts. This is easily implemented.

$$(8) \quad \begin{aligned} M, i, g \models \text{belief}(x, \varphi) &\Leftrightarrow \exists h(\forall v \in VAR (g(v) \subseteq h(v)) \wedge \forall j \in \text{bel}_i(g(x)) M, j, h \models \varphi) \\ M, i, g \models \neg \text{belief}(x, \varphi) &\Leftrightarrow \forall h(\forall v \in VAR g(v) \subseteq h(v) \rightarrow \exists j \in \text{bel}_i(g(x)) M, j, h \models \neg \varphi) \end{aligned}$$

Using this semantics we can reduce our representation to (9).

$$(9) \quad \exists x(\text{belief}(\text{hob}, \text{witch}(x)) \wedge Px) \wedge \text{belief}(\text{nob}, Qx)$$

The first problem can be solved by undoing the existential import of the existential quantification. In order to be able to express existential import, we have an existence predicate with the semantics as in (10).

$$(10) \quad \begin{aligned} M, i, g \models Ex &\Leftrightarrow g(x)(i) \text{ defined} \\ M, i, g \models \neg Ex &\Leftrightarrow g(x)(i) \text{ is undefined} \end{aligned}$$

The normal existential quantifier representing the "there is" of natural language will be now rendered by  $\exists x(Ex \wedge \dots)$ . Existential quantification is defined by (11).

$$(11) \quad \begin{aligned} M, i, g \models \exists x\varphi &\Leftrightarrow \text{there is a } d \in IC_i \text{ such that } M, i, g[d/x] \models \varphi \\ M, i, g \models \neg \exists x\varphi &\Leftrightarrow \forall d \in IC_i M, i, g[d/x] \models \neg \varphi \end{aligned}$$

With our E-predicate we can represent the *de dicto/de re*-distinction as in (12).

$$(12) \quad \begin{aligned} \exists x \text{belief}(y, \varphi) \\ \exists x(Ex \wedge \text{belief}(y, \varphi)) \end{aligned}$$

This leads to a simple treatment of the Hob-Nob sentence.

The final problem we have to face is how to link this representation to syntax. The simplest solution is to call on DRT once more. We need to add the proviso that a belief box does not have place for its own discourse referents: they drift away to the embedding box. We have to distinguish them however from other discourse

referents. This can be achieved by making sure that all other discourse referents are entered with a condition ensuring their existence. This is superfluous in case the expression responsible for their introduction also places a condition in which they appear as an argument in the box: the standard semantics for simplex conditions lets them entail the existence of all their arguments, e.g. from  $give(x, y, z)$  we can infer  $Ex$ ,  $Ey$  and  $Ez$ . But foreign discourse referents (the ones not also occurring in an argument position in a simplex condition in their box) do not need to exist. This can sometimes make it necessary to bring in a condition  $Ex$  in the other rules of the development algorithm.

Is there something wrong with representing *de dicto* belief reports as we have? I think not, in fact it is completely the reverse: there are serious problems with the traditional representation. On the picture I have been developing individual concepts also arise when it only appears to the subject that an object causes a perception or assertion. The concept will then be restricted to the belief alternatives of the subject. In traditional terminology, an internal object is created for which there is no external object. The traditional representation of *de dicto* existential sentences completely misrepresents this fact. For a *de dicto* existential to be true on the standard representation, it is sufficient that the non-satisfaction of a predicate is not consistent with whatever the subject believes.

But intuitively this not the same as the subject believing the existential sentence. Belief requires that she has noticed the incompatibility of the non-satisfaction of the predicate with her other beliefs which on the classical view would be equivalent to postulating an object satisfying the predicate. This postulation is a cognitive act, comparable to perception and believing an assertion, and it would follow that the object must be an individual concept. (On the intuitionist view, it would not even be possible to attribute an existential *de dicto* belief on the basis of the inconsistency of the non-satisfiability of the predicate with the subject's other belief.)

Our account of the emergence of individual concepts faithfully captures these reasoning processes: coming to believe in an existential sentence is the creation of an individual concept that is minimally defined over the belief alternatives of the subject.

## 5 Proper Names

The treatment of proper names is an important test case in semantical theories of belief. Rather than starting from the premiss that a name is an individual term with an autonomous meaning—the assumption in most of the philosophical literature—I will start from the opposite point of view, namely that names are a special kind of anaphoric device, much like anaphoric pronouns. This starting point makes it possible to explain some of the properties of names that do not come into the picture if we follow the philosophical literature, such as their presuppositional properties and the possibility of using the same name for different people in the same context. Haas-Spohn's 1994 shows some of the difficulties that are encountered when we start from the other premiss. Let me point out in addition that the anaphoric theory is the only practical option in natural language interfaces for the computer.

Names and anaphoric devices such as personal pronouns and definite descriptions have in common that they are all devices that find a suitable antecedent in the common ground and pass the value that they found to the construction at hand. Names and pronouns of course differ in the way in which they do this. Personal

pronouns search the immediate context of their occurrence for a suitable antecedent that matches it in gender and number and that does not directly c-command it, and definite descriptions (in their anaphoric use) search the larger context for an object that meets the content. In just that way names search in an even wider context for an object that is so-called. Names can have a rather permanent status and they can well be part of the wider common ground given by the participation in a larger culture, but they can also be extremely limited in their applicability. They can be introduced in the context of a game and disappear thereafter or they can be part of a logically embedded context, witness such locutions as (13).

(13) If we call this number  $n$ , then let  $m$  be  $n^2$ .

The behaviour of names can be described by saying that their use presupposes the existence of something called so and so. Like other presuppositional triggers, the presupposition will—in some contexts and only by default—leave its content behind in their local context. To be precise, we have to split the presupposition in two, one the statement introducing the discourse marker and its existence (e.g.  $x \wedge Ex$ ), second the condition that that discourse marker is named by the name (e.g.  $name("john", x)$ ). The reason for this division is that they have different resolution and accommodation properties. The existence presupposition must at least be locally accommodated, the name presupposition need not. Further, the name presupposition needs to be resolved to some part of the common ground (which does not rule out local and intermediate accommodations which are however not necessary) and it even holds that it can find its antecedent in contexts which are not accessible for e.g. pronoun resolution. These are not unusual properties for presupposition triggers. The name presupposition is in respect of obligatory resolution and access to inaccessible contexts comparable to the presupposition of the particle *too* and the existence presupposition (necessary also for the pronoun *he* and for anaphoric definite descriptions) behaves similar to the sortal presuppositions generated by triggers like *bachelor*.

The causal theory of names explains how a certain name comes to be the name of a person. The fact that the person has the name is exploited by the interpretation mechanism to identify the object the speaker is stating something about.

In this theory names do not have a formal counterpart which is their meaning. There is only the contingent relation:  $x$  has the name  $w$ . But we can enquire what sort of meaning we should give to a formal name occurring in situ to obtain the same results. What we ask for is an individual constant interpreted by an individual concept that denotes in an alternative what the bound variable denotes in the current theory.

Pursuing this line, we find a number of problems which to my mind suffice as an argument to abandon the Millian theory of proper names defended by Kaplan. But let us first see what is correct about that theory.

First, it follows that names are directly referential from the fact that both presuppositions of the name normally end up in the global context. That means that it is decided by the world of the global context (the actual world, if we are not reading a novel, that also partially determines the context of utterance) who the bearer of the name is and not by the worlds that are relevant if the name's context is intensional.

Second, if we limit ourselves to modal alternatives the name is also rigid. This follows from the way we set up the set of individual concepts: we initialised them as containing (next to the selves) only concepts rigid on the modal hull of the actual

world.

Third, the account of the causal theory of reference for proper names is fully consistent with what I have said so far. (I am in full agreement with it but to conceive differently of how names have a referent does not seem to have much consequences: all that is needed that  $name(x, y)$  is a contingent relationship).

So I think the present theory endorses all the predictions of the classical account on names. But it does not give up at points where the classical theory fails to predict or gives the wrong predictions.

I already mentioned the marginal cases where the name is locally introduced in a subordinate context. Here we get failure of direct reference and rigidity. Standard presuppositional examples can be adapted to construct cases where the resolution is local and other alternatives are involved, as in (14).

- (14) Bill believes that one of your colleagues is called “Tom” and he believes that Tom is a spy.  
It is possible that there is a person called “Tom” and that Tom is a spy.

Tom is here neither directly referential nor rigid. In the first example, it is Bill’s belief alternatives that determine who Tom is and, following our earlier considerations, and it is plausible that Bill does not know have a particular colleague in mind. In the second case, it is the value of Tom with respect to a modal alternative that determines the referent of the unquoted occurrence of Tom in that same alternative. It is also fully consistent with Tom actually being somebody else.

There is also failure of the causal theory.

- (15) Bill believes that that man there is called Tom and that Tom is a spy.

If we assume Bill is wrong nobody baptised that man “Tom”. Yet the second occurrence of “Tom” refers to him. Of course, in the belief alternatives of Bill, there must be baptisements but they cannot cause reference in the actual world.

The classical theory also fails to account for legitimate occurrences of names without a reference, such as (16)

- (16) Odysseus worshipped Zeus

or (17)

- (17) John believes Vulcan is covered with craters.

which are easily treated in the present theory. Most disastrous are however the cases of Pierre and Paderewski.

- (18) Pierre believes London is ugly and Londres is pretty.

As London and Londres are faithful translations of each other they should count as the same name. I.e., even in Pierre’s belief alternatives they have to refer to the

same thing (on the premise that the name has an individual concept as a meaning). It follows that there cannot be such alternatives. Yet we feel that Pierre does not have an internal contradiction and —more importantly— that there are worlds which Pierre cannot discover to be different from the actual world. (The possible world where there are two British cities called London and Londres respectively and where “Londres” is not a translation of “London” and where Londres is nice and London is not.) The conclusion here follows from the assumptions that “London” and “Londres” translate each other and the assumption that names are interpreted by an individual concept.

In the Paderewski case, somebody believes that Paderewski is a gifted musician and at the same time that Paderewski is not a gifted musician, due to a failure to realise that they are the same person after meeting them at different occasions. The case is similar to the Pierre case but does not rely on translation. Here we have clearly one name which cannot be interpreted as one name in the subject’s belief alternatives. If Paderewski is taken as a single name interpreted by an individual concept, there can be no belief alternative for the subject, contrary to our intuition. An alternative that works has two people called Paderewski, a musical one and a non-musical one. Such alternatives are ruled out by Paderewski being a name in the classical sense.

In our theory such beliefs are possible by different local resolutions: one to one counterpart of Paderewski one to another counterpart of Paderewski<sup>7</sup>. The subject is assumed to know the name Paderewski as a name of each counterpart.

Let us briefly examine what are the alternatives for one who would want to maintain a single meaning for the name Paderewski. One alternative would be to give up the idea that the meaning is an individual concept. It will refer to one thing in the actual world but will have a multiple reference in our subject’s belief alternatives. This will not do as we do not have a criterion anymore for applying a predicate to a name in a world. We can say that all referents must meet the predicate perhaps but this does not do: neither of the two beliefs in the example will hold. If we say that only some must meet the predicate, the negative version of the belief will be false. This is just not a way out.

We can also not let the name denote a set of individual concepts for much the same reasons.

The only things that comes close is to let the name (the name of one particular referent) be ambiguous, selecting a different concept at every occurrence of the name. That boils down to a version of the present theory. But the problem would be that all names would start to be ambiguous. Consider your own name. It is certainly possible that you are like Paderewski in that somebody has been introduced to you on two different occasions and has built up contradictory opinions about you due to a failure to recognise you on he second occasion. If we consider the belief alternatives of that person in that possible world, it follows that your name is ambiguous in

---

<sup>7</sup>And an explanation of why we need two belief attributions. A single one would make the local resolution to the first Paderewski strongly preferred. We need to explain why the formulations in (19) have the intended interpretation,

- (19) John believes that Paderewski is musical and that Paderewski is not musical.  
John believes of Paderewski that he is musical and that he is not musical.

while formulations as in (20) do not have this interpretation.

- (20) John believes that Paderewski is musical and not musical.  
John believes that Paderewski is musical and Paderewski is not musical

precisely the way “Paderewski” would be if we follow this theory.

A proper name occurrence can give information on the present view. It may be the case that the presupposition is (also) locally accommodated in a belief sentence. This will then add the information that (also) the subject of the belief report knows that the person is named in that way. If the common ground did not already contain this information, it will do so now.

For a simple belief sentence it seems to be the case that the default reading is one where the name is externally resolved and accommodated internally (proper name accommodation has more the characteristics of repair than of a normal device of introduction of a referent. One has to know who the bearer of the name is before one can use it.) . Only when the name cannot be externally resolved (it is common ground that there is no such thing) external resolution fails and only when the common ground has a reason for the subject not believing that the referent is so-called, the internal accommodation or resolution is omitted.

The content added by accommodation offers the basis for making distinctions between:

(21) John believes that Hesperus is Phosphorus

and

(22) John believes that Hesperus is Hesperus.

For a fuller discussion see (Zeevat 1994).

The process we have sketched so far offers an account of the content of fictional names. If Bill believes that Vulcan is a planet within a common ground in which it is well-known that Vulcan does not exist, it follows that the name presupposition is only locally accommodatable (if it is not already locally resolvable). This then means that in Bill’s belief alternatives there is a planet that bears the name Vulcan. The fact that we —as non-believers— use the name Vulcan and that it is common knowledge between us that certain astronomers in the past postulated the existence of a planet with that name makes the name also resolvable to the indirect context given by the beliefs of these astronomers. It is resolution to that context and the ensuing demand that there is an individual concept that fulfills the demand that it is called “Vulcan” within that context and the one given by Bill’s beliefs, that enforces that Bill’s belief is grounded in a counterpart of the object of the earlier belief.

How does this all compare with the Muskens’ theory of names? What Muskens suggests is that names (considered in their behaviour over belief alternatives) are definite descriptions. These definite descriptions have however an elusive character: it is hard to state their content. It would also —one cannot help feeling— be dangerous to state the content of any particular name. If one does so, it follows from the theory that it is impossible for any subject to believe that the person that is so called lacks the property from which the definite description is constructed. Muskens must therefore restrict himself to making a categorical claim: the name has a meaning which is formally an individual concept in that it denotes one single object in every world for which it is defined. Such meanings are known as the meanings of definite descriptions. My discussion of the Paderewski case led to the conclusion that it is not correct to describe proper names as having a meaning that

is one single individual concept. In that respect, the theory of this paper disagrees with the position defended by Muskens. In another respect however, we agree: the individual concepts indeed are indistinguishable from definite description meanings as traditionally conceived<sup>8</sup>.

The theory developed in this paper however agrees with Muskens about the central role of non-rigid individual concepts, i.e. definite description meanings. With a particular concept, in general there are associated many of these description, one for each of the belief subjects carrying it. Here, we are in the position to say something about the descriptions themselves. The basic individual concepts, the ones rigidly extending the objects in the actual world to its modal hull, can be taken to be actualist definite descriptions (the object that is actually so and so). Extensions by perception are again connected to a definite description (“what I see in this perception as being so and so”). Extensions by communication finally are again connected to a definite description (“what so and so describes to me as being so and so”). What seems to follow is that we get an object that is a definite description meaning that —if it spans a number of subjects— is generated by *different* definite descriptions, one for each subject. This then entails that it is misconceived to ask for *the* definite description that belongs to an individual concept.

I have argued that names do not have an eternal meaning. Rather they contextually select a subsisting individual concept from a range of candidates. The existence presupposition further demands that the concept be defined in the belief alternatives that are relevant for the evaluation of the interpretation of the sentence at hand (the alternatives relevant for the interpretation of the contexts accessible from the position of the name and which allow the accommodation of this presupposition).

This statement is more radical than it seems at first sight, as it throws doubt on the existence of eternal meanings in general. Many words in natural language (the ones in the open classes: adjectives, adverbs, nouns and verbs) can naturally be thought of as names of objects in other ontological categories: colours, kinds, properties, activities, states etc. *Mutatis mutandis*, we will have to deny that there are fixed eternal meanings for these words as well. What happens in our squared worlds is that the contingencies on which the semantic properties of e.g. a kind term depend can be absent or different in the other world. The “individual concepts” relating a subjective concept of the kind to the real kind can be many and diverse. It follows that there is little place for an eternal notion of meaning spanning both the real world and the minds that live therein.

## 6 Logical Consequence and Other Issues

Davidson’s event semantics has a variant where we associate an event argument with every natural language sentence, where this argument is always existentially quantified.

This event must be more complicated when we have a sentence that is a quantifica-

---

<sup>8</sup>It would be wrong to conclude that the traditional conception of definite description meanings is correct. On the contrary, the assumptions made in this paper suggest a different approach. Definite descriptions must be taken as expressions that find a referent in the context of utterance, i.e. as expressions presupposing the subsistence of an individual concept that meets the description and the existence of the individual concept. The only difference with names is in the content of the presupposition. This makes definite descriptions as rigid as names are, and leads to problems with traditional puzzles (The coach changes. The temperature rises.) for which a new solution must be found.

tion, a negation, a disjunction or an implication. The precise nature of the values taken by the event argument in these cases is a subject of ongoing debate involving a consideration of aspect and the proper analysis of negation in natural language. What is important in the current context is however that the position can underpin the following thesis (23).

- (23) A judgment is always a belief that something specific to the judgment exists.

The thesis has two aspects. It first makes all natural judgments existential and thus connects to our earlier considerations about existential *de dicto* beliefs. Second, a possible object satisfying the judgment must be specific to the judgement. That is, it must make this judgment true and not others<sup>9</sup>

Another possible underpinning for the thesis comes from the Brouwer-Heyting-Kolmogorov interpretation of intuitionistic logic and its successors<sup>10</sup>. According to this interpretation, each correct judgment has its own proof-object. To explore this further, we need to extend the interpretation to empirical proof-objects (evidences) and to connect the proof-objects to the theory of belief. I will not pursue this alternative here.

If we combine the principle with our treatment of belief, we see that we have an approach to stop the closure of belief sentences under logical consequence. Let it be given that a person believes  $A$  and that  $B$  is a logical consequence of  $A$ . By the principle  $B$  is of an existential nature. By our treatment of *de dicto* existential belief it is necessary that a cognitive act has created an internal object specific for  $B$  if the subject is to believe it. This is only so if there has been a cognitive act of the subject creating the object. So belief in  $A$  and  $B$  being a logical consequence of  $A$  is not a sufficient condition for a belief in  $B$ .

The same reasoning applies to mathematical truths and to closure under logical equivalence. It is an empirical fact whether the cognitive act required for the creation of a correct individual concept has taken place.

## 7 Conclusion

The speculative treatment of logical consequence would suggest that the present theory deals with all problems of belief. This is not so. A possible world semantics of the kind I sketched, with a set belief alternatives cannot deal with internally inconsistent beliefs, e.g. mathematical mistakes. As such beliefs occur, it follows that the present theory must, in the end, give way to a better one. One option would be to have a set of sets of epistemic alternatives, but space prevents me from a full discussion.

The present theory of belief however gives an adequate way of representing consistent information. We have three notions. One is the classical theory where an information state is a set of possible worlds. This is certainly an important notion, given that it constitutes our picture of the actual world and is directly relevant for

---

<sup>9</sup>There is some latitude here in the delineation of judgements. One can consider the judgment that John's car broke down in Paris as being composed of two different judgments, one that John's car broke down and one that this happened in Paris, which would both be true because of existence of the same event. This is tolerable. What would not be tolerable is that the same object would also satisfy that Harry lost his keys in Amsterdam.

<sup>10</sup>This interpretation can be extended to classical logic.

action. Our action is grounded in what we think the world is like. This notion of information only constrains our actions and cannot explain action or the motives for actions. The second notion takes the information together with the set of internal objects (the individual concepts restricted to the set of belief alternatives of the subject of the information). This is a finer notion and can be equated with the DRS notion of information. (Heim's slogan: a proposition is a set of possible world plus a set of discourse referents). This information explains motives for actions. Finally, we have situated information: the possible worlds with the bridges and chains. This information explains action in the world as it presents the subject related to the objects in the world.

One of the interesting aspect of the present theory is that it, as far as I know for the first time, gives a direct interpretation of discourse referents. They are neither psychological objects or "theoretical" objects, but they are normal semantic entities. There is no need for revision of first order logic or for representationalism in order to give an adequate interpretation to this central notion in current semantic theories.

The tradition which stands behind the existence of a name in the language community has been called a causal-intentional net. We can recover this notion within the present theory. Such a net is nothing else but the set of individual concepts of the actual world which are associated with the name. That is, they have the property of denoting in all elements of the diagonal over which they are defined an individual that is so-called and the further property that they denote the bearer of the name in the actual world. Such sets are nets, as numerous of the individual concepts cross each other.

The theory leads the way to a new conception of intensional semantics, where we have, roughly speaking, classical semantics in the centre, on the actual world and its modal hull, categorial meaning postulates, characterising the form of the meaning of words, over the whole model and a theory of contextual interpretation covering the connection between the two. But the model is not committed to this classicism. It is fully consistent to assume that the use of certain or even all words does not go back to successful baptisements, which have successfully individuated in the actual world objects, properties and relations. We would so make contact with the more sceptical traditions about meaning, such as Wittgenstein or deconstructivism. At the same time however, we have a model for constructing truth. We can after all rebaptise and fix meanings as we require.

## References

- Davidson, D. The Logical Form of Action Sentences. In: N. Rescher (ed.) *The Logic of Decision and Action*. Pittsburgh 1967, pp 81-95.
- Geurts, Bart. *Presupposing*. PhD, Universität Stuttgart. 1995.
- Hintikka, Jaakko. Semantics for Propositional Attitudes. In: Linsky (ed.) *Reference and Modality*. Oxford 1971, pp. 145-167.
- Muskens, Reinhard. Names. Chapter 8 of: R. Muskens, *Meaning and Partiality*, PhD. University of Amsterdam 1989.
- Haas-Spohn, U. *Versteckte Indexikalität und subjektive Bedeutung*. PhD Universität Tübingen 1994.
- Heim, Irene. File Change Semantics and the Familiarity Theory of Definiteness. In: Bäuerle, Schwarze & Von Stechow (eds.) *Meaning, Use and Interpretation*. Berlin 1983.

- Kamp, Hans and Uwe Reyle. *From Discourse to Logic*. Dordrecht 1993.
- Kamp, Hans. Prolegomena to a Structural Account of Belief and Other Attitudes. In C. Anthony Anderson & J. Owens (eds.). *Propositional Attitudes*. CSLI Lecture Notes 20, 1994.
- Kaplan, David. Quantifying In. In: D. Davidson & J. Hintikka (eds.) *Words and Objections*. Dordrecht, Pp. 206-242.
- Kaplan, David. Demonstratives. In: J. Almog, J. Perry, H. Wettstein (eds.) *Themes from Kaplan*. Oxford 1989, pp. 481-563.
- Karttunen, Lauri. Discourse Referents. In: McCawley (ed.) *Notes from the Linguistic Underground. Syntax and Semantics 7* New York 1976.
- Kripke, S. Meaning and Necessity. In: D. Davidson & G. Harman. (eds.) *Semantics of Natural Language*, Dordrecht 1972, pp. 253-355.
- Kripke, S. A Puzzle about Belief. In: A. Margalit (ed.) *Meaning and Use* Dordrecht 1979., pp. 239-283.
- Stalnaker, R. *Inquiry*. Cambridge (Mass.) 1984.
- Stalnaker, R. Assertion. In: Cole, P. (ed.) *Pragmatics. Syntax and Semantics 9*. New York 1978, pp. 315-332.
- Van der Sandt, Rob. Presupposition Projection as Anaphora Resolution. *Journal of Semantics* 9, 1992, pp. 179-196.
- Zeevat, Henk. The Mechanics of the Counterpart Relation. ILLC Report 1994. (to appear in *Langage*).
- Zeevat, Henk. Presupposition and Accommodation in Update Semantics. *Journal of Semantics* 9, 1992, pp. 379-412.

## Appendix

The following is an attempt at sketching a formal system for the theory described in the paper. It comes with no warranty and does not deal with extending individual concepts. Its sole purpose is to facilitate comparison.

The language has no individual constants, a modal operator, an existence predicate and a two-place predicate *belief* taking an individual term and a formula as arguments.

Models are of the form  $M = \langle U, W, nec, bel, IC, E, I \rangle$ , where  $U$  is a set of objects,  $W$  is a set of worlds, *bel* assigns to a world  $i$  and an individual  $u$  a set of worlds, *nec* maps worlds to the set of worlds modally accessible to them, *IC* gives the set of individual concepts in a world, *E* assigns to a world the set of objects existing in that world.

There are some connections between these:

If  $\langle u_1, \dots, u_n \rangle \in I(P)(i)$  then  $u_j \in E_i$  for  $1 \leq j \leq n$ .

$IC_i$  contains all constant functions mapping  $nec_i$  to  $E_i$ .

$g$  is correct for  $i$  iff  $\forall x \in VAR \ g(x) \in IC_i$ ,

$g$  extends  $h$  iff  $\forall v \in VAR$  if  $h(v)$  is defined then  $h(v) \subseteq g(v)$  and if  $h(v)$  is undefined, then  $g(v)$  is undefined.

$M, i, g \models Px_1 \dots x_n \Leftrightarrow g(x_j)(i)$  is defined for  $1 \leq j \leq n$  and  $\langle g(x_1)(i), \dots, g(x_n)(i) \rangle \in M(P)(i)$

$M, i, g \models \neg Px_1 \dots x_n \Leftrightarrow g(x_j)(i)$  is defined for  $1 \leq j \leq n$  and  $\langle g(x_1)(i), \dots, g(x_n)(i) \rangle \notin M(P)(i)$

$M, i, g \models \varphi \wedge \psi \Leftrightarrow M, i, g \models \varphi$  and  $M, i, g \models \psi$

$M, i, g \models \neg(\varphi \wedge \psi) \Leftrightarrow M, i, g \models \neg\varphi$  or  $M, i, g \models \neg\psi$

$M, i, g \models \neg\neg\varphi \Leftrightarrow M, i, g \models \varphi$

$M, i, g \models \exists x\varphi \Leftrightarrow$  there is a  $d \in IC_i$  such that  $M, i, g[d/x] \models \varphi$

$M, i, g \models \neg\exists x\varphi \Leftrightarrow$  for all  $d \in IC_i \ M, i, g[d/x] \models \neg\varphi$

$M, i, g \models \Box\varphi \Leftrightarrow$  for all  $j \in Nec_i \ M, j, g \models \varphi$

$M, i, g \models \neg\Box\varphi \Leftrightarrow$  there is a  $j \in Nec_i \ M, j, g \models \neg\varphi$

$M, i, g \models belief(x, \varphi) \Leftrightarrow$  there is an assignment  $h$  correct for  $i$  and extending  $g$  such that for all  $j \in bel_i(g(x)) \ M, j, h \models \varphi$

$M, i, g \models \neg belief(x, \varphi) \Leftrightarrow$  for no assignment  $h$  correct for  $i$  and extending  $g$  there is a  $j \in bel_i(g(x))$  such that  $M, j, h \models \varphi$