UNIVERSITY OF AMSTERDAM

Introduction Actuarial Science
Computer assignment 2: Life Insurance

Period life expectancy and actuarial valuation

In this exercise you will learn to work with interest rates and mortality rate forecasts. Using the mortality prognosis as published by the Dutch Actuarial Society[1], you will calculate life expectancies and perform some standard life insurance calculations.

Specifically, in this assignment you will learn about:

- Working in Excel spreadsheets;
- Working with interest rates and mortality rates;
- Calculating the period life expectancy;
- Determine the premium for a simple life insurance product.

First, you will have to construct discount factors $v(t)$ using a fixed interest rate r. Given the mortality prognosis from the Dutch Actuarial Society, you will calculate life expectancies at different points in time, and calculate an actuarially fair premium for a pensions product.

NB: all coloured words are clickable and redirect you to a webpage providing more information on a subject.

Practical information

- Download the Excel spreadsheet CP2 - Life Insurance.xlsx from Canvas;
- You do NOT have to hand in this assignment.

Practical information on the use of Excel

Excel may be new to some students. Some relevant information for life insurance calculations – and in general for using Excel – is listed below with references to where you may find more information.

The sum of a product: Often we need to calculate something of the form $\sum_i v_i \cdot p_i$. In Excel this can be achieved using the function SUMPRODUCT().

[1] See AG2024 in English.

Looking up values from a range of cells: When you need to look up some value from a range of cells you can either use the OFFSET()-function or the VLOOKUP()-function (or HLOOKUP(), depending on what direction your search is, horizontal or vertical). The OFFSET()-function can be used if you know where the range of cells starts, and you know how many rows and columns you have to shift from the starting point, i.e. if the data is ordered. The VLOOKUP()-function is preferred if you do not know where in the data range the value is that you look for, i.e. if the data is not ordered.

Building formulas with cell references: When we are building spreadsheets with calculations, we need to define formulas in many cells. Often the formulas in cells are very similar, and we do not want to type each formula by hand; it is easier to copy cells with formulas to other cells. When your formula refers to cells, you need to be aware of the different ways in which you can refer to a cell. You can completely fix a cell reference, you can either keep the column or the row fixed, or you can let the reference be completely free. This is achieved by including \$-signs in a reference to a cell. See the following link for more information and examples on cell referencing.

Be careful when you copy contents from one cell to other cells. Only copy the contents to those cells where it is needed. If you are not careful, the contents may be copied to an entire row or column. This may result in the file size of the spreadsheet increasing significantly, and may cause the spreadsheet to become unstable.

Further, in the green tabs in the spreadsheet some illustrations are included on how to use the functionalities that are discussed above.

1 Preparing the discount factors

In the lecture notes we introduced the notion of discount factors $v(t)$. In this assignment, we assume a fixed interest rate of 2%.

Q_1 The tab `1. Interest rates` is set up for the construction of discount factors. The fixed interest rate of 2% is defined in cell H7. Calculate the discount factors $v(t)$ for $t = 0, \dots, 100$ in the cells `C8:C108`, and ensure that the discount factors depend on cell H7. □

2 Predicting and analyzing life expectancies

In the tab `2. Life expectancy` we will calculate period life expectancies $\mathring{e}_x$ for the ages 0 and 65, and for the years 2025, 2045 and 2065.[2] Since we will calculate the life expectancies for a number of age-year combinations, we will try to set up the calculations such that age and starting year are input in the calculations. This way, we can easily compute life expectancies for different age-year combinations. We define the age in cell `D10` and the (starting) year in cell `D11`.

We are interested in the period life expectancy which is given by

$$\mathring{e}^{p}_{t,x} = \tfrac{1}{2} + \sum_{k=1}^{\infty} \prod_{j=0}^{k-1} p_{t,x+j}. \tag{1}$$

[2] In the reader we defined curtate and complete life expectancies. In this assignment we only consider the complete life expectancy, and refer to it as *the* life expectancy.

The relevant ages are already defined in the cells B15:B135, dependent on the age in cell D10. Note that the ages shown do not exceed 120, because no mortality rates are available for ages higher than 120. Can you verify how we made sure no ages higher than 120 are shown?

In the columns C and D we will pick up the one-year mortality rates that we need to calculate the period life expectancy. For that purpose, it is helpful to use the `OFFSET`-function, and the ages for which we need mortality rates are already defined in the cells `B15:D135`.

The tabs `AG2024_M` and `AG2024_F` contain the mortality forecasts as published by the Dutch Actuarial Society in 2024.

Q_2 For a specific year t, fill the columns C and D with the one-year mortality rates $q_{t,x}$ from the tabs `AG2024_M` and `AG2024_F`. The year t is an input parameter that is defined in cell `D11`, and the mortality rates should thus change if this cell is changed. Some hints / suggestions:

- Use the OFFSET(reference, a, b)-function to pick up the mortality rates from the tabs `AG2024_M` and `AG2024_F`;
- From those two tabs, use cell B2 as the reference cell, and keep this fixed (recall the cell referencing mentioned at the beginning of this assignment);
- The parameters a and b represent how may rows and columns respectively you move relative to the reference cell;
- From the tab `2. Life expectancy`, use the cells D7 and D8 in combination with the information from column B and cell D11 in defining the number of rows or columns that should be shifted relative to the reference cell;
- More information and some examples on the OFFSET-function can be found here.

Again, make sure that if no information is needed anymore that the cells are *shown* empty (i.e. when the age is higher than 120). □

Q_3 For the male period life expectancy, calculate the cumulative survival probability ${}_kp_{t,x}$ in the cells G15:G135 using the one-year mortality rates from column E, and calculate the complete remaining period life expectancy in cell G11.

Do the same for the female period life expectancy in column H. □

Q_4 Calculate the life expectancies for $x = 0$ and for $t = 2025, 2045, 2065$ and paste the results into the cells L17:N18. Do the same for $x = 65$ and for $t = 2025, 2045, 2065$ and paste the results into the cells L23:N24. □

Dutch retirement age

In the Netherlands, the official retirement age was 65 years for a very long time. Due to the increasing life expectancy, the Dutch government decided that the retirement age should be increased. The formula that is now used to determine whether the retirement age should be increased is given by[3]

$$V = \begin{cases} 0, & \text{if } (L - 18.26) - (P - 65) < 0.25 \\ 0.25, & \text{if } (L - 18.26) - (P - 65) \geq 0.25, \end{cases}$$

[3] See the relevant legislation in Dutch.

with L the remaining period life expectancy at age 65 in year t, P the retirement age in year t. The outcome of this formula, V, is the number of years by which the retirement age should be increased in year $t+1$, and the retirement age is thus either not increased or it is increased by 3 months.

You can interpret this formula as follows. The Dutch government aims to keep the expected remaining life expectancy at the retirement age equal, namely at 18.26. Thus, if the life expectancy increases, so should the retirement age. Because of this legislation, people aged around 20 in 2019 have an estimated retirement age of over 70 years.

Q_5 What is your opinion on the current legislation by the Dutch government on adjusting the official retirement age this way? Name one advantage and one disadvantage. □

3 Solving actuarial questions

We now turn to sheet `3. Annuities`. Susan is a 20-year old woman, and she is investigating her possibilities with respect to her pension. Her predicted retirement age is 67, and she wants a reasonable level of income once she retires, i.e. she wants to accrue pension benefits equal to €24,000 per annum.

In this section we calculate the premium that she needs to deposit annually in order to save for her pension, we analyze the possibilities of a high-low construction for her pension benefits, and we investigate what early retirement will cost her.

Q_6 In the cells D10:D14 we have predefined the input parameters, and in column J the relevant mortality rates are already shown. Your assignment is to calculate the annual premium that Susan will need to pay every year until her retirement at age 67 to ensure an annual pension benefit of €24,000.

Some assumptions on the premium payments:

- Premium payments are paid at the beginning of the year;
- Premium payments are only paid if Susan is alive;
- Premium payments have to be paid from the current age until she reaches the retirement age. Specifically, she does not have to pay the premium once she has reached the retirement age.

Some assumptions on the pension benefits:

- Pension benefits are receivable at the beginning of the year;
- Pension benefits are received only if Susan is alive;
- Pension benefits are receivable starting from the retirement age until death.

It is recommended to first calculate the present value of 1 unit premium payments (define this by a_p), and the present value of 1 unit pension benefits (define this by a_b). Define the desired pension benefit by b (which in this case equals €24,000). The actuarial premium p can then be determined by equating cash inflows and cash outflows:

$$p \cdot a_p = b \cdot a_b \Rightarrow p = b \cdot a_b / a_p.$$

Note that an actuarial present value a is calculated as something of the form $\sum_t v(t) \cdot c(t) \cdot Pr(t)$, where $v(t)$ is a discount factor, $c(t)$ is a cash flow vector with ones and zeroes depending on whether a cash flow is applicable in year t, and $Pr(t)$ is the probability that the cash flow $c(t)$ has to be made in year t.

- Derive the expression for a_b and a_p.

The cells where you should perform calculations are already formatted.

- Fill the cells in the columns M:P with the formulas needed to calculate the premium;
- The premium itself can be calculated using the cells AC15:AC20.

□

Q_7 Susan wishes to travel much during the first years of her retirement. Therefore, she wants to increase the pension benefit in the first ten years, and she accepts that she will therefore receive less in the years thereafter.

In the previous question you calculated the present value of a pension benefit of €24,000. Susan wants a benefit b^H in the first ten years of her retirement and a benefit $b^L = b^H - 5000$ in the years thereafter. Can you derive the benefit b^H?

For this, you may want to use the relationship $b \cdot a_b = b^H \cdot a_b^H + b^L \cdot a_b^L$, with a_b^H the present value of 1 unit that is paid in the first ten years of retirement, and a_b^L the present value of 1 unit paid in the years thereafter. Please use the cells in columns Q and R and the cells AC26:AC35 for your calculations. □

Q_8 After consideration, Susan does not want to wait until the official retirement age; she wants to retire two years early (i.e. at age 65). Assuming she has built up the right to retire at age 67 with an annual pension benefit of €24,000, what benefit can she receive if she retires two years early? Please use the cells in column S and the cells AC41:AC43 for your calculations. □